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This paper contains preliminary findings from research work still in progress and should not be quoted without prior approval of the author.

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This paper has benefitted from the comments of participants in an IMSSS workshop. In addition, this research was conducted in part while I was a visitor at the University of California at Santa Barbara (on leave from Carnegie-Mellon University), and in part while I was a consultant for the Federal Reserve Bank of Minneapolis. I would like to thank those institutions for their support.

Introduction

This paper is related to topics that are discussed in three different sets of literature. The first of these is the literature on the relation between economies that display stationary sunspot equilibria and economies that display equilibria with 2 period, deterministic cycles (e.g., Azariadis-Guesnerie (1987) or Grandmont (1986)). The second is the literature discussing "legal restrictions" on asset trades, and the kinds of monetary equilibria that can arise when such legal restrictions are imposed (e.g., Bryant-Wallace (1984)). The third is the literature on whether it is desirable to "separate" money from credit markets via legal restrictions in order to prevent "excessive" fluctuations in the price level, or even price level indeterminacy (e.g., Friedman (1960), Sargent-Wallace (1982), Smith (1987)).

The objective of the paper is to demonstrate that legal restrictions meant to prevent private agents from creating "close money substitutes", of a form that is commonly observed in practice, can substantially enhance the scope for stationary sunspot equilibria to exist. Or, phrased differently, the paper demonstrates that economies exist in which there are no stationary sunspot equilibria absent legal restrictions. However, when legal restrictions intended to separate money from credit markets are imposed, stationary sunspot equilibria can be constructed if the legal restrictions take the proper form.

In some sense such a result should not be surprising, since legal restrictions on asset trades are simply a device for limiting the participation of economic agents in different markets. It is this type of limited market participation that permits sunspot equilibria to arise, as emphasized by Cass and Shell (1983). However, the result still seems to be of interest, for several reasons.

Consider first the literature on when stationary sunspot equilibria (evolving according to a two-state Markov chain) can exist in overlapping generations models. When no legal restrictions are present such equilibria exist only if the aggregate savings function is decreasing in the gross rate of interest (at the monetary steady state), as shown by Azariadis (1981). On empirical grounds this might be viewed as limiting the appeal of such equilibria. Moreover, stationary sunspot equilibria exist (if and) only if an economy has an equilibrium with a 2 period cycle (see Azariadis-Guesnerie (1987) or Grandmont (1986)), which might or might not be viewed as an attractive feature of these equilibria.

This paper demonstrates that when a legal restriction is imposed that prevents private agents from purchasing debt obligations in amounts below some minimum amount, specified in dollar terms,[1] two state stationary sunspot equilibria exist in economies that otherwise have no stationary sunspot equilibria. Moreover, such equilibria can be constructed even though all agents have savings functions that are non-decreasing in the rate of interest, and even though aggregate savings increase when interest rates increase. Finally, stationary sunspot equilibria exist even though the economies in question may have <u>no</u> equilibria displaying 2 period cycles.[2]

A second reason why the construction here is of interest relates to "legal restrictions" theories of money. For instance, Bryant and Wallace (1984) show that legal restrictions shielding money from competition with higher yielding assets can enhance government revenue from debt issues, and hence may be part of an optimal tax scheme. However, as demonstrated here, some legal restrictions that shield money from competition with assets that dominate it in rate of return will result in the existence of sunspot equilibria, as well as deterministic equilibria of the form that Bryant and Wallace consider. Hence such legal restrictions need not have exactly their intended effects.[3]

Third, the result that legal restrictions meant to separate money from credit markets (that is, meant to prevent private agents from creating close money substitutes) can cause sunspot equilibria to exist is very relevant to an old debate in monetary economics. In particular, economists of what Sargent and Wallace (1982) term the "quantity theory school" have long argued that a failure to prevent private agents from issuing close money substitutes would result in "excessive" fluctuations in the price level and the stock of inside money, and perhaps in an indeterminate price level.

One example of a legal restriction meant to prevent the creation of too close a substitute for money is the prevention of banks from issuing liabilities bearing competitive interest rates in amounts less than $100,000. This paper examines the consequences of exactly this legal restriction in a model that, absent legal restrictions, has no stationary sunspot equilibria. It is shown that, if the minimum denomination is selected appropriately, stationary sunspot equilibria will exist when the legal restriction is imposed. In this instance, then, following the quantity theory proposal with respect to separating money from credit markets opens the economy to "excessive" fluctuations in the price level and the stock of inside money, as well as to price level indeterminacy (even in stationary equilibria).[4]

The scheme of the paper is as follows. Section I lays out the economic environment to be considered, and examines its equilibria in the absence of legal restrictions. Section II introduces a legal restriction that prevents agents from acquiring assets, other than money, in dollar denominations less than some minimum amount. Deterministic equilibria are then examined, and conditions that rule out the existence of 2 period cycles are derived. Section III constructs stationary sunspot equilibria, and demonstrates that conditions sufficient to rule out 2 period cycles are also sufficient to guarantee the existence of stationary sunspot equilibria. Section IV concludes. An appendix demonstrates that it is important for legal restrictions to take the form of a minimum nominal amount in which assets (other than money) can be acquired.

I. The Model without Legal Restrictions

We consider a world inhabited by a sequence of 2 period lived, overlapping generations, plus an initial old generation. Time is discrete, and indexed by $t=0,1,\ldots$ We focus on a pure exchange environment in which there is a single non-storable consumption good at each date.

Each young generation is identical in size and composition, and consists of two types of agents. Type 1 agents, who will be referred to as savers, constitute a fraction λ of each young generation, while type 2 agents, called borrowers, are a fraction $1-\lambda$ of each generation.

All young savers are identical at each date. Letting c_i $(i=1,2)$ denote age i consumption, savers have preferences given by the utility function $\ell n\ c_1 + \ell n\ c_2$, and an endowment stream $(y,0)$; that is, savers are endowed with $y > 0$ units of the single good when young, and zero units when old. Similarly, young borrowers are all identical. Their preferences are given by the utility function c_1 (i.e., borrowers care only about young period consumption), and borrowers have the endowment stream $(0,w)$, $w > 0$.[5]

The nature of trading is as follows. The initial old agents are endowed with a fixed stock of fiat money. Let $M = 1$ be the (constant) per capita stock of money. Then at each date old agents can buy goods with money, and young agents can engage in borrowing and lending. As is conventional in the sunspot equilibrium literature, there are no markets in state contingent claims.

Notice that nothing in the environment of this economy is stochastic. Nevertheless, it is assumed that there are 2 possible states of nature that can occur at each date. Let e_t denote the current period state at t. Then $e_t \in \{1,2\}$. Moreover, it is assumed that the state evolves according to a stationary Markov chain denoted as follows: let $\text{prob.}[e_{t+1}=1:e_t=e] = q(e)$; $e = 1,2$.

Having described the environment, it is now possible to lay out the notation employed. Under the assumptions above, young savers can either sell goods in exchange for currency, or lend to young borrowers. Let p_t denote the price level at t, let z_t denote the per capita accumulation of real balances by young savers at t, and let x_t be the amount lent by a representative young saver at t. One unit lent at t repays R_t units at t+1; i.e., R_t is the gross (real) rate of interest. Also, let L_t be the amount borrowed by each borrower at t. Finally, when we wish to focus on sunspot equilibria we will let these variables depend on the current period state and use the following notation: $p_t(e)$ will be the time t price level if the time t state is e, etc.

Equilibrium

Young savers at t solve the problem[6]

$$\max \ \ln(y - z_t - x_t) + \ln[R_t x_t + (p_t/p_{t+1})z_t]$$

subject to $z_t \geq 0$ and $z_t + x_t \leq y$. This problem gives rise to the savings function $x_t + z_t = f(R_t) = y/2$ and to the condition $p_t/p_{t+1} = R_t$ if money is to have value. (Clearly there will be positive loan demand at any finite value of R.) It is assumed that borrowers are never allowed to default on loans. Then loan demand of a young borrower at t is simply w/R_t.

It will be convenient to think of things in the following way: each young saver either holds money or makes loans, but does not do both. While clearly it is not necessary to think in this way, it is innocuous to do so, and this convention facilitates a comparison of equilibria with and without legal restrictions. Then let μ_t denote the fraction of savers at t who hold money, while fraction $1-\mu_t$ of savers make loans.

The equilibrium conditions for this economy are now as follows. For the fraction μ_t of savers who acquire money at t, $z_t = y/2$. Savers are a fraction λ of the population, and the per capita money supply is one, so money market clearing requires that

(1) $\lambda\mu_t(y/2) = 1/p_t; \; t \geq 0$

Similarly, a fraction $(1-\mu_t)$ of savers make loans, and the desired lending of these agents is $x_t = y/2$. Savers are a fraction λ of the population, while borrowers are a fraction 1-λ. The loan demand of borrowers is w/R_t, so loan market clearing requires that

(2) $\lambda(1-\mu_t)(y/2) = (1-\lambda)\,(w/R_t); \; t \geq 0.$

Finally, savers must be indifferent between making loans and holding money, so that

(3) $R_t = \dfrac{p_t}{p_{t+1}}; \; t \geq 0.$[7]

Collapsing (1)-(3) into a single equation yields the equilibrium law of motion for μ_t:

(4) $\mu_{t+1} = 2(1-\lambda)w\mu_t/\lambda y(1-\mu_t); \; t \geq 0$

There are 2 steady state equilibrium values for μ; $\mu_t = 0 \; \forall \; t$ and $\mu_t = \hat{\mu} \equiv 1 - 2(\frac{1-\lambda}{\lambda})(\frac{w}{y}) \; \forall \; t$. Also

(5) $$\frac{d\mu_{t+1}}{d\mu_t} = 2(\frac{1-\lambda}{\lambda})(\frac{w}{y})(\frac{1}{1-\mu_t})^2$$

so that

(6) $$\left.\frac{d\mu_{t+1}}{d\mu_t}\right|_{\mu_t=\hat{\mu}} = \frac{1}{2(\frac{1-\lambda}{\lambda})(\frac{w}{y})} > 1,$$

by the assumption of footnote 6. Then the steady state equilibrium with $p < \infty$ is locally unstable.

It is probably apparent that this economy has no stationary sunspot equilibrium. However, since this heterogeneous agent, multiple asset environment does not fit into existing discussions of when stationary sunspot equilibria exist, appendix A demonstrates formally that this economy does not possess a stationary sunspot equilibrium.

II. A Legal Restrictions Regime[8]

The following legal restriction on asset trades is now imposed on the economy of section I: loans can be made only in dollar denominations greater than or equal to $\$\bar{x}$. This restriction is meant to prevent private agents from issuing liabilities that are too close a substitute for money, and is intended to resemble restrictions such as minimum denominations on certificates of deposit, for instance. Finally, to emphasize the nature of the restriction, this is viewed as a legal restriction placed on the portfolio of savers.

In all other respects the economy is identical to that of section I. Clearly, then, the description of borrowers' behavior above does not require modification. The description of savers' behavior must be modified as follows. If $\bar{x}/p_t \leq y/2$, then the restriction does not bind on the decisions of savers, and their behavior is as discussed above. If $\bar{x}/p_t > y/2$, however, the legal restriction is binding. Then no saver will voluntarily hold money and make loans; rather some savers (a fraction μ_t of savers) hold money, while others (a fraction $1-\mu_t$) make loans in real amount $\bar{x}/p_t$. As above, R denotes the gross (real) rate of interest on loans.[9]

Suppose $\bar{x}/p_t > y/2$ at t, then. In this case a fraction μ_t of savers (who are a fraction λ of the population) acquire real balances in amount y/2 per person, and a fraction $1-\mu_t$ of savers make loans at t with a real value of $\bar{x}/p_t$. As above, per capita loan demand is $(1-\lambda)w/R_t$ at t. Then money market clearing requires that

(7) $\lambda\mu_t(y/2) = p_t^{-1}$; $t \geq 0$,

while loan market clearing requires

(8) $\lambda(1-\mu_t)(\bar{x}/p_t) = (1-\lambda)w/R_t$; $t \geq 0$.

Finally, since savers can voluntarily choose to make loans or hold money, but cannot do both, any saver must be indifferent between the two. A saver who holds money has $z_t = y/2$, $c_1 = y - z_t = y/2$, and $c_2 = (p_t/p_{t+1})z_t = (p_t/p_{t+1})(y/2)$. These decisions yield the utility level $\ln[(p_t/p_{t+1})(y/2)^2]$. On the other hand, a saver who makes loans has $c_1 = y-(\bar{x}/p_t)$ and $c_2 = R_t(\bar{x}/p_t)$. This results in utility $\ln[y - (\bar{x}/p_t)] + \ln[R_t(\bar{x}/p_t)]$. Then, since savers must be indifferent between making loans and holding money,

$$(9) \quad \ln[(p_t/p_{t+1})(y/2)^2] = \ln[y-(\bar{x}/p_t)] + \ln(R_t\bar{x}/p_t)$$

must hold $\forall$ t if money is to have value.

An equilibrium is a sequence of prices $\{p_t\}_{t=0}^{\infty}$, interest rates $\{R_t\}_{t=0}^{\infty}$, and values of μ_t, $\{\mu_t\}_{t=0}^{\infty}$, that satisfy (1)-(3) for all t such that $\bar{x}/p_t \leq y/2$, and that satisfy (7)-(9) for all t such that $\bar{x}/p_t > y/2$.

<u>Equilibrium</u>

Equations (7)-(9) can be collapsed into the single condition

$$\ln[(\mu_{t+1}/\mu_t)(y/2)^2] = \ln[y - \bar{x}\lambda(y/2)\mu_t] + \ln[(\frac{1-\lambda}{\lambda})(\frac{w}{1-\mu_t})]$$

which in turn can be written as

$$(10) \quad \mu_{t+1} = \phi\mu_t[\frac{1 - \bar{x}(\lambda/2)\mu_t}{1 - \mu_t}],$$

where $\phi \equiv 4(\frac{1-\lambda}{\lambda})(\frac{w}{y})$. (10) governs the equilibrium law of motion for μ if $\bar{x}/p_t > y/2$. Since p_t is given by (7), $\bar{x}/p_t > y/2$ is equivalent to the condition

(11) $\bar{x}\lambda\mu_t > 1$.

If (11) holds the legal restriction is binding. If (11) fails to hold then $\bar{x}/p_t \leq y/2$, and the equilibrium law of motion for μ is given by equation (4). Finally, if $\bar{x}/p_t > y$, then clearly it is not feasible for any saver to make loans. Thus the above conditions can be satisfied only if $\bar{x}/p_t \leq y$, which in light of (7) is equivalent to the condition

(12) $\bar{x}\lambda\mu_t \leq 2;\ t \geq 0$.

Equilibria which satisfy (12) are the focus of discussion here.

Define the function $H(\mu)$ by

$$H(\mu) \equiv \frac{1 - \bar{x}(\lambda/2)\mu}{1 - \mu}$$

and the function $G(\mu)$ by $G(\mu) \equiv \mu H(\mu)$. Finally, define the function $T(\mu)$ by

$$(13)\quad T(\mu) \equiv \begin{cases} 2(\frac{1-\lambda}{\lambda})(\frac{w}{y})(\frac{\mu}{1-\mu});\ \bar{x}\lambda\mu \leq 1 \\ \phi G(\mu);\ 2 \geq \bar{x}\lambda\mu > 1 \end{cases}$$

Then it is easy to check that $T(\mu)$ is a continuous function (and in particular is single valued) on $[0, 2(\bar{x}\lambda)^{-1}]$, and that $T(\mu)$ is continuously differentiable on $(0, 2(\bar{x}\lambda)^{-1})$ except at $\mu = (\bar{x}\lambda)^{-1}$. Moreover, the equilibrium law of motion for μ is given by

(14) $\mu_{t+1} = T(\mu_t)$; $t \geq 0$.

Steady State Equilibria

Suppose that there is a steady state equilibrium level of μ, μ^*, such that the legal restriction is binding; i.e., such that $\bar{x}\lambda\mu^* > 1$. Then, solving (10) for μ^* yields

(15) $$\mu^* = \frac{\phi - 1}{\phi\bar{x}(\lambda/2) - 1}.$$

In Section III, it will be shown that stationary sunspot equilibria exist only if $\bar{x}(\lambda/2) > 1$ holds. This condition is henceforth assumed. Then, given that $\bar{x}(\lambda/2) > 1$, μ^* satifies $0 < \mu^* < 1$ iff $\phi > 1$, so that $\phi > 1$ is henceforth assumed.[10] (The assumption of footnote 6 implies that $\phi < 2$.) Finally, in the construction of μ^*, it was assumed that $\bar{x}\lambda\mu^* > 1$. Using (15), this condition is satisfied if

(16) $$\frac{1}{2-\phi} > \bar{x}(\lambda/2).$$

If $\phi > 1$ and $(2-\phi)^{-1} > \bar{x}(\lambda/2) > 1$ hold, then, there is a steady state equilibrium in which the legal restriction is binding. Also, $\mu_t = 0$

for all t continues to constitute a steady state equilibrium. Finally, under the parameter restrictions imposed, $\mu_t = \hat{\mu}\ \forall\ t$ (with $\hat{\mu}$ given by $\hat{\mu} = 1 - (\phi/2)$) is also a steady state equilibrium; i.e., there is a steady state equilibrium in which the legal restriction does not bind. To see this let $\hat{p}^{-1} = \lambda\hat{\mu}(y/2)$ (i.e., $\hat{p}$ is the steady state equilibrium price level from section I). Then

$$(\bar{x}/\hat{p}) = \bar{x}\lambda\hat{\mu}(y/2) = \bar{x}\lambda(y/2)(1 - \frac{\phi}{2}) < \frac{y}{2}$$

by (16). In short, the steady state equilibrium of section I continues to be an equilibrium.

Dynamics

In the region where the legal restriction binds (which is known to exist if (16) holds), we have from (13) that

$$\frac{d\mu_{t+1}}{d\mu_t} = T'(\mu_t) = \phi G'(\mu_t) = \phi H(\mu_t) + \phi\mu_t H'(\mu_t). \tag{17}$$[11]

From (13), μ^* satisfies $\mu^* = \phi G(\mu^*) \equiv \phi\mu^* H(\mu^*)$, so $\phi H(\mu^*) = 1$. In addition,

$$H'(\mu) = \frac{1 - \bar{x}(\lambda/2)}{(1-\mu)^2} < 0$$

when $\bar{x}(\lambda/2) > 1$. It follows, then, that

$$\left.\frac{d\mu_{t+1}}{d\mu_t}\right|_{\mu^*} = \phi H(\mu^*) + \phi\mu^* H'(\mu^*) < 1.$$

Therefore, a sufficient condition for the local stability of this steady state equilibrium is that $G'(\mu^*) > 0$.

From (17), $G'(\mu^*) > 0$ is equivalent to the condition $H(\mu^*) > -\mu^* H'(\mu^*)$. It is straightforward but tedious to verify that $H(\mu^*) > -\mu^* H'(\mu^*)$ if $\bar{x}(\lambda/2) > 1/\phi(2 - \phi) \geq 1$. Thus a locally stable steady state equilibrium exists if

$$(18) \quad \frac{1}{2-\phi} > \bar{x}(\lambda/2) > \frac{1}{\phi(2-\phi)},$$

where the first inequality in (18) is simply condition (16), and where it will be recalled that $\phi > 1$.

<u>Non-existence of 2 period cycles</u>

It is now demonstrated that, if the legal restriction $\bar{x}$ satisfies (18), there are no 2 period cycles for this economy. This is shown in two steps. First, it is shown that satisfaction of (18) implies that $T(\mu)$ is a nondecreasing function for $\mu \epsilon [0,\mu^*]$. Then it will be apparent that this economy has no 2 period cycle.

<u>Lemma</u>. If (18) holds then $T(\mu)$ is non-decreasing for $\mu \epsilon [0,\mu^*]$, where μ^* is the steady state equilibrium value given by equation (15).

<u>Proof</u>. Clearly for $\mu \epsilon [0,(\bar{x}\lambda)^{-1}]$, $T(\mu)$ is an increasing function. For $\mu \epsilon ((\bar{x}\lambda)^{-1}, 2(\bar{x}\lambda)^{-1})$, $T'(\mu)$ exists and is a continuous function of μ. Then if $T'(\mu) \geq 0$ for all $\mu \epsilon ((\bar{x}\lambda)^{-1}, \mu^*]$ the lemma is proved.

If (18) holds, then $T'(\mu^*) = \phi G'(\mu^*) > 0$. Moreover,

$$(19) \quad \lim_{\substack{\epsilon \to 0 \\ \epsilon > 0}} T'[(\bar{x}\lambda)^{-1} + \epsilon] = \left(\frac{\phi}{2}\right) \frac{\bar{x}\lambda}{(\bar{x}\lambda - 1)^{2}} > 0.$$

Then suppose that $T'(\mu) < 0$ for some value of $\mu\epsilon((\bar{x}\lambda)^{-1},\mu^*)$. Since $T'(\mu)$ is continuous on this interval, and since (19) and $T'(\mu^*) > 0$ hold, $T'(\mu) < 0$ for some $\mu\epsilon((\bar{x}\lambda)^{-1}, \mu^*)$ implies that there exist at least 2 distinct values, say μ_1 and μ_2, such that $\mu_1, \mu_2\epsilon((\bar{x}\lambda)^{-1},\mu^*)$, and $T'(\mu_1) = T'(\mu_2) = 0$ (by the intermediate value theorem).

Now $T'(\mu) = \phi G'(\mu) = \phi H(\mu) + \phi\mu H'(\mu)$ for $\mu\epsilon((\bar{x}\lambda)^{-1},2(\bar{x}\lambda)^{-1})$.

Then $T'(\mu) = 0$ exactly when $H(\mu) = -\mu H'(\mu)$, or when

$$(20) \quad \frac{1 - \bar{x}(\lambda/2)\mu}{1 - \mu} = \frac{\mu[\bar{x}(\lambda/2) - 1]}{(1 - \mu)^2}$$

Solving (20) for μ yields

$$\mu = 1 \pm [1 - (2/\bar{x}\lambda)]^{½}.$$

But $2/\bar{x}\lambda < 1$, so (20) has only one solution in the unit interval. This, in turn, contradicts the existence of distinct values μ_1 and μ_2 satisfying $T'(\mu_1) = T'(\mu_2) = 0$. Thus assuming that $T'(\mu) < 0$ for some $\mu\epsilon((\bar{x}\lambda)^{-1},\mu^*)$ leads to a contradiction, so $T(\mu)$ is non-decreasing on $[0,\mu^*]$.

If (18) is satisfied, then, the equilibrium law of motion $\mu_{t+1} = T(\mu_t)$ looks as depicted in Figure 1. While it is probably apparent from inspection that there is no equilibrium 2 period cycle for this economy, we state this as a proposition which is proved in Appendix B.

Proposition. There is no equilibrium displaying a 2 period cycle; i.e., there are no values μ_1 and μ_2 satisfying $\mu_1 \neq \mu_2$ and $\mu_2 = T(\mu_1)$, $\mu_1 = T(\mu_2)$.

III. Stationary Sunspot Equilibria

We now construct stationary sunspot equilibria in which the legal restriction binds in each possible state; i.e., in which $\bar{x}/p(e) > y/2$; $e = 1,2$. This equilibrium is constructed under the assumption that any individual saver either accumulates money or makes loans, but does not do both. After the equilibrium is constructed it is then verified that no agent wishes to make loans and hold money simultaneously.

Suppose each individual saver either holds money or makes loans, but does not do both. Then young savers who are born in state e will accumulate real balances equal to $z(e) = y/2$; $e = 1,2$ if they accumulate money, or will make loans with a real value of $\bar{x}/p(e) > y/2$. Let $\mu(e)$ denote the fraction of savers who hold money in state e. Then money market clearing requires that

$$(21) \quad \lambda\mu(e)(y/2) = \frac{1}{p(e)}; \ e = 1,2.$$

Since per capita loan demand is $(1-\lambda)w/R(e)$ in state e, loan market clearing requires that

$$(22) \quad \lambda[1-\mu(e)][\frac{\bar{x}}{p(e)}] = (1-\lambda)\ \frac{w}{R(e)}; \ e = 1,2.$$

Finally, as before, savers must be indifferent between making loans and holding money. A young saver who lends $\bar{x}/p(e)$ when young receives

$R(e)\bar{x}/p(e)$ when old (recall that loans are indexed, so that there is no uncertainty about loan repayments), and obtains (expected) utility

$\ln[y - \bar{x}/p(e)] + \ln[R(e)\bar{x}/p(e)]$. A young saver who accumulates real balances in amount $z(e)$ receives $p(e)z(e)/p(1)$ when old with probability $q(e)$, and $p(e)z(e)/p(2)$ with probability $1-q(e)$. Then for young savers to be indifferent between making loans and holding money,

$$(23) \quad \ln(y/2) + q(e)\ln[(y/2)(\frac{p(e)}{p(1)})] + [1-q(e)]\ln[(y/2)(\frac{p(e)}{p(2)})] =$$

$$\ln[y-\bar{x}/p(e)] + \ln[R(e)\bar{x}/p(e)]; \; e = 1,2$$

must hold.

A stationary sunspot equilibrium (with the legal restriction binding in each state) is a set of values $\mu(1)$, $\mu(2)$, $p(1)$, $p(2)$, $R(1)$, $R(2)$, $q(1)$ and $q(2)$ satisfying (21)-(23), $0 < q(e) < 1$; $e = 1,2$, and

$$(24) \quad 1 < \bar{x}\lambda\mu(e) < 2; \; e = 1,2.$$

(As above, $\bar{x}\lambda\mu(e) > 1$ implies that $\bar{x}/p(e) > y/2$, and $\bar{x}\lambda\mu(e) < 2$ implies that $\bar{x}/p(e) < y$.)

The primary result of this section is the following theorem.

<u>Theorem</u>. Suppose that (18) is satisfied, so that

$$\frac{1}{2-\phi} > \bar{x}(\lambda/2) > \frac{1}{\phi(2-\phi)}.$$

Then for any values $\mu(1)$ and $\mu(2)$ in an appropriately chosen open neighborhood of μ^*, with $\mu(2) > \mu^* > \mu(1)$, a stationary sunspot equilibrium exists with $p(1)$ and $p(2)$ given by (21), $R(1)$ and $R(2)$ given by (22), and with $q(1)$ and $q(2)$ given by

$$(25)\quad q(1) = \frac{\ell n\{\phi H[\mu(1)][\frac{\mu(1)}{\mu(2)}]\}}{\ell n[\frac{\mu(1)}{\mu(2)}]}$$

and

$$(26)\quad q(2) = \frac{\ell n\{\phi H[\mu(2)]\}}{\ell n[\frac{\mu(1)}{\mu(2)}]}$$

Remarks. Equations (25) and (26) are obtained by solving (23) for q(1) and q(2). It will be recalled that

$$H(\mu) \equiv \frac{1 - \bar{x}(\lambda/2)\mu}{1 - \mu}$$

Also, it will be noted that the theorem assets that (18), which is sufficient to rule out the existence of 2 period cycles in equilibrium, is also sufficient to guarantee the existence of stationary sunspot equilibria.

The theorem will be proved by construction. To begin, choose $\mu(1)$ and $\mu(2)$ satisfying $\mu(2) > \mu^* > \mu(1)$, where it will be recalled that μ^* is the steady state equilibrium value of μ when the legal restriction is binding. Since under the parameter restrictions of section II μ^* satisfies $1 < \bar{x}\lambda\mu^* < 2$, $\mu(1)$ and $\mu(2)$ satisfy (24) if they are chosen sufficiently close to μ^*. Thus the legal restriction is binding in each state.

Now select $p(e)^{-1} = \lambda\mu(e)(y/2)$ and $R(e) = (1-\lambda)wp(e)/\lambda[1-\mu(e)]\bar{x}$ for $e = 1,2$. Then, since the values q(1) and q(2) given by (25) and (26) satisfy (23), values constructed in this way satisfy (21)-(23).

It only remains, then, to verify that q(1) and q(2) are probabilities. Since $\mu(1) < \mu(2)$, $0 < q(1) < 1$ iff

(27) $1 > \phi H[\mu(1)][\frac{\mu(1)}{\mu(2)}] > \frac{\mu(1)}{\mu(2)}$,

while $0 < q(2) < 1$ iff

(28) $1 > \phi H[\mu(2)] > \frac{\mu(1)}{\mu(2)}$.

Rewrite (27) as

(29) $\mu(2) > \phi\mu(1)H[\mu(1)] \equiv \phi G[\mu(1)]$

and

(30) $\phi H[\mu(1)] > 1$.

Then it is easy to see that (29) and (30) hold. In particular, μ^* satisfies $\mu^* = \phi G(\mu^*)$, or equivalently, $\phi H(\mu^*) = 1$. Since $\bar{x}\lambda/2 > 1$, $H'(\mu^*) < 0$, so $\mu(1) < \mu^*$ implies that $\phi H[\mu(1)] > \phi H(\mu^*) > 1$ for $\mu(1)$ sufficiently close to μ^*.[12] Thus (30) holds. Similarly, (18) implies that $G'(\mu^*) > 0$. Then (29) holds, since $\mu(2) > \mu^* = \phi G(\mu^*) > \phi G[\mu(1)]$ for $\mu(1)$ selected sufficiently close to μ^*.

To check that (28) is satisfied rewrite it as

(31) $1 > \phi H[\mu(2)]$

and

(32) $\phi\mu(2)H[\mu(2)] \equiv \phi G[\mu(2)] > \mu(1)$.

Then, as above, the fact that $H' < 0$ implies that $1 = \phi H(\mu^*) > \phi H[\mu(2)]$, so (31) is satisfied. Also, since $G'(\mu^*) > 0$, for $\mu(2)$ sufficiently close to μ^*, $\phi G[\mu(2)] > \phi G(\mu^*) = \mu^* > \mu(1)$. Thus (32) is satisfied, so $0 < q(e) < 1$, $e=1,2$.

It remains to verify that no saver has an incentive to simultaneously make loans and acquire money in the equilibrium just constructed. To do so, consider the expected utility of a saver who makes a loan with a real value of $\bar{x}/p(e)$, and acquires real balances in amount $m(e) \geq 0$. This is

$$V(e) \equiv \ell n[y - \frac{\bar{x}}{p(e)} - m(e)] + q(e)\ell n[R(e) \frac{\bar{x}}{p(e)} + \frac{p(e)}{p(1)} m(e)] +$$

$$[1 - q(e)]\ell n[R(e) \frac{\bar{x}}{p(e)} + \frac{p(e)}{p(2)} m(e)], \; e = 1,2$$

where, for given values $\mu(1)$ and $\mu(2)$, $p(e)$ is given by (21), $R(e)$ by (22), and $q(1)$ and $q(2)$ by (25) and (26). Since from the point of view of an agent $q(e)$, $p(e)$, $R(e)$ and $\bar{x}$ are parameters, no person making a loan will acquire positive real balances if

$$(33) \quad \left.\frac{dV(e)}{dm(e)}\right|_{m(e) = 0} < 0; \; e = 1,2.$$

Condition (33) is equivalent to

$$(34) \quad [y - \frac{\bar{x}}{p(e)}]^{-1} > [R(e) \frac{\bar{x}}{p(e)}]^{-1} \{q(e) \frac{p(e)}{p(1)} + [1-q(e)] \frac{p(e)}{p(2)}\}; \; e = 1,2.$$

Now $p(e)$ in (34) is given by (21), $R(e)$ is given by (22), and $q(1)$ and $q(2)$ are given by (25) and (26). Each of these terms is a continuous function of $\mu(1)$ and $\mu(2)$ (in the neighborhood of the theorem), so define the continuous functions $P_e[\mu(1),\mu(2)]$ on the neighborhood of the theorem by

$$P_e[\mu(1),\mu(2)] = [y - \frac{\bar{x}}{p(e)}]^{-1} - [R(e) \frac{\bar{x}}{p(e)}]^{-1} \{q(e) \frac{p(e)}{p(1)} +$$

$$[1 - q(e)] \frac{p(e)}{p(2)}\}; \; e = 1,2.$$

Now

$$P_e(\mu^*,\mu^*) = (y - \frac{\bar{x}}{p^*})^{-1} - (R^*\bar{x}/p^*)^{-1}; \; e = 1,2$$

where $p^* = [\lambda\mu^*(y/2)]^{-1}$ and $R^* = (1-\lambda)wp^*/\lambda\bar{x}(1-\mu^*)$. Then $P_e(\mu^*,\mu^*) > 0$;

since

$$(y - \frac{\bar{x}}{p^*})^{-1} > (\bar{x}/p^*)^{-1} > (R^*\bar{x}/p^*)^{-1}$$

where the first inequality follows from the fact that the legal restriction is binding, and the second from $R^* > 1$.

Since $P_e(\mu^*,\mu^*) > 0$; $e = 1,2$, $P_1[\mu(1),\mu(2)] > 0$ and $P_2[\mu(1),\mu(2)] > 0$ for $\mu(1)$ and $\mu(2)$ sufficiently near μ^*. Then (34) holds for some open subset of the neighborhood of the theorem and savers who make loans will not hold money for $\mu(1)$ and $\mu(2)$ chosen in this neighborhood of μ^*. Redefining the neighborhood of the theorem if necessary, this verifies the validity of the construction above for some appropriate open neighborhood of μ^*.

It is the case, then, that conditions sufficient to rule out 2 period cycles are also sufficient to imply the existence of stationary sunspot equilibria. (18) is not a necessary condition for such equilibria to exist, however, as the following example demonstrates.

<u>Example</u>. Let $y = 3$, $w = 1$, $\lambda = ½$ and $\bar{x} = 25/6$. Then $\bar{x}(\lambda/2) = 25/24$, while $\phi = 4(\frac{1-\lambda}{\lambda})(\frac{w}{y}) = 4/3$. Therefore $[\phi(2-\phi)]^{-1} = 9/8 > 25/24$, so (18) is violated.

This economy has a steady state equilibrium where the legal restriction binds, however. The steady state value of μ is $\mu^* = .857$. This economy also has a stationary sunspot equilibrium with $\mu(1) = .6$, $\mu(2) = .9$, $1/p(1) = .45$, $1/p(2) = .675$, $R(1) = 4/3$, $R(2) = 3.5556$, $q(1) = .449$, and $q(2) = .449$. (It is straightforward to check that these values satisfy (21)-(24). It is also straightforward to check that (33) is satisfied, so that no agent makes loans and holds money simultaneously.)

This economy also has an equilibrium with a 2 period cycle. To see this, compute

$$\left.\frac{d\mu_{t+1}}{d\mu_t}\right|_{\mu_t=.857} = \phi G'(.857) = -1.327.$$

Then consider the iterated map $\mu_{t+2} = T_2(\mu_t) \equiv T[T(\mu_t)]$ drawn in Figure 2. This map is continuous and clearly it intersects the 45° line at $\mu_t = \hat{\mu}$ and $\mu_t = \mu^*$. Moreover, $T_2'(\hat{\mu}) > 1$ and $T_2'(\mu^*) > 1$ both hold, as $T_2'(\hat{\mu}) = [T'(\hat{\mu})]^2 > 1$ and $T_2'(\mu^*) = [T'(\mu^*)]^2 = (1.327)^2$. Then, since $T_2(\mu)$ is continuous, it must cross the 45° line at least once from above, as at $\tilde{\mu}$ in the Figure. Thus there are distinct values $\tilde{\mu}$ and $T(\tilde{\mu})$ which constitute a 2 period cycle. It is not the case, then, that the absence of a 2 period cycle is necessary for the existence of a stationary sunspot equilibrium where the legal restriction binds in each state.

Aggregate Savings

As has been seen, the economy considered here can have a stationary sunspot equilibrium without displaying 2 period cycles in equilibrium. It also avoids the feature that the aggregate savings function is necessarily decreasing in the rate of interest, in almost any way one might choose to define an aggregate savings function here. First, clearly savers save in amounts that are independent of the rate of return (either y/2 or $\bar{x}/p_t$), and borrowers have loan demand functions that are decreasing in R (w/R). Thus fixing the composition of the population and aggregating will not produce an aggregate savings function that is decreasing in the rate of interest.

Second, consider aggregate savings of each generation (which here is equal to real balances) directly. When the legal restriction binds, which is clearly the case of interest, real balances are given by $p_t^{-1} = \lambda\mu_t(y/2)$. From (8)

$$\mu_t = 1 - (1-\lambda)wp_t/\lambda\bar{x}R_t$$

Equating μ_t between the two expressions yields

$$\frac{2}{\lambda y p_t} = 1 - \left(\frac{1-\lambda}{\lambda}\right)\left(\frac{w}{\bar{x}}\right)\left(\frac{p_t}{R_t}\right).$$

Whether $1/p_t$ and R_t are positively or negatively related is obviously ambiguous. However, for the economy of the example, the state e=2 has both high interest rates and high levels of real balances. Thus interest rates and aggregate savings can be positively correlated here.

This kind of situation can arise because the expected return on real balances can be low when interest rates on loans are high, and conversely. This is possible because the legal restriction specifies a minimum <u>dollar</u> denomination on loans. Then a high current period price level reduces the severity of the legal restriction, permitting interest rates on loans to fall even though the expected return on money is high. This argument suggests, in turn, that it is crucial to the results obtained here that legal restrictions state a minimum nominal value on loans, rather than a minimum real value. Appendix C demonstrates formally that, if legal restrictions specify a minimum real loan value, no stationary sunspot equilibrium exists in which an individual saver either holds money or makes loans, but does not do both. This appears to be the only tractable situation to consider here.

IV. Conclusion

In the presence of legal restrictions intended to separate money from credit markets, it is relatively straightforward to produce stationary sunspot equilibria despite the absence of 2 period cycles in equilibrium. It is also straightforward to produce stationary sunspot equilibria even though there is no obvious sense in which aggregate savings are decreasing (anywhere) in the rate of interest. These results may enhance the empirical plausibility of such equilibria.

In addition, these results call into question some traditional claims about why it is desirable to prevent private agents from creating "close" substitutes for money. Such claims, labelled the "quantity theory" by Sargent and Wallace (1982), assert that if money and credit markets are not separated, then the economy is exposed to the possibility of "excessive" fluctuations in the price level (presumably excessive relative to "fundamentals") and the stock of inside money, and possibly to price level indeterminacy.

The results of section III indicate that, contrary to this claim, legal restrictions intended to separate money from credit markets (of a form that is common in practice) can open the possibility of "excessive" price level fluctuations, fluctuations in the stock of inside money (which here equals

$\lambda\mu(e)\bar{x}$, in per capita terms), and price level indeterminacy (stationary sunspot equilibria). Thus, when money and credit markets are separated by legal restrictions specifying minimum dollar denominations on certain kinds of asset acquisitions, quantity theory proposals can be counterproductive in the extreme.

Moreover, legal restrictions of the kind that permit sunspot equilibria to exist here are common, and have been common historically. The requirement that bank certificates of deposit be in denominations of at least $100,000 is one example of such a restriction. In addition, in historical episodes where paper currency and specie co-existed (with specie allegedly dominating paper currency in rate-of-return), it was common for specie to circulate in relatively large denominations while paper currency was issued in smaller denominations (see, e.g., Hanson (1979, 1980)). Such an arrangement would be quite similar to the one analyzed above. Thus as a practical matter, legal restrictions of the kind discussed here are and have been of considerable importance, suggesting the relevance of analyzing sunspot equilibria in these contexts.

Appendix A

This appendix demonstrates that the economy of Section I (no legal restrictions) does not have a stationary sunspot equilibrium. To demonstrate this, the existence of such an equilibrium is supposed, and a contradiction derived.

The kinds of transactions that occur here are the same as in Section III, except that agents face no restrictions on their asset trades. In particular, if the current period state is e_t, savers choose a quantity of real balances $z(e_t)$ and a quantity of loans $x(e_t)$, taking $p(e_t)$, $p(e_{t+1})$, $q(e_t)$, and $R(e_t)$ as given, to maximize their expected utility; i.e., to solve the problem

$$\max \ell n[y - x(e_t) - z(e_t)] + q(e_t) \ell n[R(e_t)x(e_t) + \frac{p(e_t)}{p(1)} z(e_t)] + [1-q(e_t)] \ell n[R(e_t)x(e_t) + \frac{p(e_t)}{p(2)} z(e_t)]$$

subject to $z(e_t) \geq 0$ and $x(e_t) + z(e_t) \leq y$. (Notice that a unit lent at t repays $R(e_t)$ at t+1; i.e., loan repayments are indexed.) The first order conditions associated with this problem are

$$\text{(A.1)}\quad [y - x(e_t) - z(e_t)]^{-1} = q(e_t)R(e_t)[R(e_t)x(e_t) + \frac{p(e_t)}{p(1)} z(e_t)]^{-1} + [1-q(e_t)]R(e_t)[R(e_t)x(e_t) + \frac{p(e_t)}{p(2)} z(e_t)]^{-1};\ e=1,2,$$

and

$$\text{(A.2)}\quad [y - x(e_t) - z(e_t)]^{-1} = q(e_t)\frac{p(e_t)}{p(1)}[R(e_t)x(e_t) + \frac{p(e_t)}{p(1)}z(e_t)]^{-1}$$

$$+ [1-q(e_t)]\frac{p(e_t)}{p(2)}[R(e_t)x(e_t) + \frac{p(e_t)}{p(2)}z(e_t)]^{-1};\ e=1,2.$$

Also, borrowers choose loan demand $L(e_t) = w/R(e_t)$. Then loan market clearing requires that

$$\text{(A.3)}\quad \lambda x(e_t) = (1-\lambda)\frac{w}{R(e_t)};\ e = 1,2$$

which can also be written as

$$\text{(A.3')}\quad R(e_t)x(e_t) = (\frac{1-\lambda}{\lambda})w \equiv \eta.$$

Finally, money market clearing requires that

$$\text{(A.4)}\quad \lambda z(e_t) = p(e_t)^{-1};\ e = 1,2.$$

As in Azariadis (1981), a stationary sunspot equilibrium is a set of values p(1), p(2), R(1), R(2), q(1), q(2), x(1), x(2), z(1), and z(2) satisfying (A.1)-(A.4). Now suppose that such an equilibrium exists with $p(1) > p(2)$. Substitution of equations (A.3), (A.3'), and (A.4) into (A.2) yields (for $e_t = 1$)

$$\text{(A.5)}\quad [y - \frac{\eta}{R(1)} - \frac{1}{\lambda p(1)}]^{-1} = q(1)[\eta + \frac{1}{\lambda p(1)}]^{-1} +$$

$$[1 - q(1)]\frac{p(1)}{p(2)}[\eta + \frac{1}{\lambda p(2)}]^{-1}.$$

Since

$$[\eta + \frac{1}{\lambda p(e_t)}]^{-1} = \frac{\lambda p(e_t)}{\lambda\eta p(e_t)+1}$$

(A.5) can be written as

$$\text{(A.5')}\quad \frac{\lambda p(1)R(1)}{\lambda p(1)R(1)y - \lambda\eta p(1)-R(1)} = q(1)[\frac{\lambda p(1)}{\lambda\eta p(1)+1}] + [1 - q(1)][\frac{\lambda p(1)}{\lambda\eta p(2)+1}].$$

Solving (A.5') for q(1) yields

$$\text{(A.6)}\quad q(1)[\frac{\lambda p(1)}{\lambda\eta p(2)+1} - \frac{\lambda p(1)}{\lambda\eta p(1)+1}] = \frac{\lambda p(1)}{\lambda\eta p(2)+1} - \frac{\lambda p(1)R(1)}{\lambda p(1)R(1)y-\lambda\eta p(1)-R(1)}$$

Similar operations on (A.2) for $e_t = 2$ yield

$$\text{(A.7)}\quad q(2)[\frac{\lambda p(2)}{\lambda\eta p(2)+1} - \frac{\lambda p(2)}{\lambda\eta p(1)+1}] = \frac{\lambda p(2)}{\lambda\eta p(2)+1} - \frac{\lambda p(2)R(2)}{\lambda p(2)R(2)y-\lambda\eta p(2)-R(2)}$$

Now, since q(1) and q(2) are probabilities, $0 \leq q(1) \leq 1$ and $0 \leq q(2) \leq 1$. Then, from (A.6), for $0 \leq q(1) \leq 1$ it is necessary that

$$\text{(A.8)}\quad \frac{\lambda p(1)}{\lambda\eta p(2)+1} \geq \frac{\lambda p(1)R(1)}{\lambda p(1)R(1)y-\lambda\eta p(1)-R(1)} \geq \frac{\lambda p(1)}{\lambda\eta p(1)+1}$$

(since, by hypothesis, $p(1) > p(2)$). From (A.7), for $0 \leq q(2) \leq 1$, it is necessary that

$$\text{(A.9)}\quad \frac{\lambda p(2)}{\lambda\eta p(2)+1} \geq \frac{\lambda p(2)R(2)}{\lambda p(2)R(2)y-\lambda\eta p(2)-R(2)} \geq \frac{\lambda p(2)}{\lambda\eta p(1)+1}$$

In addition, since $p(1) > p(2)$, it is the case that

$$\text{(A.10)}\quad \frac{\lambda p(1)}{\lambda\eta p(1)+1} \geq \frac{\lambda p(2)}{\lambda\eta p(2)+1}\ .$$

Therefore, (A.8)-(A.10) imply that

$$\text{(A.11)}\quad \frac{\lambda p(1)R(1)}{\lambda p(1)R(1)y-\lambda\eta p(1)-R(1)} \geq \frac{\lambda p(2)R(2)}{\lambda p(2)R(2)y-\lambda\eta p(2)-R(2)}\ .$$

(Both sides of (A.11) are positive, as (A.11) just states that $y - x(1) - z(1) \geq y - x(2) - z(2)$.)

Tedious algebra implies that (A.11) is satisfied iff

$$\text{(A.12)} \quad \lambda^2 \eta p(1)p(2)R(2) + \lambda p(2)R(1)R(2) \geq \lambda^2 \eta p(1)p(2)R(1) + \lambda p(1)R(1)R(2).$$

Since $p(1) > p(2)$, clearly (A.12) can be satisfied only if $R(2) > R(1)$. However, (A.1) and (A.2) imply that

$$\text{(A.13)} \quad q(1)R(1)\left[\eta + \frac{1}{\lambda p(1)}\right]^{-1} + [1-q(1)]R(1)\left[\eta + \frac{1}{\lambda p(2)}\right]^{-1} =$$

$$q(1)\left[\eta + \frac{1}{\lambda p(1)}\right]^{-1} + [1-q(1)]\frac{p(1)}{p(2)}\left[\eta + \frac{1}{\lambda p(2)}\right]^{-1}$$

and

$$\text{(A.14)} \quad q(2)R(2)\left[\eta + \frac{1}{\lambda p(1)}\right]^{-1} + [1-q(2)]R(2)\left[\eta + \frac{1}{\lambda p(2)}\right]^{-1} =$$

$$q(2)\frac{p(2)}{p(1)}\left[\eta + \frac{1}{\lambda p(1)}\right]^{-1} + [1-q(2)]\left[\eta + \frac{1}{\lambda p(2)}\right]^{-1}$$

From (A.13) and $p(1) > p(2)$, clearly $R(1) > 1$. From (A.14) and $p(1) > p(2)$, clearly $R(2) < 1$. But then $R(1) > R(2)$, contradicting that (A.12) holds. Thus this case is impossible. An identical contradiction arises if it is assumed that $p(2) > p(1)$. Thus this economy has no stationary sunspot equilibrium.

Appendix B (Proof of Proposition)

Suppose, contrary to the proposition, that there are distinct values μ_1 and μ_2 (say with $\mu_1 < \mu_2$) satisfying $\mu_1 = T(\mu_2)$ and $\mu_2 = T(\mu_1)$. Then there are three possible cases.

Case 1. $(\mu^* \geq \mu_2 > \mu_1)$. By hypothesis $\mu_2 = T(\mu_1) > \mu_1 = T(\mu_2)$. But $T(\mu)$ is non-decreasing for $\mu \leq \mu^*$. Thus this case is not possible.

Case 2. $(\mu_2 > \mu^* \geq \mu_1)$. In this case $\mu_2 = T(\mu_1) \leq T(\mu^*)$, since $T(\mu)$ is non-decreasing for $\mu \leq \mu^*$. Since by hypothesis $\mu_2 > \mu^* = T(\mu^*)$, this case is also impossible.

Case 3. $(\mu_2 > \mu_1 \geq \mu^*)$. For all $\mu_1 \geq \mu^*$, $(\mu_1, T(\mu_1))$ lies on or below the 45° line in Figure 1. Then $\mu_2 = T(\mu_1) \leq \mu_1$, contradicting the supposition. Thus this case is also not possible, establishing the proposition.

Appendix C

Suppose that, in place of the legal restriction in the text, a legal restriction is imposed that requires loans to have a real value not less than $\hat{x}$, with $\hat{x} > y/2$. Then all behavior of agents is as described in the text, except that savers who make loans make them in real amount $\hat{x}$. Then money market clearing requires that

$$\text{(A.13)}\quad \lambda\mu(e)(y/2) = \frac{1}{p(e)};\ e = 1,2$$

in any stationary sunspot equilibrium, while loan market clearing requires that

$$\text{(A.14)}\quad \lambda[1-\mu(e)]\hat{x} = (1-\lambda)\,\frac{w}{R(e)};\ e = 1,2.$$

Also, savers must be indifferent between making loans and holding money, which requires

$$\text{(A.15)}\quad \ln(y/2)+q(e)\ln[(\frac{y}{2})(\frac{p(e)}{p(1)})] + [1-q(e)]\ln[(y/2)(\frac{p(e)}{p(2)})] =$$

$$\ln(y-\hat{x}) + \ln[R(e)\hat{x}];\ e = 1,2.$$

Collapsing (A.13)-(A.15) into one equation and rearranging terms yields

$$\text{(A.16)}\quad q(1)\ln[\frac{\mu(1)}{\mu(2)}] = \ln\{4(\frac{1-\lambda}{\lambda})(\frac{w}{y^2})(y-\hat{x})[\frac{\mu(1)}{\mu(2)}][\frac{1}{1-\mu(1)}]\}$$

and

$$\text{(A.17)}\quad q(2)\ln[\frac{\mu(1)}{\mu(2)}] = \ln\{4(\frac{1-\lambda}{\lambda})(\frac{w}{y^2})(y-\hat{x})[\frac{1}{1-\mu(2)}]\}.$$

Now suppose that $\mu(2) > \mu(1)$. For $0 \leq q(1) \leq 1$ it is necessary that

$$(A.18) \quad 1 \geq 4\left(\frac{1-\lambda}{\lambda}\right)\left(\frac{w}{y^2}\right)(y-\hat{x})\left[\frac{\mu(1)}{\mu(2)}\right]\left[\frac{1}{1-\mu(1)}\right] \geq \frac{\mu(1)}{\mu(2)},$$

while for $0 \leq q(2) \leq 1$ it is necessary that

$$(A.19) \quad 1 \geq 4\left(\frac{1-\lambda}{\lambda}\right)\left(\frac{w}{y^2}\right)(y-\hat{x})\left[\frac{1}{1-\mu(2)}\right] \geq \frac{\mu(1)}{\mu(2)}.$$

However (A.18) and (A.19) imply that

$$4\left(\frac{1-\lambda}{\lambda}\right)\left(\frac{w}{y^2}\right)(y-\hat{x})\left[\frac{1}{1-\mu(1)}\right] \geq 1 \geq 4\left(\frac{1-\lambda}{\lambda}\right)\left(\frac{w}{y^2}\right)(y-\hat{x})\left[\frac{1}{1-\mu(2)}\right],$$

which in turn implies that $\mu(1) \geq \mu(2)$, contradicting the original supposition. A similar contradiction arises if it is assumed that $\mu(1) > \mu(2)$. Thus this economy has no stationary sunspot equilibrium in which savers either make loans or hold money, but do not do both.

Notes

1. For instance, agents can buy certificates of deposit only in amounts equal to or exceeding $100,000.

2. Woodford (1986b) has demonstrated the existence of sunspot equilibria that depend on the history of the economy for economies that have no two-period cycles. However, I am aware of no similar result for stationary sunspot equilibria.

3. The environment described below is one in which there are no government expenditures, and hence no need to raise revenue. However, it should be clear that the absence of any need to finance a government deficit is not crucial to the arguments that follow.

4. The role of other kinds of legal restrictions in preventing the existence of stationary sunspot equilibria is a theme of Smith (1987).

5. The assumed form of borrowers' utility functions merits comment. It will be assumed below that borrowers must always hold a portfolio that enables them to repay their loans when old in each possible state of the world (no default). This assumption, coupled with the assumption that borrowers do not care about their old age consumption, implies that borrowers will never borrow and hold money in the sunspot equilibria to be constructed. If borrowers had more general preferences, e.g., $\ell n c_1 + \ell n c_2$, this would not be the case. Construction of sunspot equilibria would then become prohibitively difficult. The assumed form of borrowers' preferences makes the construction of sunspot equilibria here resemble such constructions in more standard homogeneous agent models. Another modelling device that would have the same effect would be to give borrowers the utility function $\ell n c_1 + \ell n c_2$, the endowment

stream (0,2w), and to assume that borrowers (agents with net indebtedness when young) are precluded from holding money. This would be in the spirit of Woodford (1986), where "workers" are, in effect, precluded from holding capital. It is easy to check that the modification just described would produce equilibria identical to those constructed below.

6. As will be shown (Appendix A), absent legal restrictions there are no stationary sunspot equilibria here. Hence only deterministic equilibria are discussed in the text.

7. In order for there to be a steady state equilibrium with valued fiat money it is necessary that $(\frac{1-\lambda}{\lambda})(\frac{w}{y}) < 1/2$ hold. This condition is henceforth assumed to be satisfied.

8. This section considers deterministic equilibria under the legal restriction discussed. Section III discusses sunspot equilibria in this economy.

9. In some respects this resembles the situation in Woodford (1986), where a subset of agents hold money while other agents hold higher yielding assets. In Woodford (1986), however, individuals are in effect exogenously divided into holders of money ("workers") and holders of capital ("capitalists"). Here it is the fact that agents must voluntarily make loans and hold money that enables sunspot equilibria to be constructed.

10. It can be verified by direct computation that $\phi > 1$ and $\bar{x}(\lambda/2) > 1$ imply that $\bar{x}\lambda\mu^* < 2$.

11. $T'(\mu)$ exists and is continuous for $\mu\varepsilon((\bar{x}\lambda)^{-1}, 2(\bar{x}\lambda)^{-1})$.

12. Notice that if $H'(\mu) \geq 0$ held, (30) would necessarily fail, and there could be no stationary sunspot equilibrium. $H'(\mu) < 0$ is equivalent to $\bar{x}(\lambda/2) > 1$, so this condition is necessary for there to be a sunspot equilibrium, as asserted above.

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Figure 1

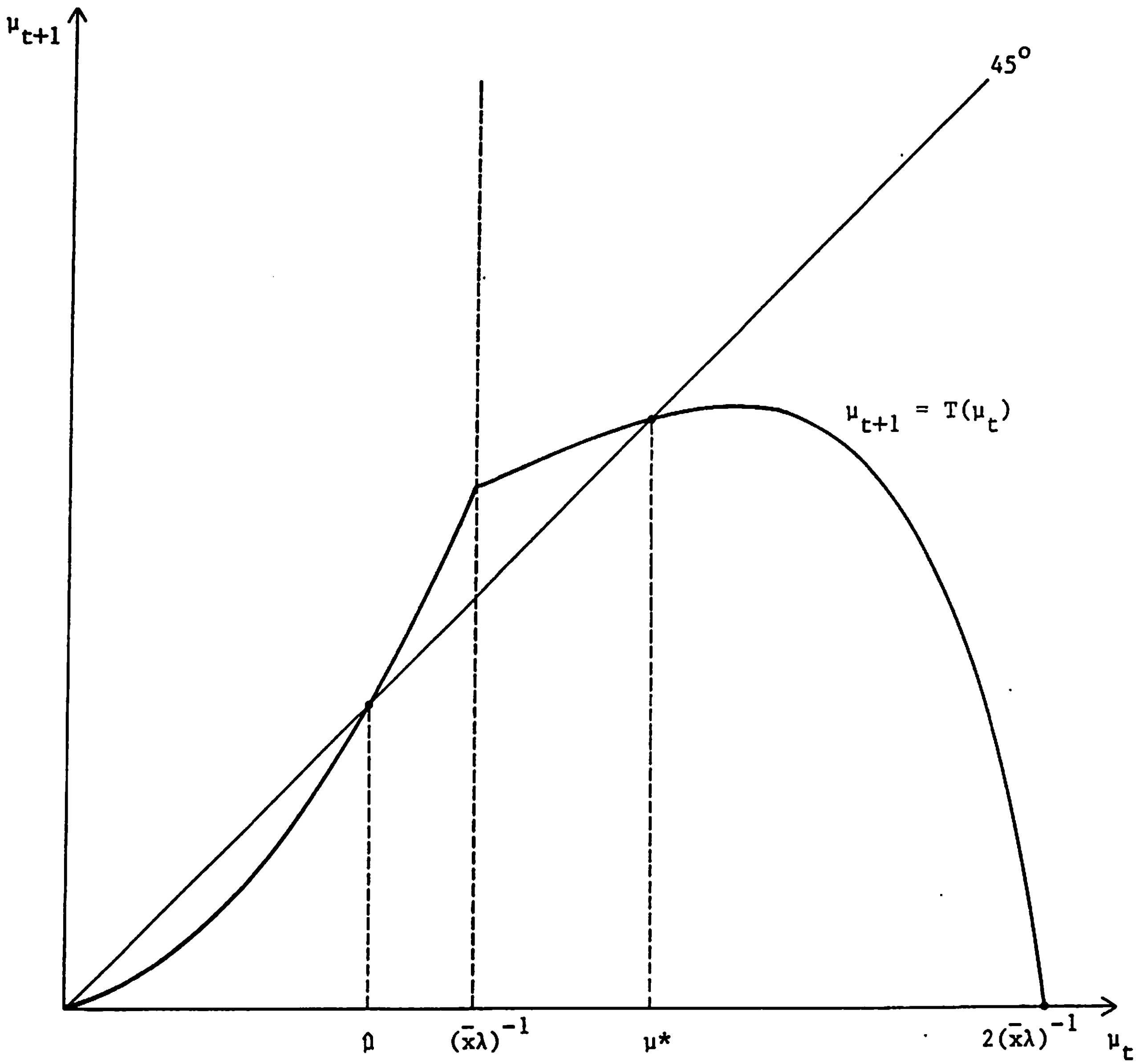
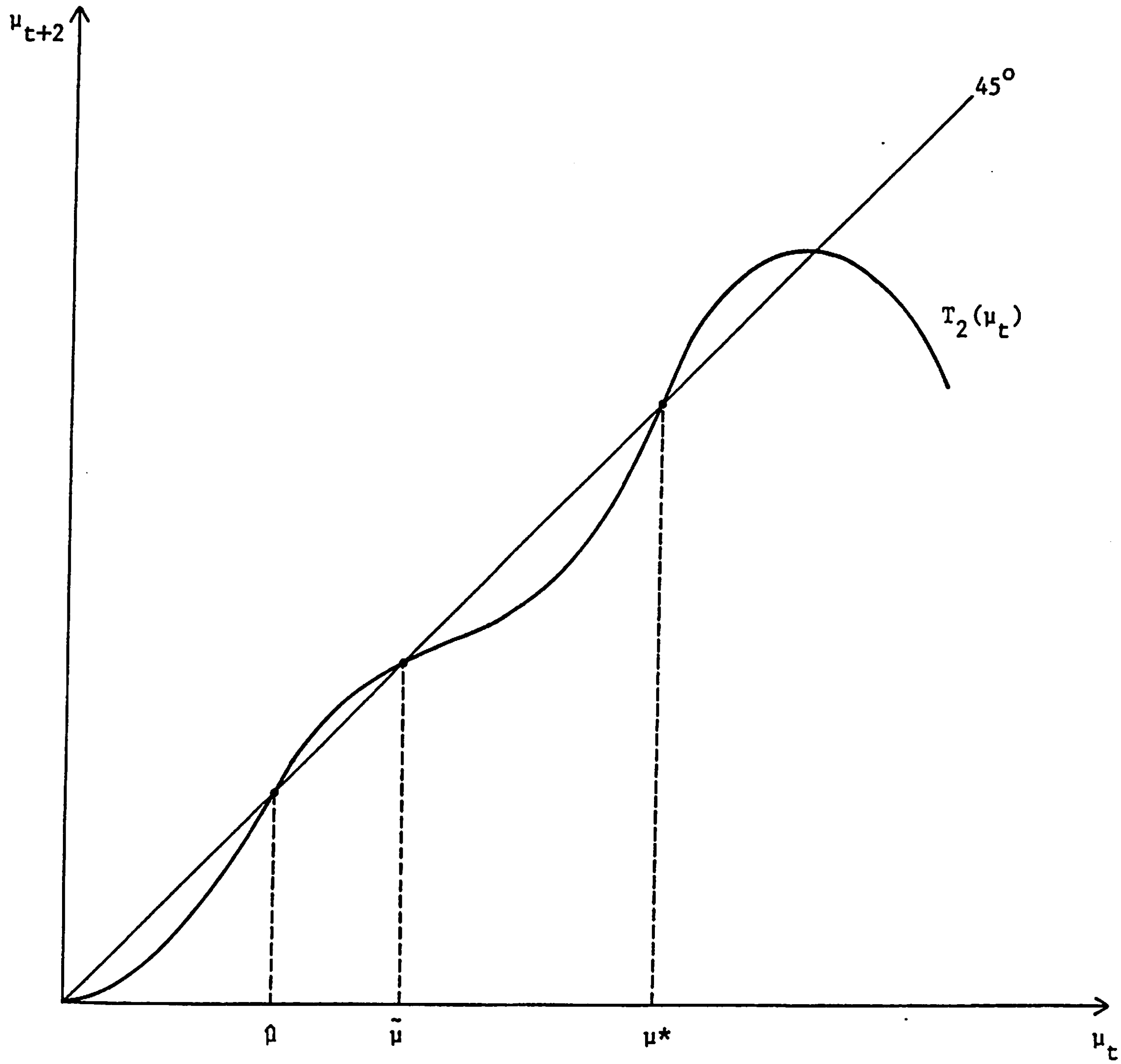


Figure 2



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<u>1985</u>

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1986

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1987

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