Western University Scholarship@Western

Department of Economics Research Reports

Economics Working Papers Archive

1973

Economic Development and Income Distribution: Theory, and Application to Ghana

Kul B. Bhatia

Follow this and additional works at: https://ir.lib.uwo.ca/economicsresrpt



Part of the Economics Commons

Citation of this paper:

Bhatia, Kul B.. "Economic Development and Income Distribution: Theory, and Application to Ghana." Department of Economics Research Reports, 7320. London, ON: Department of Economics, University of Western Ontario (1973).

Bentomas Heroldy

NOV 21873 Research Report 7320

ECONOMIC DEVELOPMENT AND INCOME DISTRIBUTION: THEORY, AND APPLICATION TO GHANA

by

Kul B. Bhatia*

October 1973

Assistant Professor of Economics, University of Western Ontario, London, Canada. I am indebted to J. Clark Leith, R. A. Berry, James R. Melvin and J. Abbey for many helpful discussions, to Ake Blomqvist for an intensive reading of the first draft of the paper, and to other members of the Trade and Development workshops at Western for their comments and suggestions. F. Bourguignon derived some of the equations reported in Section III. Research support from the Ghana project, funded by the Canadian International Development Agency, is also gratefully acknowledged.

I. Introduction

This paper is a general equilibrium analysis of factor income distribution in a developing economy. Economic development in such an economy usually entails accumulation of capital in the industrial sector, often aided by the government, transfer of labour from agriculture to industry, and improvement in agricultural capital stock and technology. Policies directed at any of these objectives will have far-reaching effects on income distribution, and these effects will not always be in accord with the avowed distributional goals of a developing country. The problem of income distribution, however, has not received much attention in the development literature.

A two-factor two-commodity model, the traditional workhorse of international trade theory, has often been used to analyze the effects of levying a tariff, and changes in factor endowments. The Stolper-Samuelson and Rybcynzki theorems, which respectively deal with these matters, are well enshrined in the literature (see, for example, Jones (1965)). This model has also been applied to several propositions in public finance such as tax-incidence (Harberger (1962), Mieszkowski (1965)). Many of these standard results can be readily extended to a developing country. The two-by-two model can thus be used to discuss the distributional effects of certain types of foreign aid (e.g., gifts of industrial capital from abroad), and many developmental policies such as employment and production subsidies, technological change in one or both sectors, etc. In discussing problems of economic development, however, dual economy models are often used (Zarembka (1972)). These are similar in spirit to the

two-by-two model, but they replace its two sectors by a modern industrial sector and a traditional, agricultural sector, and also incorporate concepts such as labour surplus, shadow wages, etc.

While models of this type, whether dualistic or not, are very attractive for their simplicity and other properties, their underlying assumptions might be far too unrealistic for most developing countries. The division of the economy into two sectors, the starting point of such models, will not be very insightful in many cases. Primary, secondary, and tertiary sectors are well-defined in most advanced countries. Even in developing countries, more than two sectors, which differ in structure and play distinct economic roles, can be identified sometimes. In Ghana, for instance, cocoa exports are the main source of foreign exchange which is used to import industrial machinery, food, and other items. Farming begins with food crops which gradually make way for cocoa trees in a few years. The two activities differ in resource allocation and involve rather dissimilar economic calculations. Moreover, they have very little in common with manufacturing regarding inputs, outputs, and technology. For a meaningful analysis, therefore, manufacturing, cocoa, and other farming should be distinguished. Also perfect mobility and full employment of factors, the key assumptions in the two-by-two model, will not be typical of developing countries where factor mobility is often hampered by traditional, social, and structural forces, and unemployment is common.

In this paper we relax some of the assumptions of the traditional two-sector model of general equilibrium. Three sectors are specified and four factors of production are recognized. Three of the four factors, however, are specific--one to each sector--so there is one mobile factor, and only two factors are employed in the production of each commodity. The model is similar to that of Jones (1971) who analyzed the three-factor, two-commodity case. The focus of the paper is on the functional, or factor income distribution, as distinguished from personal or size distribution.

The theoretical structure of the model is presented in Section II. It is generalized to the (n+1) by n case in Section III. The model is illustrated by examples from the Ghanaian economy in Section IV, the income-distribution effects of several policies are discussed in Section V, and some of the theoretical results are compared with the Ghanaian experience in Section VI. Although the Ghanaian economy forms the backdrop of much of the analysis, the model is quite general and should be applicable to other developing countries also.

II. The Theoretical Structure

The output of the three sectors is denoted by X_1 , X_2 , and X_3 . X_i employs V_i , the factor specific to it, and some V_m , the mobile factor shared by the three sectors. Let a_{ij} denote the quantity of input i required for producing one unit of X_j . We assume a perfectly competitive economy in which production is subject to constant returns to scale. Firms maximize profits, which are reduced to zero in equilibrium. With constant returns to scale, the product of the a's and the level of output determine total factor demands. The allocation of factors to the three sectors therefore is given by equations (1.1) to (4.1).

$$a_{11} X_1 = V_1$$
 (1.1)

$$a_{22} X_2 = V_2$$
 (2.1)

$$a_{33} X_3 = V_3$$
 (3.1)

$$a_{m1} X_1 + a_{m2} X_2 + a_{m3} X_3 = V_m$$
 (4.1)

If R_{j} is the return per unit of factor i, and p_{j} is the average price of commodity j, equations (5.1) to (7.1) hold in a competitive economy.

$$a_{11}R_1 + a_{m1}R_m = p_1 (5.1)$$

$$a_{22}R_2 + a_{m2}R_m = p_2$$
 (6.1)

$$a_{33}R_3 + a_{m3}R_m = P_3$$
 (7.1)

These three equations merely state that price per unit of a commodity equals the average cost of producing it under perfect competition. The production coefficients, a;, depend on relative factor prices in each sector:

$$a_{ij} = a_{ij} \left(\frac{R_m}{R_j}\right)$$
 (8.1)

There will be six equations like (8.1) to determine all the a 's. These and equations (1.1) to (7.1) provide 13 relations to determine 13 unknowns:

 X_1, X_2, X_3 ; R_1, R_2, R_3, R_m ; and a_{ij} (6). The factor endowments V_1, V_2, V_3, V_m , and commodity prices p_1, p_2 , and p_3 are the parameters in the model. If the production coefficients were technologically fixed, equations (1.1) to (7.1) could be used to determine factor returns and commodity outputs, still treating factor endowments and commodity prices as parameters.

The Determination of Factor Returns:

Substituting equations (1.1) to (3.1) into (4.1) we derive:

$$\frac{a_{m1}}{a_{11}} V_1 + \frac{a_{m2}}{a_{22}} V_2 + \frac{a_{m3}}{a_{33}} V_3 = V_m$$
 (4.1')

Equations (5.1) to (7.1) and (4.1') can be solved for R_1 , R_2 , R_3 , and R_m , for given factor endowments, production coefficients, and commodity prices. Of

course for variable production coefficients, (8.1) will also be needed.

It is clear from these equations that the returns to factors of production do not depend solely on commodity prices; (4.1') ensures that factor endowments will also affect the R's. This is an important characteristic of models of this type in which there are specific factors, and factor inputs exceed the commodities in number.³

The Effects of Changes in Parameters:

We consider the effects of small, exogenous changes in factor endowments and commodity prices. Denote relative changes by an asterisk (*); thus $p_1^* = dp_1/p_1$. By totally differentiating equations (4.1') and (5.1) - (7.1), p_1^* , V_1^* and R_1^* can be interrelated via distributive shares (0's), factor proportions (λ 's), and elasticities of substitution (σ 's). We get:

$$\lambda_{m1} \sigma_{1} R_{1}^{*} + \lambda_{m2} \sigma_{2} R_{2}^{*} + \lambda_{m3} \sigma_{3} R_{3}^{*} - \{\lambda_{m1} \sigma_{1} + \lambda_{m2} \sigma_{2} + \lambda_{m3} \sigma_{3}\} R_{m}^{*}$$

$$= V_{m}^{*} - \lambda_{m1} V_{1}^{*} - \lambda_{m2} V_{2}^{*} - \lambda_{m3} V_{3}^{*}$$

$$(4.2)$$

$$\theta_{11} R_1^* + \theta_{m1} R_m^* = p_1^* \tag{5.2}$$

$$\theta_{22} R_2^* + \theta_{m2} R_m^* = P_2^*$$
 (6.2)

$$\theta_{33} R_3^* + \theta_{m3} R_m^* = P_3^*$$
 (7.2)

 θ_{ij} , the share of factor \underline{i} in industry $j = \frac{a_{ij} R_j}{p_i}$,

 λ_{ij} , the proportion of factor <u>i</u> used by the jth industry = $\frac{a_{ij}X_j}{V_i}$,

and σ_j , the elasticity of substitution between the mobile and the specific factor in industry $\underline{j} = \frac{V_{mj}^* - V_j^*}{R_j^* - R_m^*}$,

$$(i = 1, 2, 3, m, and j = 1, 2, 3)$$
.

These equations can be rewritten in matrix notation as:

$$\theta^* \cdot R^* = p^*$$

where

$$\theta^* \cdot R^* = p^*$$
ore
$$\theta^* = \begin{bmatrix} \theta_{11} & 0 & 0 & \theta_{m1} \\ 0 & \theta_{22} & 0 & \theta_{m2} \\ 0 & 0 & \theta_{33} & \theta_{m3} \\ \lambda_{m1} \sigma_1 & \lambda_{m2} \sigma_2 & \lambda_{m3} \sigma_3 & -\{\lambda_{m1} \sigma_1 + \lambda_{m2} \sigma_2 + \lambda_{m3} \sigma_3 \} \end{bmatrix}$$

and R* and p* are column vectors $\{R_1^*, R_2^*, R_3^*, R_m^*\}$ and $\{p_1^*, p_2^*, p_3^*, (V_m^* - \lambda_{m1}, V_1^* - k_{m1}, V$ $\lambda_{m2} v_2^* - \lambda_{m3} v_3^*$) } respectively. Expansion of θ^* by cofactors of the first row yields

$$|\theta^*| = -\theta_{11}\theta_{22}\theta_{33}\Delta$$

$$\Delta = \lambda_{m1} \frac{\sigma_1}{\theta_{11}} + \lambda_{m2} \frac{\sigma_2}{\theta_{22}} + \lambda_{m3} \frac{\sigma_3}{\theta_{33}}.$$

The λ 's, θ 's, and σ 's are all positive. Therefore, $\Delta > 0$.

Using Cramer's rule we derive the following results:

$$R_{1}^{*} = \frac{1}{\Delta} \left[\left(\lambda_{m1} \frac{\sigma_{1}}{\theta_{11}} + \frac{1}{\theta_{11}} \lambda_{m2} \frac{\sigma_{2}}{\theta_{22}} + \frac{1}{\theta_{11}} \lambda_{m3} \frac{\sigma_{3}}{\theta_{33}} \right) p_{1}^{*} - \frac{\theta_{m1}}{\theta_{11}} \lambda_{m2} \frac{\sigma_{2}}{\theta_{22}} p_{2}^{*} - \frac{\theta_{m1}}{\theta_{11}} \lambda_{m3} \frac{\sigma_{3}}{\theta_{33}} p_{3}^{*} + \frac{\theta_{m1}}{\theta_{11}} (v_{m}^{*} - \lambda_{m1} v_{1}^{*} - \lambda_{m2} v_{2}^{*} - \lambda_{m3} v_{3}^{*}) \right]$$

$$(9.1)$$

$$R_{m}^{*} = \frac{1}{\Delta} \left[\lambda_{m1} \frac{\sigma_{1}}{\theta_{11}} p_{1}^{*} + \lambda_{m2} \frac{\sigma_{2}}{\theta_{22}} p_{2}^{*} + \lambda_{m3} \frac{\sigma_{3}}{\theta_{33}} p_{3}^{*} + (\lambda_{m1} v_{1}^{*} + \lambda_{m2} v_{2}^{*} + \lambda_{m3} v_{3}^{*} - v_{m}^{*}) \right]$$

$$+ (\lambda_{m1} v_{1}^{*} + \lambda_{m2} v_{2}^{*} + \lambda_{m3} v_{3}^{*} - v_{m}^{*})$$

$$(10.1)$$

The solutions for R_2^* and R_3^* are similar to that of R_1^* and can be derived by changing the subscripts in (9.1). These equations clearly show how changes in commodity prices and factor endowments will affect the returns to factors of production. An increase in V_m , the mobile factor, other things being equal, will lower its reward but raise the return to specific factors. Any increase in the endowment of specific factors on the other hand will reduce their returns and raise R_m . These and other results are rigorously derived in Section V.

It should be noted that factors of production might be specific due to their inherent nature or reasons of technology. In most cases, at least some inter-sectoral mobility takes place in the long run. In the very short run almost any factor will assume some characteristics of a specific factor. In general, therefore, our model will be better suited for short and medium, rather than long run analysis.

III. A Generalization: The $(\underline{n+1})$ x \underline{n} Case

The four-by-three model can be readily extended to the general case of (n+1) factors and n sectors. Retaining all the assumptions stated in Section 2, let us assume that there are n specific factors, one in each sector, and one mobile factor (denoted by subscript m). As before, each sector uses the mobile factor and a specific factor. Equations (1.1) to (4.1), which represent technology and factor endowments, can be stated as

$$A \cdot X = V \tag{11}$$

where

$$A = \begin{bmatrix} a_{11} & 0 & --- & 0 \\ 0 & a_{22} & 0 \\ \vdots & \vdots & \vdots \\ 0 & --- & a_{nn} \\ a_{m1} & --- & --- & a_{mn} \end{bmatrix}, \quad X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} \text{ and } V = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_m \end{bmatrix}$$

The output and input prices, equations (5.1) to (7.1) can be written as

$$A' \cdot R = p \tag{12}$$

where A' is the transpose of A, and R and p, respectively, are column vectors of factor rewards and output prices. The technological coefficients, aij, will continue to depend on relative factor prices as stated in (8.1).

Now we have (n+1) equations in expression (11), n in (12), and 2(n+1) in (8.1). Together they determine (n+1) factor prices, n output levels, and 2(n+1) coefficients of technology. Factor endowments and output prices, the V and p vectors, respectively, are treated as parameters.

Using the symbols defined above (λ , θ , and σ), effects of small, exogenous changes in parameters can be derived as follows:

$$\sum_{i=1}^{n} \lambda_{mi} \sigma_{i} R_{i}^{*} - \left(\sum_{i=1}^{n} \lambda_{mi} \sigma_{i}\right) R_{m}^{*} = V_{m}^{*} - \sum_{i=1}^{n} \lambda_{mi} V_{i}^{*}$$
(13)

$$\theta_{ii} R_{i}^{*} + \theta_{mi} R_{m}^{*} = p_{i}^{*}$$
 $i = 1,...,n$ (14)

Equations (13) and (14) can be rewritten in matrix notation as:

$$\theta^* \cdot R^* = p^*$$

where

$$\theta^* = \begin{bmatrix} \theta_{11} & 0 & \cdots & \theta_{m1} \\ 0 & \theta_{22} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \theta_{nn} & \theta_{mn} \\ \lambda_{m1}\sigma_{1} & \lambda_{m2}\sigma_{2} & \cdots & \sum_{i=1}^{n} \lambda_{mi}\sigma_{i} \end{bmatrix}$$

and R^* and p^* are column vectors $\{R_1^*, R_2^*, ----, R_m^*\}$

and
$$\{p_1^*, p_2^*, ---p_n^*, (v_m^* - \sum_{i=1}^n \lambda_{mi} v_i^*)\}$$
,

respectively. Expansion of θ^* by cofactors of the first row gives

$$|\theta^*| = -\prod_{i=1}^n \theta_{ii}$$
 D

where

$$D = \sum_{i=1}^{n} \lambda_{mi} \frac{\sigma_i}{\theta_{ii}} \qquad i = 1, ---, n.$$

Using Cramer's rule:

$$R_{k}^{*} = \frac{1}{D} \left[\left(\lambda_{mk} \frac{\sigma_{k}}{\theta_{kk}} + \frac{1}{\theta_{kk}} \sum_{i \neq k}^{n} \lambda_{mi} \frac{\sigma_{i}}{\theta_{ii}} \right) p_{k}^{*} - \frac{\theta_{mk}}{\theta_{kk}} \sum_{i \neq k}^{n} \lambda_{mi} \frac{\sigma_{i}}{\theta_{ii}} p_{i}^{*} \right]$$

$$+ \frac{\theta_{mk}}{\theta_{kk}} \left(v_{m}^{*} - \sum_{i=1}^{n} \lambda_{mi} v_{i}^{*} \right)$$

$$R_{m}^{*} = \frac{1}{D} \begin{bmatrix} \sum_{i=1}^{n} \lambda_{mi} \frac{\sigma_{i}}{\theta_{ii}} p_{1}^{*} + \sum_{i=1}^{n} \lambda_{mi} v_{i}^{*} - v_{m}^{*} \end{bmatrix}$$

It is easily verified that for n=3 and k=1, the above results are reduced to (9.1) and (10.1) respectively.

Although the general case is of considerable theoretical interest, for all practical purposes, the four-by-three model is more than sufficient to deal with income distribution effects of a wide range of policies. In most developing countries like Ghana, to which the model is applied in this paper, data, in their present state, will not permit reasonable estimates for the parameters of even the smaller model. Larger models therefore will be definitely intractable. For the rest of the paper, we revert to the four-by three model which is illustrated by the Ghanaian case in the next section.

IV. An Illustration: The Ghanaian Economy

It is difficult to approximate any real life economy by a simple theoretical model. The difficulties increase enormously in the case of developing countries where a host of conceptual and empirical problems arise. For example, it is often difficult to delineate sectors because of overlapping production activities, and data problems almost preclude even reasonable guesses about parameters such as the elasticity of substitution between factors of production. Moreover, in many cases, beside its more conventional functions, the government is involved in foreign trade, manufacturing, farming, etc., and the profit-maximizing calculus of a competitive economy will not be very relevant to these governmental activities. Any "realistic" model, consequently, is bound to be overly complicated, and perhaps devoid of all empirical content. In the present state of the arts, one can only take a step or two towards reality by incorporating just the main features of an economy in a theoretical model.

How well does the model outlined in Section II correspond to the Ghanaian case? The Ghanaian economy is dominated by primary production, which employs more than 60 percent of its labour force and contributes the bulk of GNP. Cocoa is the main activity in the primary sector. Although value added in cocoa is less than 10 percent of the GNP in most years, cocoa accounts for more than 60 percent of Ghanaian exports. The manufacturing sector is small in size but growing fast; it recorded an average compound growth rate of 27.6 percent during 1955-69. The Ghanaian economy can thus be usefully divided into three sectors: cocoa, other primary production, and manufacturing. The first two are the mainstay of the 'colonial' economic structure, and the third is the harbinger of modernism and industrialization in Ghana.

Specific factors, which play an important role in the theoretical model, are a significant feature of the three sectors in Ghana. There are trees in cocoa production, land in the case of farming, and machines and other capital goods in manufacturing. In the short run, these factors hardly move between sectors. In the long run, however, most factors can be transferred from one sector to another. Labour appears to have been the most mobile factor in Ghana. Considerable migration of labour between regions and industries has been recorded in both the 1960 and the 1970 censuses. 9

Of the other factors of production, due to vastly different technologies, there can be hardly any exchange of specific factors between manufacturing and the rest of the economy, except in the long run. Cocoa trees by their very nature cannot be shifted to any other sector. There has been a steady transfer of land from farming to cocoa. In some regions, it is fairly common to start with food cultivation, plant cocoa trees simultaneously, and in a few years, completely switch to cocoa production. There is a long gestation period: trees up to 7 years of age bear nothing, and those between 7 and 15 years of age produce less than half of a full yield. Thus it is doubtful that much land could be moved back into farming, even if it were technically feasible, in response to say a decline in the relative price of cocoa.

The dominant structural feature of the Ghanaian economy is that intermediate goods play a rather unimportant role. An input-output study, based on 1960 data, revealed that the various sectors had little dependence on each other; most of their output went directly to final uses.

"The economy of Ghana, relatively to the typical industrialized economies, has a low degree of complexity. The flows of output are mainly directed to export and consumption, or to investment in the case of construction, and are not processed or transformed to any great extent by other productive activities. The economy has weak linkages, weak structural connections between the sectors."

The study, in spite of being somewhat outdated, suggests that conclusions derived from a theoretical model which left out intermediate products could be readily applied to the Ghanaian case.

The above discussion demonstrates the relevance of the model set forth in Section II to the Ghanaian economy. The four-by-three framework, with both specific and mobile factors, conforms well to the Ghanaian case. Ghana is a price-taker in the world market for most goods. The producer price of cocoa is fixed by the Ghana Cocoa Marketing Board, and therefore can be treated as exogenous. ¹² For short run analysis, the assumption that other commodity prices also are exogenous, is not very unrealistic. Many results of our theoretical analysis can thus be usefully applied to Ghana.

V. Economic Policies and Income Distribution

Industrialization is high on the agenda of most developing countries.

Toward this goal, governments often grant tax incentives, pursue protectionist policies, subsidize industrial employment and in some cases provide capital, either in the private or the public sector. Policies of price support and subsidized production are also adopted in the farm sector. This section deals with the income distribution effects of some of these measures.

At the outset, let us write some expressions for changes in relative factor rewards. From (9.1) and (10.1) we have:

Again, by solving (4.2) to (7.2) for R_2^* and subtracting the solution from (10.1), we get:

$$(R_{1}^{*} - R_{2}^{*}) = \frac{1}{\Delta} \left[\left(\frac{1}{\theta_{11}} \lambda_{m2} \frac{\sigma_{2}}{\theta_{22}} + \frac{1}{\theta_{22}} \lambda_{m1} \frac{\sigma_{1}}{\theta_{11}} \right) (p_{1}^{*} - p_{2}^{*}) \right.$$

$$+ \frac{1}{\theta_{11}} \lambda_{m3} \frac{\sigma_{3}}{\theta_{33}} (p_{1}^{*} - p_{3}^{*}) + \frac{1}{\theta_{22}} \lambda_{m3} \frac{\sigma_{3}}{\theta_{33}} (p_{3}^{*} - p_{2}^{*})$$

$$+ (v_{m}^{*} - \lambda_{m1} v_{1}^{*} - \lambda_{m2} v_{2}^{*} - \lambda_{m3} v_{3}^{*}) \left(\frac{1}{\theta_{11}} - \frac{1}{\theta_{22}} \right) \right]$$

$$(12.1)$$

Expressions for $(R_m^* - R_2^*)$, $(R_2^* - R_3^*)$, etc. can be similarly determined. For brevity, the results derived below are stated in terms of (11.1) and (12.1), but these can be readily extended to other factor rewards.

Production Subsidies:

An excise tax or a production subsidy drives a wedge between prices paid by consumers and those received by the producers. Let p_j be the market price of the jth commodity, and t_j the ad valorem rate of subsidy. Then the price received by producers is $s_j p_j$, where $s_j = (1+t_j)$. Equations (5.2) to (7.2) will be altered to show the effects of the subsidy.

$$\theta_{11} R_1^* + \theta_{m1} R_m^* = p_1^* + s_1^* \tag{5.3}$$

$$\theta_{22} R_2^* + \theta_{m2} R_m^* = p_2^* + s_2^*$$
 (6.3)

$$\theta_{33} R_3^* + \theta_{m3} R_m^* = p_3^* + s_3^*$$
 (7.3)

In the solutions for factor rewards (R^{*} 's) presented in (9.1) and (10.1), $(p_1^* + s_1^*)$ will replace p_1^* . It is obvious that, as a result of a subsidy, returns to both factors in the subsidized industry will go up. Furthermore, for given factor endowments and unchanged commodity prices, changes in relative factor rewards can be expressed by (11.2) and (12.2):

$$(R_{m}^{*}-R_{1}^{*}) = \frac{1}{\theta_{11}^{\Delta}} \left[\lambda_{m3} \frac{\sigma_{3}}{\theta_{33}} (s_{3}^{*}-s_{1}^{*}) + \lambda_{m2} \frac{\sigma_{2}}{\theta_{22}} (s_{2}^{*}-s_{1}^{*}) \right]$$
(11.2)

$$(R_{1}^{*} - R_{2}^{*}) = \frac{1}{\Delta} \left[\left(\frac{1}{\theta_{11}} \lambda_{m2} \frac{\sigma_{2}}{\theta_{22}} + \frac{1}{\theta_{22}} \lambda_{m1} \frac{\sigma_{1}}{\theta_{11}} \right) (s_{1}^{*} - s_{2}^{*}) + \frac{1}{\theta_{11}} \lambda_{m3} \frac{\sigma_{3}}{\theta_{33}} (s_{1}^{*} - s_{3}^{*}) + \frac{1}{\theta_{22}} \lambda_{m3} \frac{\sigma_{3}}{\theta_{33}} (s_{3}^{*} - s_{2}^{*}) \right]$$

$$(12.2)$$

If $s_1^* > s_3^* > s_2^*$, (11.2) is negative and (12.2) is positive. Therefore $R_1^* > R_m^*$ and $R_1^* > R_2^*$: returns to both the mobile and specific factors of a subsidized industry will increase, and the factor specific to the most heavily subsidized industry will gain more than the specific factor in the least subsidized sector. 14

Increase in Industrial Capital:

As part of a program to industrialize, governments try to increase the endowment of industrial capital. Witness, for example, the growth of publicly owned industrial establishments in India, Ghana, and many other developing countries. In our model let us assume that sector 1 is the manufacturing sector. Increase in industrial capital is then tantamount to changing V_1 , the factor specific to that sector. As a result, the marginal product of the mobile factor in industry will rise, that of the specific factor will fall. Other things being equal, R_m will rise, attempts will be made to increase the ratio of specific to mobile factor in other sectors also; consequently, the returns to all specific factors will fall. Obviously, $(R_m^* - R_1^*)$ will be positive. From (9.1), (10.1), and (11.1):

$$\frac{\partial R_1^*}{\partial V_1^*} = -\frac{\lambda_{m1}}{\Delta} \frac{\theta_{m1}}{\theta_{11}} < 0, \quad \frac{\partial R_m^*}{\partial V_1^*} = \frac{\lambda_{m1}}{\Delta} > 0,$$

and
$$\frac{\partial (R_m^* - R_1^*)}{\partial V_1^*} = \frac{1}{\theta_{11}^{\Delta}} \lambda_{m1} > 0$$
.

The effect on relative rewards of specific factors is somewhat complicated and depends on factor shares. Differentiating (12.1) with respect to V_1^* we get:

$$\frac{\partial (R_1^* - R_2^*)}{\partial V_1^*} = -\frac{1}{\Delta} \left(\frac{1}{\theta_{11}} - \frac{1}{\theta_{22}} \right) \lambda_{m1},$$

which is $\stackrel{>}{<} 0$ as $\theta_{11} \stackrel{>}{<} \theta_{22}$. Thus if capital's share in manufacturing is greater than that of the specific factor in industry 2, R_2 will fall more than R_1 when V_1 increases. The extension of this result to other specific factors is obvious.

Change in Commodity Prices:

The assumption crucial to the above findings, as also to the model in general, is that commodity prices are exogenously determined, which is true, for example, for a small country that is a price-taker in the world market. A change in commodity prices can significantly affect the development program and the income distribution of a country. In Ghana's case, for instance, cocoa has been the main export. Changes in cocoa prices will affect Ghana's balance of payments, its ability to import capital, and other aspects of its economy. The effects on income distribution can be determined from equations (9.1) to (12.1).

From (9.1):

$$\frac{\partial R_{\underline{i}}^{*}}{\partial p_{\underline{i}}^{*}} = \frac{1}{\Delta} \left[\lambda_{mi} \frac{\sigma_{\underline{i}}}{\theta_{\underline{i}\underline{i}}} + \frac{1}{\theta_{\underline{i}\underline{i}}} \sum_{j \neq i} \lambda_{mj} \frac{\sigma_{\underline{j}}}{\theta_{\underline{j}\underline{j}}} \right] > 0, \quad \text{and}$$

$$\frac{\partial R_{\underline{i}}^{*}}{\partial p_{\underline{i}}^{*}} = -\frac{1}{\Delta} \left[\frac{\theta_{mi}}{\theta_{\underline{i}\underline{i}}} \lambda_{mj} \frac{\sigma_{\underline{j}}}{\theta_{\underline{i}\underline{i}}} \right] < 0, \quad \underline{i} \neq \underline{j}. \quad (9.2)$$

And from (10.1):

$$\frac{\partial R_{\underline{m}}^{*}}{\partial p_{i}^{*}} = \frac{1}{\Delta} \left[\lambda_{\underline{m}i} \frac{\sigma_{i}}{\theta_{ii}} \right] > 0.$$
 (10.2)

In other words, for given factor endowments, an increase in the price of a commodity will lead to a higher return for its specific factor and the mobile factor, but lower returns for other specific factors in the economy. The opposite will hold for a decline in commodity prices.

If more than one commodity price rises at the same time, the effects on relative factor rewards can be determined from (11.1) and (12.1). Assume, for example, that $p_1^* > p_3^* > p_2^*$. Then $(R_m^* - R_1^*) < 0$ and $(R_1^* - R_2^*) > 0$. Also $(R_m^* - R_2^*) > 0$, so we have the following result:

$$R_1^* > R_m^* > R_2^*$$

Nothing definite can be said, however, about R_3^* and how it relates to other factor rewards without knowing the magnitude of the price changes. ¹⁷

Factor Taxes and Subsidies

Historically, taxes and subsidies on factors of production have played an important role in economic development. After the Meiji Restoration in Japan, for instance, heavy taxes on agricultural land were levied to finance industrialization. And capital and employment subsidies are often given to the industrial sector in many developing countries of today. The objective in most cases is to modify the market allocation of resources in favour of a particular sector, or just to raise revenue.

Let $\mathbf{R}_{\mathbf{i}}^{'}$ denote the return to the \mathbf{i}^{th} factor after taxes and subsidies. Then

$$R_{i}' = b_{i}R_{i}$$
, $b_{i} = (1 + \delta_{i})$

where δ_i is the per unit subsidy on factor i. To illustrate, let us assume further that there is a tax on the specific factor and a subsidy to the mobile factor in sector 1.8 We use primes (') to denote terms in the tax case. For example, θ_{11}' is the net of tax share of the specific factor in industry 1.

Let θ_{11}^{T} and θ_{m1}^{T} be the shares of the taxes on the two factors in industry 1. Then

$$\theta_{11}' + \theta_{m1}' - \theta_{11}^{T} + \theta_{m1}^{T} = 1$$

We also assume that taxes and subsidies are small, so that $\theta_{11}^T/\theta_{11}^T$ and $\theta_{m1}^T/\theta_{m1}^T < 1$. Equations (4.2) and (5.2) are then replaced by (4.4) and (5.4).

$$\lambda_{m1}^{'}\sigma_{1}^{'} (R_{1}^{*} + b_{1}^{*}) + \lambda_{m2}\sigma_{2} R_{2}^{*} + \lambda_{m3}\sigma_{3}R_{3}^{*} - \{\lambda_{m1}^{'}\sigma_{1}^{'} + \lambda_{m2}\sigma_{2} + \lambda_{m3}\sigma_{3}\}$$

$$R_{m}^{*} - b_{m}^{*} \lambda_{m1}^{'}\sigma_{1}$$
(4.4)

$$\theta_{11}^{\prime}R_{1}^{*} + \theta_{m1}R_{m}^{*} = p_{1}^{*} - \theta_{11}^{T}b_{1}^{*} - \theta_{m1}^{T}b_{m}^{*}$$
 (5.4)

The other equations remain unchanged. In the final solution for R_1^* and R_m^* , R_1^* will be replaced by $(R_1^* + b_1^*)$ and p_1^* by $(p_1^* - \theta_{11}^T b_{11}^* - \theta_{m1}^T b_m^*)$. The following derivatives can then be readily obtained:

$$\frac{\partial R_{1}^{*}}{\partial b_{1}^{*}} = -\frac{1}{\Delta'} \left[\theta_{11}^{T} \left(\lambda_{m1}^{'} \frac{\sigma_{1}^{'}}{\theta_{11}^{'}} + \frac{1}{\theta_{11}^{'}} \lambda_{m2} \frac{\sigma_{2}}{\theta_{22}} + \frac{1}{\sigma_{11}^{'}} \lambda_{m3} \frac{\sigma_{3}}{\theta_{33}} \right) + \frac{\lambda_{m1}^{\sigma_{1}} \theta_{m1}^{'}}{\theta_{11}^{'}} \right] < 0$$

$$\frac{\partial R_{m}^{*}}{\partial b_{1}^{*}} = \frac{1}{\Delta'} \left[\lambda_{m1}' \sigma_{1}' \left(1 - \frac{\theta_{11}^{T}}{\theta_{11}'}\right) \right] > 0.$$

$$\frac{\partial R_{\underline{m}}^{*}}{\partial b_{\underline{m}}^{*}} = -\frac{1}{\Delta'} \left[\lambda_{\underline{m}1}' \sigma_{1}' + \theta_{\underline{m}1}^{T} \lambda_{\underline{m}1}' \frac{\sigma_{1}'}{\theta_{11}'} \right] < 0.$$

A tax on the specific factor, thus, lowers its return but benefits the mobile factor. A subsidy to the mobile factor will raise its return. These results are easily explained. As a result of a tax on the specific factor, industry 1 will try to substitute the mobile for the specific factor, leading to an excess demand for the mobile factor and hence an increase in $R_{\rm m}$. A subsidy to the mobile factor will also have the same effect. In both cases ratios of specific to mobile factors in the rest of the economy will rise. The rewards to other specific factors therefore will also decline. 19

VI. The Ghanaian Case Once Again

Since the model was illustrated with examples from the Ghanaian economy in Section IV, let us see how some of the theoretical results derived above apply to the Ghanaian case. Clearly, for a definitive answer, information on the parameters of the model such as σ_i , θ_{ij} , λ_{ij} , and the facts about income distribution will be needed. Our present knowledge on both these scores is very meagre. To quote Killick (1973, p. 4.30),

"The importance of income transfers, the very large numbers of self-employed, incomplete occupational specialization, and weaknesses of statistical services conspire to prevent even the roughest estimates of the overall distribution of the national income, but something can be said about particular groups within society."

What we need are data, by sectors, on factor rewards, prices, etc., which are not readily available. We can, therefore, at best check if the available facts, however scarce, are roughly consistent with the theoretical results derived in this paper.

Let us identify X_1 with manufacturing, X_2 with farming, and X_3 with cocoa. V_m , the mobile factor, is labour. The prices relevant to our model are producer prices. One of the most significant economic developments for

Ghana since 1956 has been the steady decline in the producer price of cocoa: it declined from N¢ 298.6 per long ton in 1955-56 to N¢ 149.3 in 1965-66, before starting an upward movement. No separate index of producer prices of farm products is available. A fair proxy for it, however, is an index of market prices of locally produced food, and it rose from 78.4 in 1956 to 199.0 in 1966 (base 1963=100). Not much information on prices of manufactured goods has been compiled either. The consumer price index (base 1963=100), in which local food has a weight of 0.52, rose from 78.7 in 1956 to 148.2 in 1966. This index includes manufactured goods also, but their prices are not separately mentioned. In any case, their weight does not exceed 0.01. Because of this scarcity of information, we assume that prices of farm products and manufactured goods increased at the same rate, i.e., $(p_1^* - p_2^*) = 0$. We can be sure, however, that both $(p_2^* - p_3^*)$ and $(p_1^* - p_3^*)$ were positive between 1956 and 1966.

Setting $p_2^* = p_1^*$ in (9.1) we have:

$$R_{1}^{*} = \frac{1}{\Delta} \left[\left(\lambda_{m1} \frac{\sigma_{1}}{\theta_{11}} + \lambda_{m2} \frac{\sigma_{2}}{\theta_{22}} \frac{1}{\theta_{11}} \lambda_{m3} \frac{\sigma_{3}}{\theta_{33}} \right) p_{1}^{*} - \frac{\theta_{m1}}{\theta_{11}} \lambda_{m3} \frac{\sigma_{3}}{\theta_{33}} p_{3}^{*} + \frac{\theta_{m1}}{\theta_{11}} \left(v_{m}^{*} - \lambda_{m1} v_{1}^{*} - \lambda_{m2} v_{2}^{*} - \lambda_{m3} v_{3}^{*} \right) \right]$$

$$(9.3)$$

Since $p_3^* < 0$, and population probably grew faster than the weighted average of changes in specific factors $(\sum\limits_{i=1}^3 \lambda_{mi} V_i^*)$, $R_1^* > 0$. Similarly $R_2^* > 0$, but R_3^* will be negative. Decline in cocoa prices thus definitely hurt cocoa farmers and increased the return to specific factors in other sectors.

Whether manufacturing benefited more than farming depends on the share of the specific factors. Set $p_1^* = p_2^*$ in (12.2) to obtain (12.3).

$$(R_1^* - R_2^*) = \frac{1}{\Delta} \left[\left(\frac{1}{\theta_{11}} - \frac{1}{\theta_{22}} \right) (p_1^* - p_3^*) + (v_m^* - \lambda_{m1} v_1^* - \lambda_{m2} v_2^* - \lambda_{m3} v_3^*) \left(\frac{1}{\theta_{11}} - \frac{1}{\theta_{22}} \right) \right]$$

$$(12.3)$$

For given factor endowments the sign of (12.3) obviously depends on θ_{11} and θ_{22} because $(p_1^*-p_3^*)>0$. However, there are hardly any data on factor shares in Ghana. All we know is that agriculture employs a larger fraction of labour than manufacturing $(\lambda_{m2}>\lambda_{m1})$, but that does not give any indication about factor shares. We can only conclude, therefore, that $R_1^* \gtrless R_2^*$ as $\theta_{11} \lessgtr \theta_{22}$. In this context, it should be noted that part of the increase in farm incomes (R_2) would have gone to cocoa farmers in regions where these two activities are combined.

These results are easily explained. In response to a decline in cocoa prices, resources must have moved out, especially labour, leading to higher $\frac{v_3}{v_m}$, but lower $\frac{v_1}{v_m}$ and $\frac{v_2}{v_m}$. Consequently, the marginal productivity of v_1 and v_2 would have increased, at the expense of that of v_3 . This argument also suggests that the marginal product of labour should have fallen in both manufacturing and farming, but it does not follow that wage rate (real or nominal) should also be lower. We know from (10.1) that $\partial R_m^*/\partial p_1^*>0$: decline in cocoa prices, per se, would therefore reduce wages, but the rise in manufacturing and farm prices could offset the influence of the fall in the price of cocoa. Besides, the labour force and other factors have been growing, albeit at different rates. The net effect of all these variables on wages can be determined if the parameters of the model are estimated, and data on v_1^* , v_2^* , v_3^* and v_m^* are collected.

No such data, however, are available at present. A lot more information needs to be compiled on returns to specific factors, their rate of growth, and other variables before the theoretical results of the model can be properly verified. Pending such empirical work, these results can be treated as simply testable hypotheses.

Some suggestive empirical evidence is provided by data on net payments to cocoa farmers. From NC 78.8 million in 1956-57, the payments declined to NC 71.0 million in 1959-60, increased in 1960-61 to NC 96.5 million, but declined again to NC 84.9 million in 1963-64. There was a bumper crop in 1964-65, so the net payments reached a record NC 115.3 million, although the producer price remained unchanged. During the next two years, however, the payments dropped to NC 61.1 and 63.6 million, respectively. 23 Estimating the return to specific factor in cocoa from these and other data is a formidable task in itself and lies outside the scope of this paper.

Conclusion:

We have presented in this paper a theoretical structure for analyzing the income distribution effects of several policies of economic development which are generally followed in a developing country. With Ghana as the backdrop, the structure of the economy is described by a model with four factors and three sectors. The distributional effects of various policies depend on structural parameters such as elasticities of substitution, factor shares, etc. Although data problems preclude precise estimation, we have shown that the theoretical results derived here are consistent with some of the recent economic developments in Ghana. It should be noted, however, that the model is not a complete model of income distribution, but rather a theoretical framework for discussing how economic policies, although not specifically aimed at changing the distribution of income, have distributional effects.

References

- Birmingham, Walter, Neustadt, I., and Omaboe, E. N., A Study of Contemporary Ghana, Vol. I, Evanston: Northwestern University Press, 1966.
- Cocoa Statistics, Gill and Duffus Group, England.
- Harberger, Arnold C., "The Incidence of the Corporation Income Tax," <u>Journal</u> of <u>Political Economy</u>, LXX, No. 3, June 1962, 215-40.
- Johnson, Harry G., The Two-Sector Model of General Equilibrium, London: George Allen and Unwin, 1971.
- International Monetary Fund, <u>International Financial Statistics</u>, 1971
 Supplement.
- Jones, Ronald W., "The Structure of Simple General Equilibrium Models,"

 Journal of Political Economy, LXXIII, No. 6, December 1965, 557-72.
- , "A Three-Factor Model in Theory, Trade, and History," in Bhagwati, J., Jones, R. W., Mundell, R. A., and Vanek, J. (eds.), Trade, Balance of Payments, and Growth, North-Holland, 1971.
- Killick, Tony, "Development Economics in Action: A Study of Economic Policies in Ghana," mimeographed (1973).
- Leith, J. Clark, Exchange Control, Liberalization and Development: Ghana, North-Holland for the National Bureau of Economic Research, 1973 (forthcoming).
- Mieszkowski, Peter M., "On the Theory of Tax Incidence," <u>Journal of Political</u> <u>Economy</u>, LXXIII, 1965, 250-62.
- Wehner, H. G., Jr., "The Cocoa Marketing Board and Economic Development in Ghana: A Case Study," unpublished Ph.D. dissertation, University of Michigan, 1964.
- Zarembka, Paul, <u>Toward a Theory of Economic Development</u>, San Francisco: Holden-Day, 1972.

Footnotes

These distributional goals usually refer to the size distribution of income which is related to the functional distribution. The relationship between the two distributions is not discussed in this paper although it can be expected, for example, that any policy which increases the relative share of wages will also lead to a more equal size distribution because wages are generally a larger fraction of income at the lower levels.

 2 For a detailed application of this model to the theory of distribution, see Johnson (1971).

³In the two-by-two model, for example, with two factors which are mobile between sectors, the competitive profit relationships are:

$$a_{11}^{R_1} + a_{21}^{R_2} = p_1$$

$$a_{21}^{R_1} + a_{22}^{R_2} = p_2$$

The R's thus depend directly on the p's. This is the well-known factor-price equalization theorem.

These and other equations are derived in Appendix A.

The expression $\frac{\sigma_i}{\theta_{ii}}$ occurs again and again and plays a key role in these solutions. Jones (1971) calls it the elasticity of the marginal product of the mobile factor in the ith sector. The marginal product of V_m in industry

i is
$$\frac{R_m}{P_i}$$
. Elasticity

e = percent change in inputs
percent change in marginal products

$$= \frac{a_{11}^{*} - a_{m1}^{*}}{R_{m}^{*} - P_{1}^{*}}$$

But $p_i^* = (\theta_{ii} R_i^* - \theta_{ii} R_m^*) + (\theta_{mi} + \theta_{ii}) R_m^*$.

Substituting for p_1^* in the above expression, and using the definition of σ_i ,

we get

$$\sigma_i/\theta_i = e_i$$

⁶ See, for example, Leith (1973), Tables 2.1, 3.2, and 5.7, and the data sources cited there.

⁷ Leith (1973), Table 3.2.

⁸In Ghana, most cocoa land is suitable for farming, but not all farmland can be used for growing cocoa. Farmland which is unsuited for cocoa is obviously specific to farming, but even better quality land will not be transferred to cocoa cultivation immediately in response to say, an increase in the price of cocoa. In the Ghanaian case, therefore, we can either define the farm sector to include only that land which is unsuitable for cocoa cultivation, or define short run by a period of a year or less which is too short for moving land out of farming into cocoa.

 $_{
m For\ some\ analysis\ based\ on\ the\ 1960\ Census,\ see\ Birmingham,\ {\rm \underline{et\ al.}}$ (1966), Chapter 6.

 $^{10}{
m This}$ is based on the very informative discussion in Birmingham et al., Chapter 10.

11 Birmingham, Newstadt, and Omaboe (1966), p. 66.

 12 For a detailed discussion of the structure and policies of the Cocoa Marketing Board, see Wehner (1964).

 13 We assume here that either the subsidies are financed by lump sum taxes, or the subsidies vector sums to zero. Neither assumption undermines the generality of the conclusions derived here.

$$1^{4}R_{m}^{*} > R_{2}^{*}$$
, but the signs of $(R_{m}^{*} - R_{3}^{*})$ and $(R_{2}^{*} - R_{3}^{*})$ will be ambiguous.

15 So far as the production side of the model is concerned, this case is formally equivalent to the production subsidy case discussed above. Commodity taxes and subsidies drive a wedge between the consumer and producer prices, whereas a change in commodity prices, as a result of, say, commercial policy, introduces no such distortion. The two policies, although equivalent on the production side, will have very different welfare implications.

¹⁶The expression for $(R_m^* - R_2^*)$, not derived in this paper, is

$$\frac{1}{\Delta \theta_{22}} \left[(\lambda_{m1} v_1^* + \lambda_{m2} v_2^* + \lambda_{m3} v_3^* - v_m^*) + \lambda_{m3} \frac{\sigma_3}{\theta_{33}} (p_3^* - p_2^*) + \lambda_{m1} \frac{\sigma_1}{\theta_{11}} (p_1^* - p_2^*) \right]$$

¹⁷Thus we cannot decide <u>a priori</u> the signs of $(R_m^* - R_3^*)$, $(R_1^* - R_3^*)$, and $(R_2^* - R_3^*)$. This indeterminacy is inherent to the three-commodity framework. For example, if we assume $p_1^* > p_2^* > p_3^*$, we can derive $R_1^* > R_m^* > R_3^*$, but effects on R_2 will be ambiguous.

- 18 For simplicity we assume that tax revenues just equal the subsidy payments.
- ¹⁹One difference between the taxed and untaxed sectors is that some units of the specific factor in the former will be left unused to prevent an excess supply of its output. Other specific factors will be fully employed. Since commodity prices are exogenous in this model, excess supply in the goods market can be eliminated by changing factor use.
 - 20 Cocoa Statistics, December 1972.
- The index of market price of locally produced food is reported in Ghana, Statistical Year Book, 1963 (p. 110), and 1965-66 (p. 144).
 - ²²IMF (1971), pp. 90, 91.
- 23 Based on <u>Cocoa Marketing Reports</u>, various issues. These figures must be interpreted with care. Firstly, these are total payments, not payments per farm; and secondly, they represent cash sales, not net payments to the fixed factor V_3 . Also, such payments will be significantly affected, especially in the short run, by factors such as weather.

Appendix

Derivation of (4.2):

Totally differentiating (4.1') we get:

$$\frac{a_{m1}}{a_{11}} \frac{dV_1}{V_m} + \frac{V_1}{V_m} \frac{d}{a_{11}} + \frac{a_{m2}}{a_{22}} \frac{dV_2}{V_m} + \frac{V_2}{V_m} \frac{d}{a_{22}} + \frac{a_{m3}}{a_{33}} \frac{dV_3}{V_m} + \frac{V_3}{V_m} \frac{d}{a_{33}} = \frac{dV_m}{V_m}$$

$$(4.2')$$

$$\sigma_1 = \frac{d\left(\frac{a_{m1}}{a_{11}}\right) \left(\frac{a_{m1}}{a_{11}}\right)}{d(R_1/R_m)/(R_1/R_m)} > 0$$

$$\therefore d(a_{m1}/a_{11}) = \sigma_1 (R_1^* - R_m^*) (a_{m1}/a_{11})$$

Take the first two terms on the L.H.S. in (4.2'). Substitute for $d(a_{m1}/a_{11})$ and multiply and divide by X_1 . We have:

$$\frac{a_{m1}}{a_{11}} \frac{X_1}{X_1} \frac{dV_1}{V_m} + \frac{V_1}{V_m} \sigma_1 (R_1^* - R_m^*) \frac{a_{m1}}{a_{11}} \frac{X_1}{X_1}$$

But $a_{11} X_1 = V_1$, so we can write:

$$\lambda_{m1} V_1^* + \lambda_{m1} \sigma_1 (R_1^* - R_m^*)$$

Similar manipulation of other terms yields (4.2).

Derivation of (5.2):

Totally differentiate (5.1). We get:

$$a_{11} dR_1 + R_1 da_{11} + a_{m1} dR_m + R_m da_{m1} = dp_1$$
 (5.1')

Divide each term by $p_1 X_1$, and multiply by X_1 .

The first two terms will be:

$$\frac{a_{11} X_1}{p_1 X_1} dR_1 + \frac{R_1 X_1 d a_{11}}{p_1 X_1}$$

Multiply and divide the first term by R_1 and the second by a_{11} .

$$\frac{a_{11} X_{1}}{p_{1} X_{1}} R_{1} \frac{dR_{1}}{R_{1}} + \frac{a_{11} X_{1} R_{1}}{p_{1} X_{1}} \frac{da_{11}}{a_{11}}$$

$$= \theta_{11} R_1^* + \theta_{11} a_{11}^*.$$

Thus we can rewrite (5.1') as (5.2').

$$\theta_{11}/R_1^* + \theta_{11} a_{11}^* + \theta_{m1} R_m^* + \theta_{m1} a_{m1}^* = p_1^*.$$
 (5.2')

But under perfect competition, for any given level of output, firms minimize average cost while taking factor prices as given. Thus (5.1') reduces to

$$R_1 da_{11} + R_m da_{m1} = 0$$

which in turn leads to

$$\theta_{11} a_{11}^* + \theta_{m1} a_{m1}^* = 0.$$
 (5.3')

By substituting (5.3') into (5.2'), we derive (5.2) in the text.