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BACKWARD-BENDING SUPPLY OF LABOR UNDER A CONSUMPTION OR INCOME TARGET BEHAVIOR*

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Abstract

It is widely accepted that a consumption or income target behavior will lead to a backward-bending supply of labor. In this note, we show that such results can be proved only for very special situations. If there is more than one work activity (i.e., multiple jobholding) or time is spent on "nonmarket work" activities in addition to (market) work and (pure) leisure activities, even a consumption or an income target is unlikely to ensure backward-bending supply of labor. The widely held result is due to a very special case in which there is only one work activity and time is allocated only between (market) work and (pure) leisure.

BACKWARD-BENDING SUPPLY OF LABOR UNDER A CONSUMPTION OR INCOME TARGET BEHAVIOR

I

It is widely accepted that a consumption or income target behavior on the part of an individual will yield a "backward-bending supply of labor" for him (e.g., Perlman (1969, p. 11) and Freeman (1972, p. 15)).[1] Moreover, it is expected to be of unitary elasticity (Finegan (1961, p. 232) and Vatter (1961, pp. 578-80)). Such behavior is supposed to explain the backward-bending supply of labor in underdeveloped countries (Berg (1961)). But, as noted by Perlman (1969, p. 11), a consumption (income) target behavior might be followed in developed countries as well; and there seems to be evidence to that effect for multiple jobholders (see Wilensky (1963) and Perrella (1970, p. 58)).

In this note we look for the conditions under which a consumption or an income target behavior will necessarily lead to a backward-bending supply of labor. The two targets are not identical. A consumption target means that the quantities to be purchased from the goods are given, while an income target implies only a constraint on total expenditure. We shall, therefore, discuss the two targets separately. The next section is devoted to the consumption target where it is assumed that total time is allocated between work and leisure activities. In the third section we discuss the consumption target by assuming that total time is allocated among (market) work, leisure and monetarily gainful activities at home (e.g., shopping) which are usually referred to as "nonmarket work" activities. The fourth section is devoted to a brief discussion of the income target. The results are summarized in the fifth section.

II

Let x_k denote the amount bought of the k^{th} market good, and t_i the amount of time spent in the i^{th} time activity (whether leisure or work). The individual is assumed to maximize his utility

$$(1) \qquad u(x_1,\ldots,x_K, t_1,\ldots,t_n)$$

subject to

$$(2) \qquad \sum_{k=1}^{K} p_k x_k = \sum_{i=1}^{m} w_i t_i + A \qquad \text{(money income constraint)}$$

$$(3) \qquad \sum_{i=1}^{n} t_i = T \qquad \text{(time constraint)}$$

$$(4) \qquad x_1 = x_1^o,\ldots,x_K = x_K^o \qquad \text{(consumption target)}$$

where p_k is the price of the k^{th} market good, w_i is the money wage paid in the i^{th} time activity, A is nonlabor income and T is total time available. It is also assumed, with no loss of generality, that the first m time activities are work activities for which $w_i > 0$ $(i=1,\ldots,m)$.

Substituting (3) into (1) and (4) into (1) and (2) yields a maximization of

$$(1') \qquad u(t_1,\ldots,t_{n-1}, T - \sum_{i=1}^{n-1} t_i, x_1^o,\ldots,x_K^o) \equiv V(t_i,\ldots,t_{n-1}; x_1^o,\ldots,x_K^o)$$

subject to

$$(2') \qquad \sum_{i=1}^{m} w_i t_i = \sum_{k=1}^{K} p_k x_k^o - A$$

The first order conditions for maximization, assuming that an internal solution occurs, are[2]

$$\frac{\partial V}{\partial t_i} + \lambda w_i = 0 \qquad i=1,\ldots,m$$

$$(5) \qquad \frac{\partial V}{\partial t_i} = 0 \qquad i=m+1,\ldots,n-1$$

$$\sum_{i=1}^{m} w_i t_i - \sum_{k=1}^{K} p_k x_k^o + A = 0$$

$$t_i,\ \lambda > 0$$

λ is the Lagrange multiplier interpreted as the marginal utility of money.

Differentiation of the equilibrium conditions (5) will yield, if only wages are assumed to change,

$$(6) \qquad dt_i = -\lambda \sum_{j=1}^{m} \frac{D_{ji}}{D} dw_j - \frac{D_{n,i}}{D} \sum_{j=1}^{m} t_j dw_j \qquad i=1,\ldots,n-1$$

where D is the bordered Hessian determinant and D_{ji} its cofactors. The important characteristic of D for our purposes is that it has in its last column (row) m values of $w_i > 0$, but otherwise only zeros. For simplicity let us assume that $dw_j \geq 0$ for $j=1,\ldots,m$.

Had there been only one work activity, i.e., $m = 1$, we would have got

$$(7) \qquad dt_1 = -\left(\lambda \frac{D^*_{11}}{D^*} + \frac{D^*_{n,1}}{D^*} t_1 \right) dw_1$$

where D^* is the bordered Hessian determinant (of the same type as D) when there is only one work activity. If we multiply equation (7) by w_1 we find that

$$(8) \qquad w_1 dt_1 = -\left(\lambda w_1 \frac{D^*_{11}}{D^*} + w_1 \frac{D^*_{n,1}}{D^*} t_1 \right) dw_1 = -t_1 dw_1 < 0$$

since $w_1D^*_{11}$ is an expansion of D^* in terms of alien cofactors and is equal to zero, while $w_1D^*_{n,1} = D^*$. As a result, $dt_1 < 0$. Equation (8) implies a unitary elasticity backward-bending individual supply of labor as it is usually postulated.

The definite result of equation (8) is due to the "income effect," while the substitution effect vanished. In our model $\frac{dt_1}{dA} = -\frac{D^*_{n,1}}{D^*}$, and that is the analogue of the usual income effect of a wage change. One might wonder what happened to the substitution between work and leisure activities. The explanation is that in the usual model this substitution can be viewed as a reflection of the choice between goods (income) and leisure. Since in our model no such choice takes place, the wage rate has no substitution effect within the work-leisure context. More accurately, there is no real choice between work and leisure in our model. The problem facing the individual is that of earning the consumption target with the least effort ("pain"). The amount of leisure time is derived as a residual after work decisions have already been made.

But what if the individual works in more than one activity, as do 5% of the employed in the U.S.? In addition, consumption and work decisions are family matters (see Mincer (1962)), and the utility maximization model presented above can easily be adapted to the family case.[3] Multiple jobholding from the family point of view (e.g., both husband and wife are working) is a common phenomenon, practiced by more than a third of the families. Moreover, as noted above, there are indications that multiple jobholding by individuals and families is motivated, in part at least, by consumption (or income) target considerations. Thus, our model might describe "real world" phenomena.[4]

Suppose that the individual (family) works in m activities. Multiplying equation (6) by w_i and summing over all the work activities yields

$$(9) \qquad \sum_{i=1}^{m} w_i dt_i = - \lambda \sum_{i=1}^{m} w_i \sum_{j=1}^{m} D_{ji} dw_j - \sum_{i=1}^{m} w_i \frac{D_{n,i}}{D} \sum_{j=1}^{m} t_j dw_j$$

$$= - \sum_{j=1}^{m} t_j dw_j < 0$$

since $\sum_{i=1}^{m} w_i \sum_{j=1}^{m} D_{ji} dw_j = \sum_{j=1}^{m} \left(\sum_{i=1}^{m} w_i D_{ji} \right) dw_j = 0$ ($\sum_{i=1}^{m} w_i D_{ji}$ is an expansion of D by alien factors), and $\sum_{i=1}^{m} w_i D_{n,i} = D$. This result implies that any type of rise in (real) wages will reduce the supply of labor when it is valued at the original wages. Put differently, a Laspeyre quantity index of labor supply will be smaller than unity. Thus, we find that in the case of more than one work activity, the consumption target behavior ensures a "backward-bending supply of value (in original terms) of labor." Moreover, if all wages increase by the same rate the elasticity of that supply can be easily shown to be -1.

Equation (9) would have implied a backward-bending supply of labor (measured in time units), if all wages were originally identical, i.e., $w_1 = \ldots = w_m = w$, since then $w \sum_{i=1}^{m} dt_i < 0$. Moreover, if all wages were increased by the same magnitude, $dw_1 = \ldots = dw_m = dw$, equation (9) would imply unitary elasticity backward-bending supply of labor. Since all wages increase by the same rate in this case, and one could always define the units such that w=1, time spent in all work activities could be aggregated into one composite commodity.

Thus, it is of no surprise that in this particular case the result for a multiple jobholder is the same as that obtained above for a single jobholder.

It should be clear that the result above refers to <u>aggregate</u> supply of labor by the individual (family). Change in time supplied to a particular work activity will depend on whether it is inferior (neutral or normal) with respect to nonlabor income. (This can be easily verified by substituting $dw_j = \alpha w$ into equation (6), where α is a constant.)

Wage rates are usually different at any moment of time for a husband and his wife, and even for an individual who is a multiple jobholder various wages are not likely to be identical. It is, therefore, of interest to find out the condition which ensures backward-bending supply of labor when not all wages are identical in the original situation. Summing equation (6) for total work time yields

$$\sum_{i=1}^{m} dt_i = -\lambda \sum_{i=1}^{m} \sum_{j=1}^{m} \frac{D_{ji}}{D} dw_j - \sum_{i=1}^{m} \frac{D_{n,i}}{D} \sum_{j=1}^{m} t_j dw_j$$

(10)

$$= - \sum_{i=1}^{m} \frac{D_{n,i}}{D} \sum_{j=1}^{m} t_j dw_j$$

since $$\sum_{i=1}^{m} \sum_{j=1}^{m} w_j \frac{D_{ji}}{D} \frac{dw_j}{w_j} = \sum_{j=1}^{m} \left(\sum_{i=1}^{m} w_i D_{ji} \right) \frac{dw_j}{w_j} = 0$$

as $w_i = w_j$ for $i = j$ $(i,j = 1,\ldots,m)$. The substitution terms vanish for two reasons: First, cross-substitutions among work activities cancel out when work time is aggregated. Second, as was explained above, no substitution takes place in our model between work and leisure activities. Thus, backward-bending supply of labor is ensured only if <u>all</u> income effects,

$-\frac{D_{n,i}}{D}$ (i=1,...,m), are negative. It is not unlikely that some work activities might be regarded pleasant enough to be normal with respect to (nonlabor) income. In that case, equation (10) will imply an increase in hours of work.

One might regard the last result as inconceivable, since it implies that at least some leisure activities are inferior with respect to (nonlabor) income. The result will be more in line with our intuition once we remember that among the leisure activities--defined as time activities with a zero wage rate--we find "maintenance" activities such as sleeping and commuting which might be inferior. For example, when wages increase, an individual could meet the consumption target even if he allocates a larger proportion of work time to lower paying but physically less demanding (or nearer) activities. As a result, the individual might spend less time sleeping (or commuting). If a part of the leisure time being saved is spent at relatively enjoyable work activities, total work time will increase.

III

The examples of possible inferior leisure activities given above are closely related to the so-called "nonmarket work" activities: housework, studying, health care, shopping, etc. These activities play an important role in the literature of "new home economics" (e.g., Mincer (1962) and Becker (1965)) and human capital (e.g., Becker (1964)). Their main characteristic is that they yield monetary gains (or savings) in addition to any direct (dis)utility they might have. The question is, then, whether our conclusions will change once we allow for "leisure" activities with such monetary rewards.

Without loss of generality let us assume that there is only one "nonmarket work" activity--the buying of goods (shopping)[5]--and that it is the $m+1^{th}$ time activity in our model.

It is usually claimed (Stigler (1961) and Sharir (1970, Ch. 2)) that as more time is spent on search for information concerning goods or on travelling and waiting while actually buying them, the prices paid will be (on the average) lower. To simplify the discussion, assume that all goods are bought in the same "trip", so that we can define a simple "production function"

$$(11) \qquad p_k = p_k(t_{m+1}) \qquad \partial p_k/\partial t_{m+1} < 0 \qquad k=1,\ldots,K$$

The model will now include equations (1)-(4) and (11). It is easy to see that we get all the first order conditions given by (5), except that for the $m+1^{th}$ time activity we get

$$\frac{\partial V}{\partial t_{m+1}} + \lambda w_{m+1} = 0$$

where $w_{m+1} \equiv - \sum_{k=1}^{K} x_k^o \frac{\partial p_k}{\partial t_{m+1}}$. As a result, the last column (row) of the bordered Hessian, D, will now have as its $m+1^{th}$ element $w_{m+1} > 0$ rather than a zero.

The only change required in equations (6)-(10) in the new model is that the subscript j "will go to" m+1 rather than only to m (i continues "to go to" m). But now, for the substitution terms to vanish, productivity at home must increase by the same rate as productivity in any market work activity, i.e., $\frac{dw_1}{w_1} = \ldots = \frac{dw_m}{w_m} = \frac{dw_{m+1}}{w_{m+1}} = \alpha$. Only then will we be able to expand D by alien cofactors. If this restrictive condition is not met there will be substitution between market and nonmarket work activities. Work time in the market will increase due to this substitution if productivity at home falls relative to that in the market.

The consumption target does not rule out the choice between market and nonmarket work activities, because both types of time activities contribute to the financial ability to meet that target. But, as before, the rule of minimization of total work effort (whether in the market or at home) while achieving the consumption target will still be followed.

Suppose that productivity at home and in the market increase by the same rate, so that the substitution terms vanish. The previous results concerning backward-bending supply of quantity or of value of labor, which were obtained from equations (8) and (9) under a similar condition, might not hold. The last row of D now has m+1 positive elements so that $\sum_{i=1}^{m} w_i D_{n,i} \neq D$, and a negative sign is no longer a definite result for equations like (8) and (9). The consumption target is no longer able to ensure a backward-bending supply of labor even for a single jobholder or for a multiple jobholder when all wage rates are identical. Moreover, it cannot ensure a backward-bending supply of value (in original terms) of labor for a multiple jobholder. Only if all market work activities are inferior, i.e., $-\frac{D_{n,i}}{D} < 0$ for $i=1,\ldots,m$, a backward-bending supply of labor is ensured. The rationale for that result is clear. If at least some nonmarket work activities are regarded as less pleasant than some market work activities, working time spent at the marketplace might increase due to the income effects of higher productivity (at home and in the market).[6]

IV

Let us substitute the consumption target by an income target, Y^o (a constant). For a given nonlabor income it immediately implies an earnings target as well. This might be the case of an individual who is on a pension plan or on an unemployment insurance plan which "taxes" his earnings at a 100% rate. In this case equation (2) will read $\sum_{k=1}^{K} p_k x_k = Y^o$ and equation (4) becomes $\sum_{i=1}^{m} w_i t_i = Y^o - A$.

Maximization of utility (equation (1) after substituting equation (3) into it) subject to the money income and earnings targets (equations (2) and (4) in their new form) yields a bordered Hessian determinant D' whose rank is $n + k + 1$ (instead of n for D). But D' will preserve a crucial feature of D: in the last column (row), there will be m values of $w_i > 0$, but otherwise only zeros.[7] As a result, the above discussion could be repeated with $D'_{n+k+1,i}$ replacing $D_{n,i}$. Our conclusions will be identical to those in Section II. Although there **is** free choice among goods (the constraint is only on total expenditure), prices are fixed for the consumer, and the goods can be viewed as one composite commodity. We are back in the previous model where the quantity of the composite good is given, and there is no real choice between it and leisure.

The introduction of "nonmarket work" into the income target model (i.e., including equation (11)) makes it clear that there are actually two possible income targets. If the individual (family) wants a certain (fixed) amount of income, it implies a certain nominal labor income, and we are actually back in the previous model. There is no real choice between work at home and work in the market. The results of section II will hold.[8] But if the individual (family) wants a certain standard of living to be maintained the nature of the problem changes. Suppose he wants consumption expenditures to be constant in terms of a Paasche quantity index. Equation (2) becomes $\sum_{k=1}^{K} p_k(t_{m+1})x_k = Y^* = \sum_{k=1}^{K} p_k(t_{m+1})x_k^*$ where x_k^* is the quantity of the k^{th} good purchased in the original period. The money income, Y^*, is no longer a constant. It varies inversely to the time spent in non-market work activity (shopping). It is, therefore, evident from equation (4), $\sum_{i=1}^{m} w_i t_i = Y^* - A$, that there is a choice between market and nonmarket work activities in meeting the standard of living target. The determinant D' will have m+1 positive elements in the last column (row),[9] and the conclusions will be identical to those of section III.

V

The discussion above suggests the following conclusions: (1) As long as exogenous price changes are ruled out, the consumption and income targets yield similar results. There is, however, one exception to that rule (see below). (2) If total time is allocated to (market) work and (pure) leisure activities, backward-bending supply of labor is ensured under both types of targets if (i) there is only one work activity, (ii) wages of all work activities were originally identical, or (iii) all work activities are inferior. (3) If total time is allocated among market work, leisure and nonmarket work activities, the consumption target and the standard of living income target (in terms of a Paasche quantity index) ensure backward-bending supply of labor only if productivity at home increases by at least the same rate as productivity in the market and all market work activities are inferior. A fixed income target will, however, ensure a backward-bending supply of labor even in this case if any of the conditions of conclusion (2) are met.

We were looking for the conditions which ensured backward-bending supply of labor for an individual under a consumption or an income target. The discussions in the literature which argue that such a target ensures backward-bending supply of labor utilize one of the above special cases; they assume that time is allocated between (market) work and (pure) leisure activities only, and that there is only one work activity. Under quite reasonable situations--the existence of nonmarket work activities and multiple jobholding--we have shown here that such targets will ensure backward-bending supply of labor only under very restrictive conditions. This does not imply, of course, that backward-bending supply of labor might not occur even if these conditions are not met.

Footnotes

[1]Barzel and McDonald (1973, pp. 624-6) have shown recently that even a partial income target behavior--i.e., when there is a minimum (but not a maximum) income which is required--contributes to a backward-bending supply of labor, at least in a certain range, when the utility function is of the CES type.

[2]The Lagrangean function is

$$L = V(t_1,\ldots,t_{n-1};\ x_1^o,\ldots,x_K^o) + \lambda\left(\sum_{i=1}^{m} w_i t_i - \sum_{k=1}^{K} p_k x_k^o + A\right)$$

Second order conditions for maximum are assumed to be fulfilled.

[3]Different x_k's can be viewed as consumption of goods by different members of the family. The same holds for the work and leisure activities (t_i's). The only difference in the model is that of additional time constraints according to the number of members of the family.

[4]See Sharir (1975) for a more complete discussion of this point.

[5]The other mentioned "nonmarket work" activities could be incorporated only at the price of an undesirable complication of the model. They require the introduction of various production functions at home or the use of a multi-period model.

[6]Similar results were obtained by Becker (1965, pp. 501, 506) when no target was assumed, but all time spent at home was viewed as nonmarket work time.

[7]The Lagrangean function is

$$L = V(t_1,\ldots,t_{n-1},\ x_1\ldots,x_K) + \mu\left(\sum_{k=1}^{K} p_k x_k - Y^o\right)$$

$$+ \lambda\left(\sum_{i=1}^{m} w_i t_i - Y^o + A\right).$$

[8]The only difference in the equilibrium conditions for this problem and the previous one is for the $m+1^{th}$ time activity. It becomes $\frac{\partial V}{\partial t_{m+1}} + \mu\, w_{m+1} = 0$ rather than $\frac{\partial V}{\partial t_{m+1}} = 0$. This does not change the last column (row) but the one before it.

[9]The Lagrangean function now becomes

$$L = V(t_1,\ldots,t_{n-1}, x_1,\ldots,x_K) + \mu \left(\sum_{k=1}^{K} p_k(t_{m+1})\, x_k - \sum_{k=1}^{K} p_k\,(t_{m+1})\, x_k^* \right)$$

$$+ \lambda \left(\sum_{i=1}^{m} w_i t_i - \sum_{k=1}^{K} p_k\,(t_{m+1})\, x_k^* + A \right)$$

In this case the condition for the $m+1^{th}$ time activity becomes

$$\frac{\partial V}{\partial t_{m+1}} + \mu \sum_{k=1}^{K} \frac{\partial p_k}{\partial t_{m+1}} (x_k - x_k^*) - \lambda\, w_{m+1} = 0$$

where $$w_{m+1} \equiv - \sum_{k=1}^{K} \frac{\partial p_k}{\partial t_{m+1}} x_k^* \quad .$$

References

Barzel, Yoram and McDonald, Richard J., "Assets, Subsistence, and the Supply Curve of Labor." American Economic Review 69 (September 1973), 621-33.

Becker, Gary S., Human Capital. New York: National Bureau of Economic Research, 1964.

________. "A Theory of Allocation of Time," Economic Journal 75 (September 1965), 493-517.

Berg, Elliot J., "Backward-Sloping Labor Supply Functions in Dual Economies- The African Case," Quarterly Journal of Economics 75 (August 1961), 468-492.

Finegan, T. Aldrich, "Comment: The Backward Sloping Supply Curve," Industrial and Labor Relations Review 15 (August 1961), 230-234.

Freeman, Richard B., Labor Economics. Englewood Cliffs, N.J.: Prentice-Hall, 1972.

Mincer, Jacob, "Labor Force Participation of Married Women," in Aspects of Labor Economics. New York: National Bureau of Economic Research, 1962.

Perlman, Richard, Labor Theory. New York: John Wiley and Sons, 1969.

Perrela, Vera C., "Moonlighters: Their Motivations and Characteristics," Monthly Labor Review 93 (August 1970), 57-64.

Sharir, (Sharier) Shmuel, "Consumer's Buying Behavior and the Structure of the Retail Industry," unpublished Ph.D. dissertation, Department of Economics, Columbia University, 1970.

________, "Short-Run Work Choices Under an Earnings Target: The Case of Multiple Jobholding," mimeographed (December 1974).

Stigler, George J., "The Economics of Information," Journal of Political Economy 64 (June 1961), 213-25.

Vatter, Harold G., "On the Folklore of the Backward-Sloping Supply Curve," Industrial and Labor Relations Review 14 (July 1961), 578-86.

Wilensky, Harold L., "The Moonlighter: A Product of Relative Deprivation," Industrial Relations 3 (October 1963), 105-24.