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CONSUMPTION TAXES VERSUS INVESTMENT INCOME TAXES: IMPLICATIONS FOR IMPATIENCE AND RISK-BEARING*

by

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I. INTRODUCTION

The question of the differential effects of a tax on consumption (expenditure) as opposed to a tax on income has been a major controversy in fiscal policy.[1] The existing analysis, by analytically separating consumption (saving) and investment decisions has failed to come to grips with the problem. This is true both of the applications of the Fisherian theory of saving and of the single-period theories of risk-taking behaviour for analyzing the problems of fiscal policy.[2] Both Hansen (1958) and Musgrave (1959), who develop intertemporal models of saving behaviour by households under certainty (with no asset choice), tend to agree that a proportional consumption tax does not encourage (nor discourage) a substitution of present over future consumption;[3] while an equal revenue proportional income tax encourages a substitution in favour of present consumption (i.e., against saving). This latter effect is also accompanied by an income effect thus rendering the total effect ambiguous. Nevertheless, this substitution effect (encouraging current consumption) induced by the income tax is usually taken to imply that an income tax discriminates against saving.[4] However, in an uncertain world where opportunities for investing in safe and risky assets are allowed, current saving no longer constitutes future consumption even when we require that the expected present value of life-time consumption equals the expected present value of life-time wealth: the introduction of capital risks makes future consumption a risky prospect. Thus with the introduction of uncertainty, the Hansen-Musgrave results become suspect.[5]

Kaldor, on the other hand, without formally specifying his model arrives at some interesting conclusions. He argues that a tax on consumption does not discriminate against risk-taking but in fact discriminates in favour of saving; while a tax on investment income (or, *a fortiori*, a tax on total income) discriminates against both saving and risk-taking.

In this paper, we plan to investigate the differential incidence of these taxes on household consumption and investment (risk-taking) decisions in a simple two-period temporal model. For simplicity, we compare a consumption tax with a tax on investment income rather than a tax on total income.[6]

The analysis is carried out for the balanced budget incidence of an increase in the consumption tax (or, alternatively, investment income tax) used to finance matching lump-sum transfers to the household. As Diamond (1970) has pointed out, this is equivalent to the differential incidence of a consumption tax (or, an investment income tax) increase and a lump-sum tax decrease.[7]

It is seen that for low rates of tax (less than 50%) and low expected return on the risky investment (less than 100%), the differential incidence of a consumption tax is to raise both current consumption and risk-taking. If these restrictions on tax rates and expected profit rates are satisfied, the differential incidence of a tax on investment income would also, under the additional restriction of zero rate of return on riskless investment, be to encourage current consumption and risk-taking. For the type of balanced budget operation discussed in this paper, it is also seen that under the set of conditions mentioned above, the consumption tax leaves the household with a higher level of utility than does the investment income tax. This result also requires that the propensities to consume (and to invest in the risky asset) are identical under the alternative budgetary policies. It is further seen that under these conditions and when both the consumption and the investment income tax rates are equally large, the differential incidence of a consumption tax encourages current consumption and risk-taking more effectively than a tax on investment income.

The rest of the paper proceeds as follows. Section II discusses the balanced budget incidence of a consumption tax, while an investment income tax is analyzed in section III. Section IV attempts to provide a further comparison of the two fiscal structures on the basis of equally large tax rates. The effect of these tax policies on household life-time utility is analyzed in section V. Some concluding remarks are put forward in Section VI.

II. A TAX ON CONSUMPTION

It is assumed that the individual makes the portfolio allocation decision in the first period to maximize expected lifetime utility. In the second period he dissaves, consuming entirely his wage income, the capital and the realized investment income. The intertemporal consumption allocation decision can be stated as[8]

$$\max \quad EF(C_1, C_2) = V(C_1) + EU(C_2),$$

$$\text{s.t.} \quad C_1 = (Y_1 - a - m)(1 - t) + K_1, \qquad \text{(II-1)}$$

$$C_2 = (1 - t)[Y_2 + a(1 + X) + m(1 + r)] + K_2,$$

where $C_i (i=1,2)$ is consumption in period i, Y_i is exogenous non-asset income (or, its present value, where appropriate) received in period i, while a and m denote the amounts allocated to the risky and the safe assets, respectively. The rate of return on the secure asset is r and X is the random rate of return on the risky investment such that $X \in [-1, \infty)$. The consumption tax rate is denoted by t while K_i is the matching lump-sum transfer received in period i. We further assume that both V and U are at least twice continuously differentiable with positive and diminishing marginal utilities, thus guaranteeing risk-aversion and the diminishing marginal rate of substitution between present and future consumption prospects.

Now substituting for C_2, we can eliminate the constraints and restate the maximization as

$$\max_{\{C_1, a\}} V(C_1) + E[U\{(1-t)[Y_1(1+r) + Y_2 + a(X-r)] + (K_1 - C_1)(1+r) + K_2\}] \quad \text{(II-2)}$$

The necessary conditions for the existence of an interior maximum are given by

$$V'(C_1) - (1+r)\, E\{U'(C_2)\} = 0; \quad \text{(II-3)}$$

$$(1-t)\, E\{U'(C_2)(X-r)\} = 0. \quad \text{(II-4)}$$

Since we have assumed strict concavity of both $V(C_1)$ and $U(C_2)$ these are also sufficient conditions for a utility maximum. These conditions state that at an optimum, the marginal rate of time-preference equals the rate of return on the safe asset (equation (II-3)) and that the expected marginal utility of a unit of investment in each asset is equalized (equation (II-4)).

Differentiating the first-order conditions (II-3) and (II-4), with respect to $\underline{t}$, K_1 and Y_1 we obtain the following results:

$$\frac{\partial c_1}{\partial t} = (-) \frac{[Y_1 + Y_2(1+r)^{-1}]}{(1-t)} \frac{\partial c_1}{\partial Y_1}, \tag{II-5}$$

$$\frac{\partial a}{\partial t} = \left(\frac{a}{1-t}\right)\left\{1 - \frac{1}{a}\frac{\partial a}{\partial Y_1}[Y_1 + Y_2(1+r)^{-1}]\right\}. \tag{II-6}$$

$$\frac{\partial c_1}{\partial K_1} = \left(\frac{1}{1-t}\right)\frac{\partial c_1}{\partial Y_1}, \tag{II-7}$$

$$\frac{\partial a}{\partial K_1} = \left(\frac{1}{1-t}\right)\frac{\partial a}{\partial Y_1} \tag{II-8}$$

These comparative static results can be easily interpreted. Equations (II-7) and (II-8) indicate that changes in the lump-sum transfer payment increase[9] consumption and risk-taking by generating simple income effects without influencing the intertemporal consumption substitution possibilities. Likewise, the proportional consumption tax reduces present consumption due to the income effect. However, noticing that the expression $(a/[1-t])$ in (II-6) has the interpretation of the substitution effect,[10] we observe that while present consumption is reduced by the tax (equation (II-5)), the individual is able to maintain his former level of expected lifetime utility by investing more in the risky asset. However, the income effect works in the opposite direction. The total effect on risk-taking, therefore, depends on the magnitude of the income elasticities.[11]

If the tax variables are such that the individual, in each period, receives as a lump sum exactly equal to what is expected to be taken from him by the consumption tax, then the tax variables must satisfy[12]

$$\left.\begin{aligned} K_1 &= t(Y_1 - a - m), \\ K_2 &= t[Y_2 + a(1+\bar{X}) + m(1+r)], \end{aligned}\right\} \qquad \text{(II-9)}$$

where $\bar{X}$ denotes the expected value of X. Equations (II-9) can also be rewritten as

$$K_2 + K_1(1+r) = t[Y_1(1+r) + Y_2 + a(\bar{X} - r)]. \qquad \text{(II-10)}$$

To maintain this relationship changes in the tax variables must satisfy

$$\frac{\partial K_2}{\partial t} + (1+r)\frac{\partial K_1}{\partial t} = \bar{Y} + t(\bar{X}-r)[\frac{\partial a}{\partial t} + \frac{\partial a}{\partial K_1}\frac{\partial K_1}{\partial t} + \frac{\partial a}{\partial K_2}\frac{\partial K_2}{\partial t}], \qquad \text{(II-11)}$$

where $\bar{Y} = [Y_1(1+r) + Y_2 + a(\bar{X}-r)]$. But from the first-order conditions we see that

$$\frac{\partial a}{\partial K_1} = (1+r)\frac{\partial a}{\partial K_2} \qquad \text{(II-12)}$$

and consequently (II-11) can be rewritten as

$$[\frac{\partial K_1}{\partial t} + (1+r)^{-1}\frac{\partial K_2}{\partial t}] = \frac{[\bar{Y} + t(X-r)\frac{\partial a}{\partial t}]}{[(1+r) - t(X-r)\frac{\partial a}{\partial K_1}]}. \qquad \text{(II-13)}$$

We can now evaluate the change in risk-taking and in consumption from a consumption tax rate and a lump-sum transfer change which would leave zero expected net revenue in each period. We can call these the "expected revenue compensated changes". Note that the first-order conditions, (II-3) and (II-4), implicitly define optimal first period consumption and optimal risk-taking as functions of the tax parameters:

$$a^* = a(t, K_1, K_2), \qquad \text{(II-14)}$$

$$C_1^* = C(t, K_1, K_2). \qquad \text{(II-15)}$$

From (II-14) we obtain, using (II-12)

$$\frac{\partial a^*}{\partial t} = \frac{\partial a}{\partial t} + \frac{\partial a}{\partial K_1}\left[\frac{\partial K_1}{\partial t} + (1+r)^{-1}\frac{\partial K_2}{\partial t}\right]. \qquad \text{(II-16)}$$

This together with (II-13) yields the subsidy compensated tax derivative

$$\left.\frac{\partial a^*}{\partial t}\right|_{comp} = \frac{\left(\frac{a}{1-t}\right)\left[1 + \left(\frac{\bar{X}-r}{1+r}\right)\frac{\partial a}{\partial Y_1}\right]}{\left[1-\left(\frac{t}{1-t}\right)\left(\frac{\bar{X}-r}{1+r}\right)\frac{\partial a}{\partial Y_1}\right]} . \qquad \text{(II-17)}$$

Since $(\bar{X}-r) > 0$, the numerator is always positive. The sign, therefore, depends on the denominator, i.e.,

$$\left.\frac{\partial a^*}{\partial t}\right|_{comp} \gtrless 0 \text{ as } 1 \gtrless \left(\frac{t}{1-t}\right)\left(\frac{\bar{X}-r}{1+r}\right)\frac{\partial a}{\partial Y_1} . \qquad \text{(II-18)}$$

Note that for t = 0.50 the requirement is

$$1 \gtrless \left(\frac{\bar{X}-r}{1+r}\right)\frac{\partial a}{\partial Y_1} .$$

Since $0 < \partial a/\partial Y_1 < 1$ and r is usually small (a reasonable upper bound for the real rate of return on a safe asset might be .10), a sufficient condition for $\partial a^*/\partial t\big|_{comp}$ to be positive is that $r < \bar{X} \leq 1$ (for $t \leq .50$).

We can therefore conclude that <u>so long as the expected return on the risky asset does not exceed 100% and the consumption tax rate is below 50%, the balanced budget incidence of a consumption tax is to increase risk-taking</u>. If high tax rates (i.e., $t > 0.5$) happen to be combined with high expected profit rates (i.e., $\bar{X} > 1$), risk-taking may actually decrease. The general point, however, is that the result depends on t, $\bar{X}$, r and $\partial a/\partial Y_1$, the first being a government parameter and the rest being determined in the market.

Likewise, we can evaluate the expected revenue compensated change in optimal current consumption. From (II-15) we have

$$\frac{\partial C_1^*}{\partial t} = \frac{\partial C_1}{\partial t} + \frac{\partial C_1}{\partial K_1}\frac{\partial K_1}{\partial t} + \frac{\partial C_2}{\partial K_2}\frac{\partial K_2}{\partial t} .$$

As before, together with equations (II-12), (II-13) and equations (II-5) through (II-8), the above reduces to

$$\left.\frac{\partial C_1^*}{\partial t}\right|_{comp} = \frac{\left(\frac{a(\bar{X}-r)}{(1+r)(1-t)^2}\right)\frac{\partial C_1}{\partial Y_1}}{\left[1 - \left(\frac{t}{1-t}\right)\left(\frac{\bar{X}-r}{1+r}\right)\frac{\partial a}{\partial Y_1}\right]} \qquad \text{(II-19)}$$

Notice that (II-19) admits of an interpretation similar to that of (II-17). The sign of $(\partial C_1^*/\partial t)\big|_{comp}$ is the same as that of the denominator, and thus the preceding comments apply. That is, so long as the expected return on the risky asset is small (less than 100%) and the consumption tax rate is low (less than 50%), the balanced budget incidence of a consumption tax is to encourage current consumption. It must be pointed out that this finding is not inconsistent with our earlier result, $\partial C_1/\partial t < 0$ (given by equation (II-5)): The effect of a simple change in the consumption tax rate is to discourage current consumption by generating an income effect in a straightforward manner (equation (II-5)). However, the expected revenue compensated change in the tax-subsidy rates also generate an income effect, albeit, in the opposite direction, i.e., tending to stimulate current consumption. The total effect (equation (II-19)), therefore, obviously depends on the relative magnitude of the two sets of income effects.

A special case of the above results, obtain if we happen to start from a situation of no fiscal policy ($t=0$, $K_i = 0$). Equations (II-17) and (II-19) now reduce to

$$\left.\frac{\partial a}{\partial t}\right|_{comp}^{t=0} = a\{1 + \left(\frac{X-r}{1+r}\right)\frac{\partial a}{\partial Y_1}\}, \tag{II-17a}$$

and

$$\left.\frac{\partial C_1^*}{\partial t}\right|_{comp}^{t=0} = \frac{a(\bar{X}-r)}{(1+r)}\frac{\partial C_1}{\partial Y_1}. \tag{II-19a}$$

Clearly both of these derivatives are positive (since we have assumed that both current consumption and the risky investment are normal goods).

III. A TAX ON INVESTMENT INCOME

While the model remains the same as that of the preceding section, we now introduce a tax on investment income (rather than a tax on consumption) accompanied by a matching lump-sum transfer. The individual budget constraints can now be written as

$$\left.\begin{aligned} C_1 &= Y_1 - a - m, \\ C_2 &= Y_2 + (a+m) + (1-t)[aX+mr] + K, \end{aligned}\right\} \tag{III-1}$$

where the notation is the same except that t is now the investment income tax rate. On substitution, (III-1) reduces to

$$C_2 = Y_2 + (Y_1-C_1)\{1+(1-t)r\} + (1-t)a(X-r) + K, \tag{III-2}$$

and the maximization problem reads as

$$\max_{\{C_1, a\}} \quad V(C_1) + E(U\{Y_2 + (Y_1 - C_1)[1+(1-t)r] + (1-t)a(X-r)+K\}). \quad \text{(III-3)}$$

We proceed as before and state the necessary and sufficient conditions for the existence of a maximum:

$$V'(C_1) - (r^*)\, E[U'(C_2)] = 0, \quad \text{(III-4)}$$

$$(1-t)\, E[U'(C_2)(X-r)] = 0, \quad \text{(III-5)}$$

where we have defined

$$(r^*) = [1 + r(1-t)].$$

As before, these first-order conditions upon implicit differentiation with respect to t, K and Y_1 yield the following comparative static results:

$$\frac{\partial C_1}{\partial t} = \left(\frac{C_1 r}{r^*}\right) \left\{\frac{V'(C_1)}{V''(C_1)C_1} + 1\right\} \frac{\partial C_1}{\partial Y_1} - \left(\frac{C_1 r}{r^*}\right) \left\{\frac{Y_1}{C_1} \frac{\partial C_1}{\partial Y_1} + \frac{V'(C_1)}{V''(C_1)C_1}\right\}, \quad \text{(III-6)}$$

$$\frac{\partial a}{\partial t} = \frac{a}{1-t} \left\{1 - \frac{r(1-t)}{r^*} \left[\frac{Y_1}{a} \frac{\partial a}{\partial Y_1}\right]\right\} + \left(\frac{C_1 r}{r^*}\right)\left(1 + \frac{V'(C_1)}{V''(C_1)C_1}\right)\frac{\partial a}{\partial Y_1}; \quad \text{(III-7)}$$

and

$$\frac{\partial C_1}{\partial K} = \left(\frac{1}{r^*}\right) \frac{\partial C_1}{\partial Y_1}, \quad \text{(III-8)}$$

$$\frac{\partial a}{\partial K} = \left(\frac{1}{r^*}\right) \frac{\partial a}{\partial Y_1} \;. \quad \text{(III-9)}$$

Since this is effectively a tax on future consumption, the substitution effect,[13] which equals $\{(r/r^*)(V'(C_1)/V''(C_1))[(\partial C_1/\partial Y_1)-1]\}$ in equation (III-6), tends to work in favour of current consumption, which may or may not be outweighed by the income effects. However, the substitution effect in the risk-taking function (equation (III-7)) has an ambiguous sign ((III-12) in footnote 13). Although an investment income tax with full loss offset would tend to encourage a substitution away from the riskless asset (since, effectively the investment income tax reduces the size of the bet), the three-way substitution (between current consumption, risk-taking and investing in the secure asset) may actually lead the individual to invest less in both the assets.[14]

For the special case of zero rate of return on the riskless asset (r=0), the above results are simplified into the following:

$$\left.\frac{\partial C_1}{\partial t}\right|^{r=0} = 0, \qquad \text{(III-6a)}$$

$$\left.\frac{\partial a}{\partial t}\right|^{r=0} = \left(\frac{a}{1-t}\right), \qquad \text{(III-7a)}$$

which are simply the substitution effects relevant for r=0. In other words, at r=0, the income effects of an investment income tax disappear. This is explained as follows. Postponement of a unit of current consumption, ceteris paribus, generates (1+r) units available for future consumption. However, if r=0, the physical amounts are unchanged over time (i.e., one orange given up today adds to exactly one more orange for tomorrow). Thus the loss of real income, due to the tax on investment income (or interest income), insofar as current consumption is concerned is nil, and hence no income effect.[15] The lack of a substitution effect in (III-6a) is explained by the fact that r=0 also implies that the marginal utility of a unit of current consumption equals the expected marginal utility of a unit of future consumption at the optimum (i.e., $V'(C_1) = E\{U'(C_2)\}$, see equation (III-4)). This together with the argument of no real yield on postponed consumption (at $r=0$), suggests that the investment income tax loses its distortionary property insofar as consumption is concerned (i.e., distorting the relative prices of current and future consumption). Equation (III-7a) can also be explained in a similar manner. Risk-taking is important inasmuch as it contributes to future consumption. Further, the expected marginal utility of a unit of future consumption weighted by the rate of return on the risky asset is zero at the optimum (i.e., $E\{U'(C_2)X\} = 0$, see equation (III-5)). Thus, the investment income tax, essentially, <u>a tax on X</u>, just allows the household to retain its former level of expected lifetime utility.[16]

We now analyze the differential incidence of an investment income tax increase and a lump-sum tax decrease on impatience and risk-taking. Since investment income taxes are paid only in the second period, for distributional neutrality, we require that the lump-sum is also paid out in that period such that expected net revenue is nil, i.e.,

$$K = t[a\bar{X} + mr],$$

or

$$K = t[(Y_1 - C_1)r + a(\bar{X} - r)]. \tag{III-13}$$

Again, to maintain (III-13) changes in the tax variables must satisfy

$$\frac{\partial K}{\partial t} = \frac{[(Y_1 - C_1)r + a(\bar{X} - r)] + t(\bar{X} - r)\frac{\partial a}{\partial t} - tr\frac{\partial C_1}{\partial t}}{[1 - t(\bar{X} - r)\frac{\partial a}{\partial K} + tr\frac{\partial C_1}{\partial K}]}. \tag{III-14}$$

From the solution to the first-order conditions, (III-4) and (III-5), upon implicit differentiation, we obtain

$$\frac{\partial a^*}{\partial t} = \frac{\partial a}{\partial t} + \frac{\partial a}{\partial K}\ \frac{\partial K}{\partial t}, \tag{III-15}$$

$$\frac{\partial C_1^*}{\partial t} = \frac{\partial C_1}{\partial t} + \frac{\partial C_1}{\partial K}\ \frac{\partial K}{\partial t}, \tag{III-16}$$

which together with (III-14) allows us to evaluate the expected revenue compensated changes in consumption and risk-taking. Thus, from (III-15), (III-14) and using equations (III-6) through (III-9), we obtain

$$\left.\frac{\partial a^*}{\partial t}\right|_{comp} = \frac{(\frac{a}{1-t})[1 + (\frac{tr}{r^*})\frac{\partial C_1}{\partial Y_1}] + [\frac{a(\bar{X}-r)}{r^*} + (\frac{r}{r^*})(1+\frac{tr}{r^*})\frac{V'(C_1)}{V''(C_1)}]\frac{\partial a}{\partial Y_1}}{[1 - \frac{t(\bar{X}-r)}{r^*}\frac{\partial a}{\partial Y_1} + (\frac{tr}{r^*})(\frac{\partial C_1}{\partial Y_1})]} \tag{III-17}$$

which is, in general, indeterminate. However, for r=0, this simplifies into

$$\left.\frac{\partial a}{\partial t}\right|_{comp}^{r=0} = \left(\frac{a}{1-t}\right)\left\{\frac{1 + \bar{X}(1-t)\frac{\partial a}{\partial Y_1}}{1 - t\bar{X}\frac{\partial a}{\partial Y_1}}\right\}. \tag{III-17a}$$

Clearly, the sign of (III-17a) is the same as

$$[1 - t\bar{X}\frac{\partial a}{\partial Y_1}].$$

Thus, if $t \le .5$ is combined with $\bar{X} \le 2$ risk-taking with increase.

In the preceding manner, from (III-16), (III-14) and using equations (III-6) through (III-9), we obtain

$$\left.\frac{\partial C_1^*}{\partial t}\right|_{comp} = \frac{(\frac{r}{r^*})\frac{V'(C_1)}{V''(C_1)}[\frac{\partial C_1}{\partial Y_1}-1] + \frac{a(\bar{X}-r)}{(1-t)r^*}(\frac{C_1}{Y_1})[\frac{Y_1}{C_1}\frac{\partial C_1}{\partial Y_1} + (\frac{r(1-t)}{r^*})\frac{V'(C_1)}{V''(C_1)C_1}(\frac{Y_1}{a}\frac{\partial a}{\partial Y_1})]}{[1 - \frac{t(\bar{X}-r)}{r^*}\frac{\partial a}{\partial Y_1} + (\frac{tr}{r^*})\frac{\partial C_1}{\partial Y_1}]} \qquad \text{(III-18)}$$

A set of sufficient conditions for this effect to be positive is

(i) utility is logarithmic,[17]

(ii) $\frac{Y_1}{C_1}\frac{\partial C_1}{\partial Y_1} \geq \frac{Y_1}{a}\frac{\partial a}{\partial Y_1}$, and

(iii) $\bar{X} \leq 1$,

none of which are necessary. For r=0, this again simplifies into

$$\left.\frac{\partial C_1^*}{\partial t}\right|_{comp}^{r=0} = \frac{(\frac{a\bar{X}}{1-t})\frac{\partial C_1}{\partial Y_1}}{[1-t\bar{X}\frac{\partial a}{\partial Y_1}]}, \qquad \text{(III-18a)}$$

which would be positive if $t \leq 0.5$ and $\bar{X} \leq 2$, neither being necessary. Thus we can conclude that for the case of zero rate of return on riskless investment the balanced-budget incidence of an investment income tax increase and a lump-sum tax decrease is to raise both current consumption and risk-taking if the expected rate of return on the risky asset times the tax rate is less than unity.

IV. THE TWO TAXES COMPARED

As we have indicated in the introductory remarks, it is difficult to compare two taxes directly in terms of an analytical model. Such considerations led us to attempt an indirect evaluation of these taxes by comparing each with a lump-sum tax levied on the individual (or equivalently with tax proceeds going back to the same individual paying the tax). This method allowed us to arrive at qualitative results that required information regarding certain government and market determined parameters (i.e., all being observable) and could be reasonably interpreted. One such class of results was obtained for the special case when the return on riskless investment was zero. Let us rewrite these results as follows:

$$\left.\frac{\partial a^*}{\partial t_c}\right|_{comp}^{r=0} = \frac{\frac{a}{(1-t_c)}\left[1 + \bar{X}\frac{\partial a}{\partial Y_1}\right]}{\left[1 - \left(\frac{t_c}{1-t_c}\right)\bar{X}\frac{\partial a}{\partial Y_1}\right]}, \qquad \text{(IV-1)}$$

$$\left.\frac{\partial C_1^*}{\partial t_c}\right|_{comp}^{r=0} = \frac{\left[\frac{a\bar{X}}{(1-t_c)^2}\frac{\partial C_1}{\partial Y_1}\right]}{\left[1 - \left(\frac{t_c}{1-t_c}\right)\bar{X}\frac{\partial a}{\partial Y_1}\right]}, \qquad \text{(IV-2)}$$

and,

$$\left.\frac{\partial a^*}{\partial t_i}\right|_{comp}^{r=0} = \left(\frac{a}{1-t_i}\right)\left[\frac{1 + \bar{X}(1-t_i)\frac{\partial a}{\partial Y_1}}{1 - t_i\bar{X}\frac{\partial a}{\partial Y_1}}\right], \qquad \text{(IV-3)}$$

$$\left.\frac{\partial C_1^*}{\partial t_i}\right|_{comp}^{r=0} = \frac{\left(\frac{a\bar{X}}{1-t_i}\right)\frac{\partial C_1}{\partial Y_1}}{\left[1 - t_i\bar{X}\frac{\partial a}{\partial Y_1}\right]}, \qquad \text{(IV-4)}$$

where t_c and t_i denote consumption and investment income tax rates, respectively.

Choosing equal tax rates as a basis of comparison, from (IV-1) and (IV-3), we obtain

$$\left.\frac{\partial a^*}{\partial t_c}\right|_{comp}^{r=0} > \left.\frac{\partial a^*}{\partial t_i}\right|_{comp}^{r=0} . \tag{IV-5}$$

This can be seen as follows. Assuming that both (IV-1) and (IV-3) are positive, (i.e., their denominators are positive), the denominator of (IV-3) is greater than that of (IV-1). Furthermore, the numerator of (IV-3) is smaller than that of (IV-1). This result requires that the propensities to invest in the risky asset is the same whether the individual is paying a consumption tax or an investment income tax. It is interesting to observe that while, for r=0, the simple incidence of a consumption tax (with no restriction on government's budget equation) is not as effective as the investment income tax in encouraging risk-taking, the balanced budget incidence reverses this conclusion. The former result can be seen by comparing equations (II-6) and (III-7a). In other words, the expected revenue compensation associated with the consumption tax exerts stronger income effects stimulating risk-taking than is the case with the investment income tax.

Therefore, we arrive at the interesting conclusion that <u>if the tax rate and the expected profit rates are such that the balanced budget incidence of a consumption tax and that of a tax on investment income is to encourage risk-taking (i.e., $t_i, t_c \leq 5$ and $\overline{X} \leq 1$), then for equally large tax rates, a tax on consumption encourages risk-taking more effectively than a tax on investment income</u>. On balance, this result may seem to support Kaldor's intuition that a tax on consumption does not discriminate against risk-taking, but not quite. His main contention was that a tax on consumption would be neutral so far as risk-taking was concerned, while a tax on investment income (or a fortiori, a tax on income) would discourage risk-taking. On the other hand, basic to our result, indicating the possibility of greater risk-taking under the consumption tax, is a situation where the balanced budget incidence of both these taxes were to encourage risk-taking.

To see the effects of equal tax rates on current consumption, we compare (IV-2) and (IV-4) and let $t_c = t_i$. Once again, we start from a situation where both these effects are positive. As before, the denominator of (IV-4) is greater than that of (IV-2), while the numerator of (IV-4) is smaller than that of (IV-2). Thus, under identical propensities to consume (and to invest in the risky asset)

$$\left.\frac{\partial c_1^*}{\partial t_c}\right|_{comp}^{r=0} > \left.\frac{\partial c_1^*}{\partial t_i}\right|_{comp}^{r=0} . \qquad \text{(IV-6)}$$

Once again, we recall that, for r=0, the simple incidence of a consumption tax is to discourage current consumption due to the income effect (equation (II-5)), while an investment income tax leaves it unchanged (equation (III-6a)). Therefore, the more effective encouragement of current consumption under a consumption tax than under an investment income tax (equation (IV-6)) is due to stronger income effects generated by the expected revenue compensation associated with the consumption tax than that of the investment income tax. Thus we conclude that if the initial rates of tax (equal) and the expected return on the risky asset are such that the balanced budget incidence of a consumption tax (with matching lump-sum transfers) and a tax on investment income (with matching lump-sum transfers) were to stimulate current consumption, a tax on consumption did this more effectively than a tax on investment income. This is contrary to the result under certainty. The general consensus of the literature that a tax on income (or investment income) discourages saving as compared to a tax on expenditure (consumption) is not borne out by this model which incorporates uncertainty via capital risks.[18]

V. BALANCED BUDGETS AND CHANGES IN EXPECTED UTILITY

Earlier we have indicated the conceptual difficulties involved in comparing alternative fiscal policies. The main difficulty involved is choosing a satisfactory criterion as the basis of comparison. The most common criterion is that both the fiscal structures yield equal revenue to the government. In a world of uncertainty, however, there arises the additional difficulty that the revenue raised by the government is a random variable. One of its implications is that it may not always be feasible to choose the tax parameters such that both the fiscal structures raise the same revenue in each state of nature.[19] As a partial solution to this problem, we employed, in the preceding sections, <u>expected</u> revenue compensation where changes in the actual lump-sum parameters (transfers) were equated to changes in <u>expected</u> revenue. Since the tax base differs with the type of the tax, so does expected revenue. This, in turn, implies that the compensatory lump-sum transfers are of different magnitudes under different tax structures. Evidently, then, changes in expected lifetime utility under each of the alternative fiscal policies (with expected revenue compensation) would, in general, be different. In this section, we attempt to further discover the implications of the balanced budget analysis of the preceding sections in terms of changes in expected lifetime utility (welfare).

Differentiating (II-2) totally, we obtain

$$dEF = V'(C_1)\ dC_1 - E\{U'(C_2)[Y_1(1+r) + Y_2 + a(X-r)]\}dt_c$$
$$+ (1-t_c)\ E\{U'(C_2)(X-r)\}da$$
$$+ (1+r)\left[E\{U'(C_2)\}\ dK_1 - E\{U'(C_2)\}\ dC_1\right]$$
$$+ E\{U'(C_2)\}\ dK_2.$$

In view of (II-3) and (II-4), this reduces to

$$\frac{\partial EF}{\partial t_c} = E\{U'(C_2)\}\{(1+r)\frac{\partial K_1}{\partial t_c} + \frac{\partial K_2}{\partial t_c}\}$$
$$- E\{U'(C_2)[Y_1(1+r) + Y_2 + a(X-r)]\}. \qquad \text{(V-1)}$$

However, for balanced budget we require (using (II-13)),

$$\left.\frac{\partial EF}{\partial t_c}\right|_{comp} = \left[\frac{E\{U'(C_2)\}\ a(\frac{\bar{X}-r}{1-t_c})}{1 - (\frac{t_c}{1-t_c})\ (\frac{\bar{X}-r}{1+r})\frac{\partial a}{\partial Y_1}}\right]. \qquad \text{(V-2)}$$

Similarly, for the investment income tax, we have (from (III-3))

$$dEF = V'(C_1)\ dC_1 + E\{U'(C_2)[-r^*\ dC_1 - r(Y_1-C_1)dt_i$$
$$+ (1-t_i)(X-r)da - a(X-r)dt_i + dK]\}.$$

Again, from (III-4), (III-5) and (III-14),

$$\frac{\partial EF}{\partial t_i} = E\{U'(C_2)\} \left[\frac{\frac{a(\bar{X}-r)}{1-t_i} + (\frac{rt_i}{r^*}) \frac{V'(C_1)}{V''(C_1)} [(\bar{X}-r)\frac{\partial a}{\partial Y_1} + r(1-\frac{\partial C_1}{\partial Y_1})]}{[1 - (\frac{t_i}{r^*})(\bar{X}-r) \frac{\partial a}{\partial Y_1} + (\frac{rt_i}{r^*}) \frac{\partial C_1}{\partial Y_1}]} \right] \tag{V-3}$$

Clearly, (V-3) is indeterminate. For r=0, however, we obtain

$$\left.\frac{\partial EF}{\partial t_i}\right|_{comp}^{r=0} = E\{U'(C_2)\} \left[\frac{\left(\frac{a\bar{X}}{1-t_i}\right)}{[1 - t_i \bar{X} \frac{\partial a}{\partial Y_1}]} \right], \tag{V-3a}$$

Interestingly enough, the signs of these derivatives depend on the magnitudes of t_i, t_c, and $\bar{X}$ as in the preceding sections.[20] Comparing (V-2) and (V-3a), we find that for $t_i = t_c$, <u>expected lifetime utility is higher under the consumption tax than under the investment income tax</u>, i.e.,

$$\left.\frac{\partial EF}{\partial t_c}\right|_{comp}^{r=0} > \left.\frac{\partial EF}{\partial t_i}\right|_{comp}^{r=0}. \tag{V-4}$$

VI. CONCLUSION

We have investigated the balanced budget incidence of a tax on consumption (with matching lump-sum transfers) and of a tax on investment income (with a matching lump-sum transfer) in the context of a simple two-period model of consumption and portfolio allocation decisions under uncertainty. The analysis suggests that when the rate of return on the safe investment is zero, when the tax rates (on consumption and on investment income) are low (less than 0.5) and when the expected return on the risky asset is less than 100%, both current consumption and risk-taking are encouraged under both the systems of budget policy. It is also shown that under such circumstances, if the propensities to consume (and to invest in the risky asset) are identical under the two budget policies and that the two tax rates are equally large, the differential incidence of a consumption tax is to encourage risk-taking and to discourage saving (encourage current consumption) more effectively than that of a tax on investment income such that expected lifetime utility is greater under the consumption tax than under the investment income tax. While these results are fairly restrictive, they at least point out that the introduction of uncertainty alters the "standard" results of fiscal policy in an important way.

FOOTNOTES

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[1]See, for instance, Goode (1964) and Kaldor (1955).

[2]For instance, Hansen (1958) and Musgrave (1959) analyze fiscal policy in a Fisherian model of saving behaviour; while Ahsan (1974b), Ahsan (1975), Mossin (1968) and Stiglitz (1969) discuss the effects of taxation in models of optimal composition of a portfolio of given size.

[3]Consumption in both periods is, however, reduced by the tax if they are normal goods.

[4]This view is shared, among others, by Goode and Kaldor.

[5]Indeed, Ahsan (1976) has observed that in the presence of capital risks, a proportional consumption tax is no longer neutral with respect to consumption and portfolio allocation decisions; the substitution effect encourages risk-taking which may either stimulate or reduce future consumption depending on whether the individual makes a loss or a gain in the capital market.

[6]This need not worry us unnecessarily. In terms of our model, wage income plus investment income is total income; and, in such contexts, Musgrave has pointed out that, "what matters in the case of the income tax is whether or not future interest income will be taxed." (1959, p. 260)

[7]Since a direct comparison of taxes (based, for instance, on equal expected revenue) does not appear to be analytically tractable, it seems worthwhile that an indirect comparison suggested below would convey some notion of the comparison of two taxes: we combine the analyses for each of the consumption and investment income taxes with lump-sum taxes under matching tax distributions (or, equivalently, with tax proceeds going back to the same taxpayer). In other words, we have not compared the investment income tax with a lump-sum tax in both periods; rather we compare it with a lump-sum tax falling only in the second period, because investment incomes are also realized in this period. The analogous point is made by Diamond in the context of incidence of an interest income tax in a neoclassical growth model. Incidentally, this procedure also corresponds to Kaldor's suggestion for comparing different taxes.

[8]This version of an intertemporal consumption-portfolio allocation model has been analyzed in some detail in Ahsan (1976). Notice that the additive separability of the utility function is a rather strong assumption. However, some possible justifications for making such an assumption are offered in the above mentioned paper.

[9]It can be seen that in this additively separable case, the assumptions of decreasing absolute and non-decreasing relative risk-aversions guarantee that $0 < \partial C_1/\partial Y_1,\ \partial a/\partial Y_1 < 1$. See Ahsan (1976).

[10]It can be seen that for the consumption tax

$$\frac{\partial a^*}{\partial t}\Big|_{dEF = o} = \left(\frac{a}{1-t}\right),$$ where a* denotes optimal risk-taking: dEF = o requires (from(II-2)),

$$[V'(C_1) - (1+r)E\{U'(C_2)\}]dC_1 + (1-t)E\{U'(C_2)(X-r)\}da$$

$$= [Y_1(1+r) + Y_2]E\{U'(C_2)\}dt + aE\{U'(C_2)(X-r)\}dt - [(1+r)dK_1 + dK_2]E\{U'(C_2)\}.$$

But in view of (II-3) and (II-4), this reduces to

$$\frac{\partial K_1}{\partial t} + (1+r)^{-1}\frac{\partial K_2}{\partial t} = [Y_1 + Y_2(1+r)^{-1}]. \qquad \text{...(F-1)}$$

Further, from (II-17), (II-6) and (II-8) we obtain

$$\frac{\partial a^*}{\partial t} = \left(\frac{a}{1-t}\right) - \frac{\partial a}{\partial K_1}\{Y_1 + Y_2(1+r)^{-1}\} + \frac{\partial a}{\partial K_1}\left[\frac{\partial K_1}{\partial t} + (1+r)^{-1}\frac{\partial K_2}{\partial t}\right],$$

which together with (F-1) yields

$$\frac{\partial a^*}{\partial t}\Big|_{dEF = o} = \left(\frac{a}{1-t}\right).$$

Similarly, from (II-16), (II-13), (II-7), (II-5) and (F-1) , we obtain

$$\frac{\partial C_1^*}{\partial t}\Big|_{dEF = o} = 0.$$

[11]For a more detailed discussion, see Ahsan (1976).

[12]An alternative would be to base the lump-sum transfer on the realized return on the risky asset (X) rather than on the expected return ($\bar{X}$). This would imply a different lump-sum transfer for each individual even if we assumed all individuals to be identical with respect to non-asset income and preferences. Basing the lump-sum transfer on the expected value of X (as here) results in a single lump-sum transfer which is the same for all identical individuals. Also notice that, although the expected tax payments equal the lump-sum transfer received, the individual cannot ignore the taxation policy as he does not realize that this indeed is the case. This is implicit in all balanced-budget studies.

[13]As indicated in footnote 10, this can be ascertained as follows: $dEF = o$ requires that

$$\frac{\partial K}{\partial t}\Big|_{dEF = o} = (Y_1 - C_1)r. \qquad \text{...(III-10)}$$

Substituting (III-6), (III-8) and (III-10) in the total derivative of optimal current consumption (C_1^*), i.e.,

$$dC_1^* = \frac{\partial C_1}{\partial t} dt + \frac{\partial C_1}{\partial K} dK,$$

we obtain

$$\frac{\partial C_1^*}{\partial t}\Big|_{dEF = o} = \left(\frac{r}{r^*}\right) \frac{V'(C_1)}{V''(C_1)} \left[\frac{\partial C_1}{\partial Y_1} - 1 \right]. \qquad \text{...(III-11)}$$

Likewise, from (III-7), (III-9) and (III-10), we also have

$$\frac{\partial a^*}{\partial t}\Big|_{dEF = o} = \left(\frac{a}{1-t}\right) + \left(\frac{r}{r^*}\right) \frac{V'(C_1)}{V''(C_1)} \frac{\partial a}{\partial Y_1}. \qquad \text{...(III-12)}$$

[14]The total effect on risk-taking, however, may turn out to be stimulating due to the income effects. For a more complete discussion, see Ahsan (1976).

[15]This would not be the case if the tax-base included non-asset incomes as in the case of the consumption tax (see equation (II-5)).

[16]Again, as already argued in footnote 15, this may not be the case with another tax, e.g., the consumption tax (see equation (II-6)).

[17]Notice that for a logarithmic utility function the factor

$$\left(\frac{V'(C_1)}{V''(C_1)C_1} + 1 \right) \text{ drops out.}$$

[18]At the level of the substitution effects alone (i.e., expected utility compensated changes rather than expected revenue compensated changes), from footnotes 10 and 13 it is evident that, for $r = 0$, the effects of a consumption tax and an investment income tax are identical. Under both these cases, current consumption is left unchanged while risk-taking is encouraged.

[19]Stiglitz (1969) discusses some of these difficulties.

[20]The possibility that (i.e., under $t_i, t_c \leq .5$ and $\bar{X} \leq 1$) expected lifetime utility may actually increase due to a revenue compensated increase in the tax rate is not surprising. For instance, the investment income tax at $r = 0$ does not, as indicated by equations (III-6a) and (III-7a), lead to any decrease in utility before compensation. Thus, basing the lump-sum transfers on the expected tax payments would surely raise expected lifetime utility. The main point of the exercise in this section is, however, a comparison of the utility levels under the two tax systems.

REFERENCES

Ahsan, S. M., 1974a, "Taxation and Behaviour Under Uncertainty" (unpublished doctoral thesis, McMaster University).

Ahsan, S. M., 1974b, "Progression and Risk-Taking," Oxford Economic Papers, Vol. 26, No. 3, 318-328.

Ahsan, S. M., 1975, "A Note on Capital Gains Exemption and Risk-Taking," Quarterly Journal of Economics, Vol. LXXXIX, February, 151-153.

Ahsan, S. M., 1976, "Taxation in a Two-Period Temporal Model of Consumption and Portfolio Allocation," Research Report 7413, University of Western Ontario, and forthcoming in Journal of Public Economics.

Diamond, P. A., 1970, "Incidence of an Interest Income Tax," Journal of Economic Theory, Vol 2, 211-224.

Goode, R., 1964, The Individual Income Tax (Washington, D.C.: Brookings Institution).

Hansen, B., 1958, Economic Theory of Fiscal Policy (London: Allen and Unwin).

Kaldor, N., 1955, An Expenditure Tax (London: Allen and Unwin).

Mossin, J., 1968, "Taxation and Risk-Taking: An Expected Utility Approach," Economica, N.S., 35, 74-82.

Musgrave, R. A., 1959, The Theory of Public Finance (New York: McGraw-Hill).

Stiglitz, J. E., 1969, "The Effects of Income, Wealth and Capital Gains Taxation on Risk-Taking," Quarterly Journal of Economics, 83, 263-283.