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Derived Demand Estimates of the Benefits from Public Investment in Intermediate Goods

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I

Any attempt to evaluate the social desirability of a public investment project must invariably lead us back to some form of cost-benefit analysis. We must estimate the project's productivity of social welfare by comparing all of the benefits it generates with the opportunity cost of the resources it uses. The estimation of project benefits is, however, fraught with difficulty. When commodities are supplied at zero prices or at non-market clearing prices, which bear no relation to the consumers' preferences, the present value of sales is no basis for arriving at investment decisions. Even where prices are representative of the marginal utility of the good to its consumers, a project of any significant size can be expected to alter market prices, leaving us in the ambiguous position of having a choice as to which price to use to evaluate sales. One estimate will inevitably be too high and the other too low.

The conventional wisdom suggests that the means of circumventing this impasse lies in measuring the change in the area under the demand curves for goods whose outputs are altered by the existence of the project. This change in area, on the assumption that the demand function is of the real income compensated variety, or that the demand for the good in question has zero income elasticity, is an unambiguous measure of the money value of benefits provided.[1] This area represents the income compensation that would be necessary to make consumers indifferent between the pre- and post-project situations. Even if the above stringent requirements are not met, the change in the area under the conventional "Marshallian" demand function will give us a reasonable approximation of the project benefits, if the change in output or

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[1] See D. Patinkin, [3] p. 104.

price is not large, or if expenditures on the good are a small item in the individuals' budgets.

In theory, we can construct a demand function for a publically supplied good, by asking potential consumers how much of the commodity they would purchase at different prices. But the collective nature of publically supplied goods will impeach the results of such a procedure. By "collective nature" I mean the "common-property" or "joint-supply" nature of most publically supplied goods. That is, supplying the good to one individual is not possible without supplying it to other individuals. Equal amounts need not be supplied to all individuals, but the share each person receives can not be varied once the level of output of the public good is determined.[2]

Thus when each beneficiary of the public project is asked what price he would be willing and able to pay for various amounts of the public good, with the proviso that he would actually have to make the stated payment, we can expect him to understate his demand. He does this on the expectation that he will thereby be relieved of part or all of his share of the cost without affecting the quantity he would ultimately obtain. On the other hand, if he were not required to pay he would have a strong incentive to overstate his demand.

Fortunately, we can estimate the demand function for at least one kind of public investment project without resorting to the revealed preference experiment. For the public project that generates goods or services, which are used as inputs into other production processes, we can derive a demand function from the production function of each user of this input. The vertical summation of these derived demand functions is the aggregate demand function for the public good or service.

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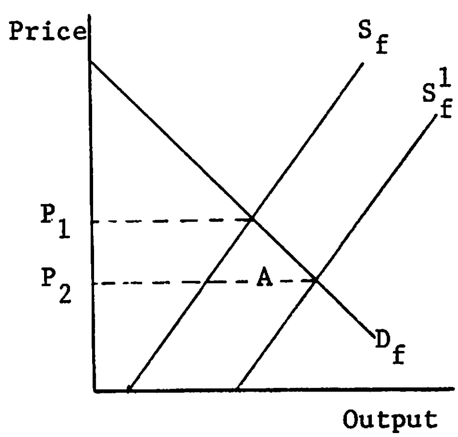
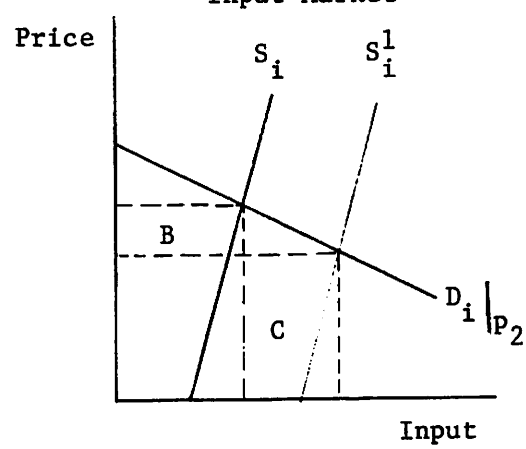
[2] See J. H. Boyd, [1], p. 21 and P. A. Samuelson, [7] p. 387.

Paraphrasing Prest and Turvey, if optimum welfare conditions are satisfied all along the line, the change in the area under the derived demand curves (the change in total revenue product) is the correct "reflector" of the benefits stemming from a public investment that increases the supply of an input.[3] In a perfectly competitive world this change in area would be equivalent to the change in the market value of the total quantity of the input employed plus any increase in the consumers' and producers' surplus in the market for any final good using this input. In the two diagrams below this can be represented as area A plus C minus B, where A represents the change in consumers' and producers' surplus, and C-B the change in the market value of the input.

Final Market



Input Market



The remainder of this paper will be devoted to a discussion of the use of the derived demand for a publically supplied input in the measurement of the social benefits to be derived from an increase in its supply. Specifically, in the next section we will follow the path implied by Prest and Turvey's comment.

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[3] A. R. Prest and R. Turvey, [5] p. 163.

II

Assume that a derived demand function is constructed holding quantities, other inputs and final product prices constant. Then, if the demands for goods are income inelastic, the prices of final goods are an index of the incremental utility consumers enjoy from an additional unit of output. Thus, the change in the total revenue product (the area under the derived demand curve) represents the increase in the consumer utility that would be brought about by the additional final good output resulting from the increased availability of the public input. In a sense, the marginal revenue product function is allowing us to map consumers' welfare from the final product market into the input market. The accuracy of our mapping and hence, the accuracy of our estimate of social benefits depends on the degree to which the demands for the final goods involved are purged of income effects.

If we are dealing with highly aggregative production functions, the additional error introduced into our estimates of social benefits by using the conventional derived demand approach will probably be relatively small. Thus, this method may offer a rough, but nonetheless, very useful estimate of the social benefits to be derived from increased investment in a publically supplied input, where we already have an estimate of an aggregate production function for its users, and the price of their final output. In this spirit let us now turn to some empirical aggregate production functions that have appeared in the literature and discuss the social benefit measures that can be derived from them.

Using the conventional derived demand method we could approximate the gross benefits to be derived from an incremental investment in the education of the farm labor force or in agricultural research and extension activities by deriving the appropriate input demand curves from Zvi Griliches' cross-sectional estimate of the aggregate United States agricultural production function.[4] Vernon W. Ruttan takes us one step further as he has estimated derived demand functions for irrigated land in each region of the country.[5] If we had all of his data we could estimate the gross benefits to be derived from an incremental investment in irrigated land in each region. Although I would prefer to measure the benefits that would accrue to an increase in the supply of irrigation water per se. It is really water that is the economic good, in whose production scarce resources are utilized. This would require disaggregating irrigated land into at least two inputs, land and water.

Now turning to a specific industry let us consider Willis L. Peterson's cross sectional estimate of the production function for commercial poultry farms and see how we would set about estimating the gross benefits to be derived from an incremental investment in poultry research.[6]

Peterson's production function is:

$$Y = A\, X_1^{b1}\, X_2^{b2}\, X_3^{b3}\, X_4^{b4}\, X_5^{b5}\, E\ .$$

Where: Y is the value of poultry products sold

A is a constant term

X_1 is the interest on land and buildings

X_2 is the expenditure for hired labor

X_3 is feed purchased

X_4 is chicks purchased

X_5 is the expenditure for research at the experiment station

E is the error term .

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[4] Zvi Griliches, [2] p. 966.

[5] V. W. Ruttan, [6] p. 23.

[6] W. L. Peterson, [4] p. 666.

Then denoting the coefficients estimated from the production function by $\hat{A}\ \hat{b}_1,\ \hat{b}_2,\ \hat{b}_3,\ \hat{b}_4,\ \hat{b}_5$:

$$\frac{\partial Y}{\partial X_5} = \hat{A}\ \hat{b}_5\ X_1^{\hat{b}_1}\ X_2^{\hat{b}_2}\ X_3^{\hat{b}_3}\ X_4^{\hat{b}_4}\ X_5^{\hat{b}_5 - 1}\ .$$

Since Y is already in dollar terms $\frac{\partial Y}{\partial X_5}$ is the marginal revenue product of X_5. Then holding X_1, X_2, X_3, X_4 and P_y constant: the change in total revenue produce = Δ

$$= \hat{A}\ \hat{b}_5\ X_1^{\hat{b}_1}\ X_2^{\hat{b}_2}\ X_3^{\hat{b}_3}\ X_4^{\hat{b}_4} \int_{\ell_1}^{\ell_2} X_5^{b_5 - 1}\ dX_5$$

$$\Delta = \hat{A}\ X_1^{\hat{b}_1}\ X_2^{\hat{b}_2}\ X_3^{\hat{b}_3}\ X_4^{\hat{b}_4}\ [X_5^{\hat{b}_5}]_{\ell_1}^{\ell_2} \quad \text{and}$$

$$\Delta = \hat{A}\ X_1^{\hat{b}_1}\ X_2^{\hat{b}_2}\ X_3^{\hat{b}_3}\ X_4^{\hat{b}_4}\ (\ell_2 - \ell_1)\ ,$$

where $(\ell_2 - \ell_1)$ is the incremental investment in poultry research. Then assuming a ten year lag before there is a payoff to the investment $(\ell_2 - \ell_1)$ and that the change in total revenue product (Δ) can be treated as an annuity that will continue into perpetuity, we can solve

$$\frac{\Delta}{r} - (\ell_2 - \ell_1)(1+r)^{10} = 0$$

for r, which will be the internal rate of return accruing to society as a result of this increased investment.

Unfortunately, a simultaneity problem arises where we use the conventional derived demand function to estimate the social benefits arising from an increase in the supply of a public input. The larger the increment in the supply of this input we are considering, the less acceptable becomes our assumption of output price constancy. Additional poultry research is bound to result in cost saving innovations, which will shift the supply function for poultry products to the right. This in turn, will lead to a decline in poultry prices. These poultry prices are shift parameters in the derived demand function for poultry research. Thus, we can expect the fall in poultry prices, resulting from the increased investment in poultry research, to produce a downward shift in our derived demand function. Furthermore, we can expect that the larger the increment of public investment, the greater will be this shift. Thus we are in the ambiguous position of having an increase in welfare as measured by the area under the poultry demand curve due to the decline in poultry prices and a conflicting decline in the total revenue product accruing to poultry research due to a downward shift in the derived demand curve. In this situation the only unambiguous gain to society is the increase in consumers' and producers' surplus in the poultry products market. This is the area Peterson attempts to measure by his "Index-Number Approach".[7]

III

The derived demand for an input we have been discussing is defined by holding other inputs constant and allowing the quantity of the output to vary with increased doses of the input in question. This is essentially the input market analog of the conventional Marshallian demand curve. Prest and Turvey, as well as every other student of cost-benefit analysis utilize this formulation where they are discussing the demand function for an input. However, it is possible to formulate another construction of the derived demand for an input, which I believe allows us to develop a more accurate measure of the

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[7]W. L. Peterson, [4] p. 657.

social benefits to be derived from an increased supply of a public input. This procedure involves deriving the demand for an input when output is held constant and the other inputs are allowed to vary with the increased use of the one input. In essence, by holding output constant and allowing all inputs to vary we are moving along an isoproduct surface. Hence, this is the input market analog of the "Hicksian" real income compensated demand curve, where we are constrained to move along a specific indifference surface.

In treating social benefits from the perspective of production, as opppsed to consumption, output becomes the analog of utility. Just as we have to move along an indifference surface, by adjusting money income, to get an unambiguous measure of social benefit when there are observed changes in the consumption goods market, so we must move along an isoproduct surface, by allowing all of the inputs to vary, in order to derive an unambiguous measure of social benefit due to changes in the input market. By fixing the quantity of output, the benefits to society from the increased availability of a publically supplied input are simply the value of the quantities of the other inputs no longer necessary to produce the given output--the value of the inputs released by the increased supply of the public input.

Let us assume that each firm using the publically supplied input can be characterized by the following Cobb-Douglas production function:

$$Q = X_1^a \, X_2^b \, X_3^c$$

where X_3 is the publically supplied input. Furthermore, assume that in aggregate the users of X_3 consume such a small proportion of the total supply of X_1 and X_2 that they have no effect on the prices of these inputs. Thus the prices of the other inputs, P_1 and P_2 are fixed, and let X_3 be supplied at a zero price. Then let Q^o be the fixed level of output and X_3^o the pre-project level of the public input. Set up the following constrained cost minimization problem:

$$Z = P_1X_1 + P_2X_2 + \lambda\ (Q^o - X_1^a\ X_2^b\ X_3^c) + \mu\ (X_3^o - X_3)\ .$$

Take the partial derivatives with respect to X_1, X_2, X_3, λ and μ. Then set these partial derivatives equal to zero:

$$\frac{\partial Z}{\partial X_1} = P_1 - \lambda\ a\ X_1^{a-1}\ X_2^b\ X_3^c = 0$$

$$\frac{\partial Z}{\partial X_2} = P_2 - \lambda\ b\ X_1^a\ X_2^{b-1}\ X_3^c = 0$$

$$\frac{\partial Z}{\partial X_3} = \lambda\ cX_1^a\ X_2^b\ X_3^{c-1} - \mu = 0$$

$$\frac{\partial Z}{\partial \lambda} = Q^o - X_1^a\ X_2^b\ X_3^c = 0$$

$$\frac{\partial Z}{\partial \mu} = X_3^o - X_3 = 0.$$

The Lagrangian multiplier μ is the total derivative of cost with respect to X_3. Since λ is the total derivative of cost with respect to output, we can assume $\lambda cX_1^a\ X_2^b\ X_3^{c-1}$ is positive and as a result μ is negative, since:

$$\mu = - \lambda\ cX_1^a\ X_2^b\ X_3^{c-1}\ .$$

Mu is the measure we are looking for, it is the reduction in total cost that arises from increasing the supply of the public input by one unit, keeping output constant. The total benefit derived from an additional unit of the public input is obtained by summing the μ s for each user of this additional unit.

Although μ is the appropriate measure of the benefits arising from a marginal change in the supply of the input, it does not suffice as a measure for larger changes. For that purpose we must derive another measure from the five efficiency conditions stated above. I will do this in general terms in order to be explicit about the procedure I am following.

Let us associate the superscript 'o' with the pre-investment situation and '1' with the post investment situation. Then solve the above system of equations to get an expression for X_1 and X_2 in terms of P_1, P_2, X_3 and Q^o:

$$X_1 = \phi^{(1)} (P_1, P_2, X_3, Q^o)$$

$$X_2 = \phi^{(2)} (P_1, P_2, X_3, Q^o) .$$

Total cost, TC, is just a function of X_1 and X_2 as P_1 and P_2 are assumed fixed and P_3 is assumed equal to zero.

Therefore:

$$TC^o = P_1 X_1^o + P_2 X_2^o$$

$$TC^1 = P_1 X_1^1 + P_2 X_2^1$$

$$TC^o - TC^1 = P_1 (X_1^o - X_1^1) + P_2 (X_2^o - X_2^1)$$

and $$dTC = P_1 dX_1 + P_2 dX_2 .$$

Differentiating our expressions for X_1 and X_2 with respect to X_3 we get:

$$dX_1 = \phi_3^{(1)} dX_3$$

$$dX_2 = \phi_3^{(2)} dX_3 .$$

Substituting these expressions into our equation for the change in total cost we have:

$$dTC = P_1 \phi_3^{(1)} dX_3 + P_2 \phi_3^{(2)} dX_3$$

or $$dTC = [P_1 \phi_3^{(1)} + P_2 \phi_3^{(2)}] dX_3 .$$

Then, if we sum the dTC's for each user of the input, we have our measure of the benefit to society from the increased supply of the public input.

Throughout this section we have assumed that the prices of X_1 and X_2 are invariant with the utilization of these factors by the users of X_3. If these factors are "specific" to a certain industry their prices will change with changes in their utilization by that industry. But if we also allow these prices to vary, we must know the supply and demand functions that will prevail in each of these factors' markets before and after the change in the supply of the public input in order to be able to derive our benefit measure. I doubt that this expensive refinement would significantly improve our benefit measure. While the decreased demand for X_1 and X_2 resulting from the increase in X_3 can be expected to result in a decline in the prices of these factors and hence a decline in the welfare of the owners of these factors, this effect will most probably be offset by the increase in the welfare of the users of these inputs as a result of the price declines.

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