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This paper contains preliminary findings from research work still in progress and should not be quoted without prior approval of the authors.

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EXTERNALITIES AND ASYMMETRIC INFORMATION

by

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Abstract

A reconsideration of the Pigovian theory of taxation for regulating externalities via taxation is undertaken for environments with private information. It is found that quantity limits may emerge as an outcome of social welfare maximization in a fairly wide range of circumstances. This may explain authorities apparent predilection for quantity limits, rather than tax-cum-subsidy schemes, to manage many externalities.

I. Introduction

Ever since Pigou's The Economics of Welfare economists have been aware of the importance of externalities in consumption and production. Everyday examples are easy to find. In consumption they run the gamut from being unable to sleep on a Saturday night because of the party next door, and sitting beside a smoker on a crowded bus to enjoying watching the opposite sex go by on the beach, and the benefits from having fellow citizens vaccinate their pets for rabies or supporting public T.V. In production they range from dumping toxic wastes into the environment, the negative impact a rival's advertising has on a firm's sales, the effects of fraud or malfeasance on the integrity of a financial system to the positive effects that occur when markets thicken such as when extra users adopt a common communication system or an industrial standard, and additional firms join a trade fair, industrial park or shopping center.[1]

Most of us were brought up on Meade's (1952) example of the apple producer living beside the beekeeper. Apple blossoms are beneficial to honey production. Yet the apple producer when deciding on his apple production, being concerned soley with his own profits, neglects to take into account the external benefits bestowed by his activity on the beekeeper. An underproduction of apples results. The Pigovian answer to this problem is simple: subsidize the output of apples to the extent required to equalize private and social benefits of apple production. More recently, Lucas (1985) and Romer (1985) have emphasized the importance of externalities in explaining the observed facts of economic growth and development. Both papers contain a role for Pigovian type industrial policies in the sense that activities with large external benefits should be subsidized. But as Lucas (1985) notes: picking winners and losers in models is easy. In real life, especially in a world where information is private, this isn't so. A basic fact of political economy seems to be when the rewards to winners are high and the penalties to losers large, everybody will claim to be former and not the latter. Even in a world with private information, however, when externalities are important enough and transaction costs low enough, institutions or coalitions should arise so as to attempt to internalize those benefits or costs of activities which are normally externalized. The design of such institutions is the subject of this paper.

Governments typically regulate with limits rather than Pigou taxes or subsidies, despite the lack of any existing economic analysis suggesting why the former may indeed by more desirable.[2] There are limits on the allowable pollution levels for factories and cars, on minimum housing quality for fire safety, on the speeds automobiles can travel, on blood alcohol levels for drivers, for food and restaurant sanitation, on the age for presidents and airline pilots, and on the daily price variation in stock and commodities markets. This paper offers a potential explanation for the policy maker's preference for limits, over Pigovian tax or subsidy schemes. In particular, it will be argued that limits can emerge as a consequence of social welfare maximization in environments with asymmetric information.

Formally, the analysis proceeds with the developments of a small two-good choice-theoretic general equilibrium model along the lines of Prescott and Townsend (1984) which can be used to analyze the issues surrounding the subject of externalities. In the model constructed there are externalities associated with an agent's consumption and production, the extent of which depend on the type of individual he is with his type being private information. Clearly, if allocations are to be made contingent upon an agent's true type, then an incentive scheme must be put in place to ensure that individuals end up effectively revealing their types truthfully. That is, an allocation scheme must be incentive compatible. As will be seen, it is precisely this incentive compatibility requirement which can lead to consumption and production limits being set at a fixed price across types of agents.

The analysis presumes that consumption is unobservable, or at least can be made unobservable at some cost. This allows for the interesting possibility that agents may try to cheat on the allocations assigned by the coalition by engaging in further secretive trading amongst themselves. That is, a black market may develop which arbitrages away differences in agent's marginal valuations for commodities. Clearly, any syndicate must take this possibility into account in its formation. The restriction that the potential emergence of black markets places on the design of an allocation scheme is analyzed.

Finally, as is common in economics, the present study is conducted at a high level of abstraction, and will not be able to explain many features of "real world" regulation, and thus will be open to usual charges of naivety. But broadly speaking the features of the world indentified in this analysis do exist, and shed some light on the policymaker's taste for quantity limits.

II. The Physical Environment

Consider a closed economy inhabited by a continuum of agents. Agents are randomly distributed by type, denoted by t, over the interval [0,1] according to the continuous density function f(t). An individual's type is private information. Agent t's goal in life is to maximize his utility, $\underline{U}(t)$, as given by the function

$$\underline{U}(t) = U(x(t),y(t),t) + \int_0^1 \varepsilon(y(s),s,t)f(s)ds. \qquad (1)$$

The first term is the direct utility agent t obtains from his personal consumption, x(t) and y(t), of two goods x and y. The direct utility function, U(•) is assumed to be increasing, concave and twice continuously differentiable in its first two arguments. The second term is the indirect utility--which may be negative--the agent realizes from the consumption of y by other agents in the economy. That is, there are externalities in consumption present. More specifically the term $\varepsilon(y(s),s,t)$ measures the utility benefit to individual t from agent s's consumption of y(s) units of y. Also, suppose that an agent's demand for good y is an increasing function of his type. It is easily demonstrable that this implies a restriction on U(•) of the form[3]

$$U_{yt}(\cdot)/U_y(\cdot) > U_{xt}(\cdot)/U_x(\cdot) \quad \text{for } t \in [0,1]. \qquad (2)$$

Each individual t is endowed with a certain amount of the x good, $\bar{x}$. It is assumed that commodity x can be transformed into y by agent t according to the following linear production technology

$$y = x/\underline{c}(t). \qquad (3)$$

The technology coefficient, $\underline{c}(t)$ is assumed to be a function of all the production being undertaken in the economy so that

$$\underline{c}(t) = \int_0^1 \gamma(y(s),s,t)f(s)ds. \qquad (4)$$

Hence, there are also production externalities in the economy. (In Section VI it will be assumed alternatively that each agent t is endowed with a certain amount of the y good, $\bar{y}$, which can be transformed into x according to the production technology $x = \underline{c}(t)y$.)

Before agents have been randomly assigned their type it is in their interest to coalesce and mutually agree on a system to govern society's future consumption allocations. Recall that an agent's type is private information, so that any allocation mechanism conditioning on an individual's true type must ensure, if it's not to be thwarted, that agents will in their own self-interest end up effectively truthfully revealing their type. That is, the allocation mechanism must be incentive compatible. The characterization of the notion of incentive compatibility in the current setting is the subject which will now be turned to.

III. Incentive Compatibility

Without any loss of generality it may be presumed that an effective allocation mechanism is incentive compatible--see Harris and Townsend (1983), and Myerson (1982). This implies that under an allocation system the utility an agent earns from revealing his type, t, honestly must be at least as great as that which would be realized if the individual claimed he was some other type, s, instead. In particular an effective mechanism must ensure that the following incentive compatibility condition is obeyed.

$$U(x(t),y(t),t) \geq U(x(s),y(s),t) \quad \text{for } s,t \in [0,1]. \qquad (5)$$

Rather than work with the above condition directly it turns out to be easier to use two equivalent conditions, which are given in the theorem below.

<u>Theorem</u>: The incentive compatibility condition (5) is <u>equivalent</u> to the following two conditions, (6) and (7), holding simultaneously

$$U_x(x(t),y(t),t)x'(t) + U_y(x(t),y(t),t)y'(t) = 0 \qquad (6)$$

and

$$y'(t) \geq 0. \qquad (7)$$

<u>Proof</u>: (Necessity). For condition (5) to hold $U(x(s),y(s),t)$ must take a local maximum at $s=t$. The first- and second-order conditions for a maximum are

$$\partial U(x(s),y(s),t)/\partial s|_{s=t} = U_x x' + U_y y' = 0 \qquad (8)$$

$$\partial^2 U(x(s),y(s),t)/\partial s^2|_{s=t} \leq 0. \qquad (9)$$

Now differentiate (8) with respect to s, while holding $s=t$ as required by incentive compatibility, to obtain

$$\partial^2 U(x(s),y(s),t)/\partial s^2|_{s=t} + \partial^2 U(x(s),y(s),t)/\partial s \partial t|_{s=t} = 0 \qquad (10)$$

Thus, (8), (9) and (10) together imply

$$\partial^2 U(x(s),y(s),t)/\partial s \partial t|_{s=t} = U_{xt} x' + U_{yt} y' = [\frac{-U_y}{U_x} U_{xt} + U_{yt}]y' \qquad (11)$$

$$\geq 0$$

Finally, the above expression forces $y'(t) \geq 0$ because the term in brackets on the right-hand side is positive since by assumption the demand for y is increasing in t [c.f. (2)].

(Sufficiency). For any given s

$$\partial U(x(s),y(s),t)/\partial s = U_x(x(s),y(s),t)x'(s) + U_y(x(s),y(s),t)]y'(s)$$

$$= [\frac{-U_x(x(s),y(s),t)U_y(x(s),y(s),s)}{U_x(x(s),y(s),s)} + U_y(x(s),y(s),t)]y'(s)$$

Thus, $\frac{\partial U(x(s),y(s),t)}{\partial s} \gtrless 0$ as $\frac{U_y(x(s),y(s),t)}{U_x(x(s),y(s),t)} \gtrless \frac{U_y(x(s),y(s),s)}{U_x(x(s),y(s),s)}$.

Furthermore note that

$$\frac{\partial}{\partial t} \frac{U_y(x(s),y(s),t)}{U_x(x(s),y(s),t)} = \frac{1}{U_x} [U_{yt} - \frac{U_y}{U_t} U_{xt}] > 0$$

Consequently, $\partial U(x(s),y(s),t)/\partial s \gtrless 0$ as $t \gtrless s$ which implies that $U(x(s),y(s),t)$ takes a global maximum at s=t, and hence that the incentive compatibility condition (5) will be satisfied. Q.E.D.

Thus, the set of incentive compatible mechanisms ensures that agents placing a relatively high value on commodity y (high t's) actually receive more y, but less x, than those individuals valuing x relatively more, a fact evident from (6), and (7).[4] Clearly, a feasible mechanism cannot give more of both goods to any particular type of agent. All individuals would claim to be this type. Similarly, a mechanism which provided y loving agents relatively large amount of x would also not be feasible. These agents (high t's) would have to be given disproportionately large amounts of x to entice them to reveal their type honestly. But then the x loving agents (low t's) would claim to be y loving ones. Finally, note that the set of feasible incentive compatible mechanisms appears to be quite large.

IV. Externalities in Consumption

A characterization of the optimal mechanism governing allocations in society when there are externalities in consumption will be provided now. To make the analysis more tractable two simplifying assumptions will be made: first, it will be assumed agents' utility functions are separable and linear in x so that $U(x(t),y(t),t) \equiv x(t) + V(y(t),t)$ and second, there are no externalities in production with $\underline{c}(t)$ being written simply as $\underline{c}(t) = c$. The optimal mechanism is designed to maximize *ex ante* societal welfare, W. It is described by the solution to the following programming problem which maximizes the expected value of an agent's utility function (12), subject to society's production possibilities as described by (13), and the incentive compatibility constraints (6) and (7).

$$\max_{x(t),y(t)} W = \int_0^1 [x(t) + V(y(t),t) + \int_0^1 \varepsilon(y(s),s,t)f(s)ds]f(t)dt \qquad (12)$$

s.t.

$$\int_0^1 x(t)f(t)dt = \bar{x} - c\int_0^1 y(t)f(t)dt \qquad (13)$$

$$x'(t) + V_y(y(t),t)y'(t) = 0 \qquad (6)$$

$$y'(t) \geq 0 \qquad (7)$$

This programming problem can be rewritten in a more simple form by substituting the constraint (13) into objective function (12) and reversing the order of integration on the $\varepsilon(\cdot)$ term to obtain

$$\underset{y(t)}{\text{Max}}\ W = \int_0^1 [\bar{x} + V(y(t),t) - cy(t) + \int_0^1 \varepsilon(y(t),t,s)f(s)ds]f(t)dt \quad (14)$$

$$\equiv \int_0^1 S(y(t),t)f(t)dt$$

s.t.

$$y'(t) \geq 0 \quad (7)$$

where the incentive compatibility condition (6) can be eliminated since x(t) no longer enters the objective function and hence given the optimal y(t) path can be innocuously chosen to satisfy this condition. The term S(y(t),t) represents individual t's contribution to social welfare, W, from his consumption of the goods x and y, both directly via his own utility level and indirectly through the utility levels of others. The properties of S(y(t),t) play a crucial role in the design of society's allocation system.

To see this consider the benchmark case where the incentive compatibility constraint (7) does not have to be incorporated into society's allocation mechanism, such as would occur in the situation where all information were public. Then the "first-best" social optimum for the consumption of y for agent t, denoted by y*(t), would be described implicitly by the solution to:[5]

$$S_y(y^*(t),t) = [V_y(y^*(t),t) + \int_o^1 \varepsilon_y(y^*(t),t,s)f(s)ds] - c = 0 \quad (15)$$

$$\text{for } t \in [0,1]$$

Clearly, the above equation sets the marginal social benefit of agent t's consumption of y, represented by the term in the brackets, equal to its marginal social cost in terms of foregone x, or c. Now in the environment being modelled where information is private, an incentive compatible mechanism may not be able to obtain this ideal "first-best" optimum. This is because the ideal solution may violate the incentive compatibility condition (7) for some

types of agents. Specifically, note from (15) that

$$y^{*\prime}(t) = \frac{-S_{yt}(y^*(t),t)}{S_{yy}(y^*(t),t)} \gtreqless 0 \text{ as } S_{yt}(y^*(t),t) \gtreqless 0 \qquad (16)$$

$$\text{for } t \in [0,1]$$

where it is also easily seen from (15) that it is always the case that $S_{yy}(y^*(t),t) < 0$. Thus the ideal "first-best" solution will only be feasible if $S_{yt}(y^*(t),t) \geq 0$ for all types of individuals. Hence in the situation where those individuals who desire the commodity most are also the ones who should have it, in the sense that they have the highest marginal social value for it, attaining the "first-best" allocation is possible.

The allocation mechanism in the above case can be supported by a price system (all prices are expressed in terms of x) which charges agent t a per unit price for y of $p^*(t)$ where

$$p^*(t) = V_y(y^*(t),t) \quad \text{for } t \in [0,1]. \qquad (17)$$

A fixed flat charge (tax), $\tau^*(t)$, is also levied on agent in the amount

$$\tau^*(t) = \bar{x} - p^*(t)y^*(t) - [x^*(0) - \int_0^t V_y(y^*(s),s)y^{*\prime}(s)ds] \qquad (18)$$

$$\text{for } t \in [0,1]$$

with

$$x^*(0) = \bar{x} - \int_0^1 [\int_0^t V_y(y^*(s),s)y^{*\prime}(s)ds]f(t)dt - c\int_0^1 y^*(t)f(t)dt. \quad (19)$$

This lump-sum payment is constructed to ensure that individual t's budget balances, a fact which is readily discernable after recognizing that the term in brackets in (18) is the solution for $x^*(t)$ implied by (6). The formula for $x^*(0)$ given by equation (19) is obtained by substituting the solution for $x^*(t)$ into (13) and solving the resulting expression. Finally, note that the price/lump-sum charge combination, $(p^*(t), \tau^*(t))$, facing agent t has been constructed to ensure that it is in his best interest to pick the assigned allocation, $(x^*(t), y^*(t))$.

The properties of the pricing schedule are easy to characterize. By differentiating (17) with respect to t one obtains

$$p^{*\prime}(t) = V_{yy}(y^*(t),t)y^{*\prime}(t) + V_{yt}(y^*(t),t) \quad (20)$$

$$= -V_{yy}(y^*(t),t)[\frac{-V_{yt}(y^*(t),t)}{V_{yy}(y^*(t),t)} - \frac{-S_{yt}(y^*(t),t)}{S_{yy}(y^*(t),t)}] \text{ [by (16)]}$$

$$\gtrless 0 \text{ as } \left.\frac{dy(t)}{dt}\right|_{p=\bar{p}} \equiv \frac{-V_{yt}(\cdot)}{V_{yy}(\cdot)} \gtrless \frac{-S_{yt}(\cdot t)}{S_{yy}(\cdot t)} = \frac{dy^*(t)}{dt}.$$

Thus the price charged can actually fall in agent type implying that it is possible for individuals with a high demand for y relative to some other group to also be paying a relatively low price for it. In particular, the price charged increases or decreases in agent type, t, depending on whether individual's private demand rises or falls in t more or less than the social demand for their consumption, i.e., on whether $dy(t)/dt\big|_{p=p} \gtrless dy^*(t)/dt$. In order to stimulate consumption over some interval of agent type a cut in price may be required.[6] Alternatively, agent t could be thought of as making a single fixed payment, $\omega^*(t)$, to the syndicate for the amount $y^*(t)$ of y. The size of this payment would be given by $\omega^*(t) = p^*(t)y^*(t) + \tau^*(t)$. It follows from (18) that $\omega^*(t)$ can be either an increasing or decreasing function in t depending on whether $y^{*\prime}(t) \gtrless 0$. That is, so long as the allocation of y is increasing (decreasing) in type, so is the total payment being made.

Next consider the situation where $S_{yt}(y^*(t),t) < 0$ for some range of agent types, say $[t_0,t_1] \subset [0,1]$. Over this interval those individuals who desire y the most are precisely the ones who shouldn't have it, in the sense that they have the lowest marginal social value for it.[7] In this case implementing the ideal "first-best" solution isn't feasible since the incentive compatibility constraint (7) will be violated, as is evident from (16). The solution to the above programming problem determining the optimal schedule for y in this situation, now denoted by $\hat{y}(t)$, is easily seen to be described by the following conditions:

$$S_y(\hat{y}(t),t) = 0 \quad \text{for } t \in [0,1] \text{ s.t. } \hat{y}'(t) \geq 0 \tag{21}$$

and

$$\int_{\hat{t}_0}^{\hat{t}_1} S_y(\hat{y}(\hat{t}_0),t)f(t)dt = 0 \tag{22}$$

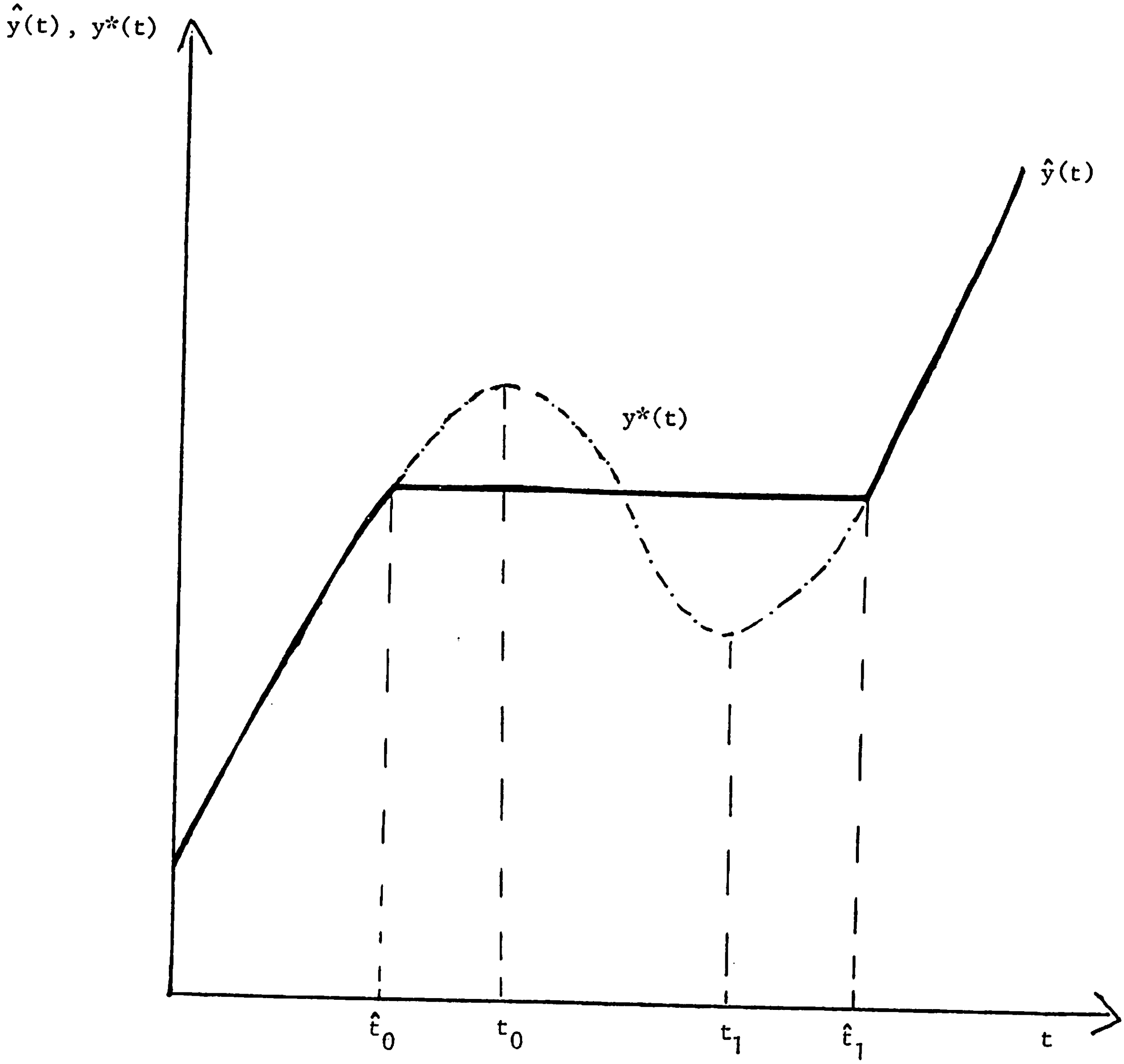
where $\hat{y}'(t) = 0$ for $t \in [\hat{t}_0, \hat{t}_1]$, and $\hat{y}(\hat{t}_0) = \hat{y}(\hat{t}_1)$ with $S_y(\hat{y}(\hat{t}_0), \hat{t}_0) = 0$. Clearly, as (21) shows, the ideal first-best solution will obtain over the regions where the incentive compatibility constraint isn't binding. Thus the preceding analysis holds for these regions. Over the range of agent types, $[\hat{t}_0, \hat{t}_1] \supset [t_0, t_1]$, for which the incentive compatibility constraint is binding, equation (22) must hold. Thus quantity limits on agents' consumptions are imposed over this interval. Note that the ceiling quantity, $\hat{y}(t_0)$, and the interval over which it applies $[\hat{t}_0, \hat{t}_1] \supset [t_0, t_1]$ are effectively chosen so that on average agent's marginal net social utility will be zero over this range.[8] An illustration of the type of situation being considered is provided by Figure 1.

It should also be noted that the pricing schedule supporting this allocation is strictly increasing over the range where quantity limits apply. This can be immediately seen by rewriting equation (20) to conform to the case under discussion to get

$$\hat{p}'(t) = V_{yt}(\hat{y}(\hat{t}_0), t) > 0 \qquad \text{for } t \in [\hat{t}_0, \hat{t}_1]. \tag{23}$$

Once again individuals over the range of agent types for which the quantity limits apply could just as easily by thought of as making a single fixed payment to the syndicate for the alloted quota of y. Specifically each agent $t \in [\hat{t}_0, \hat{t}_1]$ could be thought of paying the amount $\hat{\omega}(t) = \hat{p}(t)\hat{y}(\hat{t}_0) + \hat{\tau}(t)$ to the coalition for $y(t_0)$ units of y. It is easy to infer from the analogue to (18) for the current situation that this payment is constant across range of agent types where the quantity limits apply. Thus, an equivalent interpretation is that the allocation scheme imposes a consumption limit combined with a fixed (average) price for y.

Figure 1



Finally, suppose that $S_{yt}(y^*(t),t) < 0$ for all t. Then a single limit $\bar{y}$ is imposed whose level is determined by the condition $\int_0^1 S_y(\bar{y},t)f(t)dt = 0$. The good y will be banned outright if $\int_0^1 S_y(0,t)f(t)dt \leq 0$.

V. Black Markets

What was in an individual's ex ante best interest is not necessarily in his ex post best one. While agents may have agreed upon some distribution scheme, after the allocations have in fact been made incentives could be present for agents to trade further among themselves. That is black markets may develop, so to speak, on which agents trade outside the contracted allocation mechanism. Such black markets operate to circumvent the original agreement. Obviously then, the potential for black markets to emerge must be taken into account when designing the optimal distribution scheme. The presence of black markets can place severe limits on the ability of the allocation system to price discriminate among agents.

Suppose that a freely accessible black market production process operates where x can be transformed into y according to $y = [1/(c+\delta)]x$. Thus the black market price for y is $c+\delta \equiv b$. Individuals can also freely trade unwanted quantities of y for x among themselves on the black market at the price b, but the seller of y must incur a per unit transactions cost of δ, expressed in terms of x.[9] Clearly, an effective mechanism cannot allocate to any agent, say t, an amount of good y, or y(t), which would result in his marginal valuation of this good, $V_y(y(t),t)$, exceeding the black market price for it, $b = c+\delta$. Otherwise the agent simply purchases more y on the black market until his marginal valuation was equated to the black market marginal cost of production. It would have been better for the distribution scheme to have provided the individual with this extra quantity of y directly, since it could have been produced at a lower per unit cost c, rather than b. Equally, as clearly the allocation mechanism cannot assign a quantity y(t) of y to agent t which would result in his marginal valuation for it, $V_y(y(t),t)$, falling below the marginal cost of producing it, c. If this occurred the agent would profitably sell his y for x on the black market until his marginal valuation for it was equal to his net black market selling price, $b-\delta = c$.

Therefore a constraint on the design of the distribution system is that y(t) must be chosen such that $c \leq V_y(y(t),t) \leq b \equiv c+\delta$ for all $t \in [0,1]$. This condition can be reformulated directly in terms of bounds on y(t), or on the quantity of y that is allocated to agent t, as is shown below.

$$y_b(t) \leq y(t) \leq y_c(t), \text{ where } V_y(y_b(t),t) = b, \text{ and } V_y(y_c(t),t) = c. \qquad (24)$$

In the subsequent analysis it will be assumed that there exists intervals of agent types over which the above constraint precludes the first-best optimal allocation, $y^*(t)$, from being feasible. Specifically, let $y^*(t) > y_c(t)$ for $t \in [0,t_0')$, and $y^*(t) < y_b(t)$ for $t \in (t_1',1]$.

The optimal allocation scheme in the presence of black markets, denoted by $\tilde{y}(t)$, is given by the solution to the following programming problem.

$$\max_{y(t)} \; W = \int_0^1 S(y(t),t)f(t)dt \qquad (14)$$

s.t.

$$y'(t) \geq 0 \qquad (7)$$

$$y_b(t) \leq y(t) \leq y_c(t) \qquad (24)$$

Once again it proves useful to analyze the above problem's solution for two special cases. First, suppose that $S_{yt}(y^*(t),t) \geq 0$ for all t. The conditions governing the optimal allocation rule for y, or $\tilde{y}(t)$ are then easily determined to be

$$\tilde{y}(t) = y_c(t) \quad \text{for } t \in [0,t'_0)$$

$$\tilde{y}(t) = y^*(t) \quad \text{for } t \in [t'_0,t'_1]$$

$$\tilde{y}(t) = y_b(t) \quad \text{for } t \in (t'_1,1] \text{ where note } y^*(t'_0) = y_c(t'_0), \text{ and}$$

$$y^*(t'_1) = y_b(t'_1)$$

An illustration of the resulting $\hat{y}(t)$ schedule is provided in Figure 2. The above allocation scheme can be supported by the pricing system $\tilde{p}(t)$, where

$$\tilde{p}(t) = c \quad \text{for } t \in [0,t'_0)$$

$$\tilde{p}(t) = p^*(t) \quad \text{for } t \in [t'_0,t'_1]$$

$$\tilde{p}(t) = b \quad \text{for } t \in (t'_1,1], \text{ with } p^*(t'_0) = c \text{ and } p^*(t'_1) = b,$$

which is portrayed (in price/<u>quantity</u> space) by Figure 3.[10] As can be seen, the presence of a black market limits the range of price discrimination permissible by the allocation system. Note that as the cost of transacting on the black market approaches zero, the distribution mechanism breaks down in the sense that each agent will end up consuming the quantity of y that he would in a standard competitive equilibrium. [See Haubrich (1985) for a parallel discussion about multilateral incentive compatability undertaken within the context of a simple intertemporal exhange economy.] It may be the case that the cost of black market transacting can be influenced to some

Figure 2

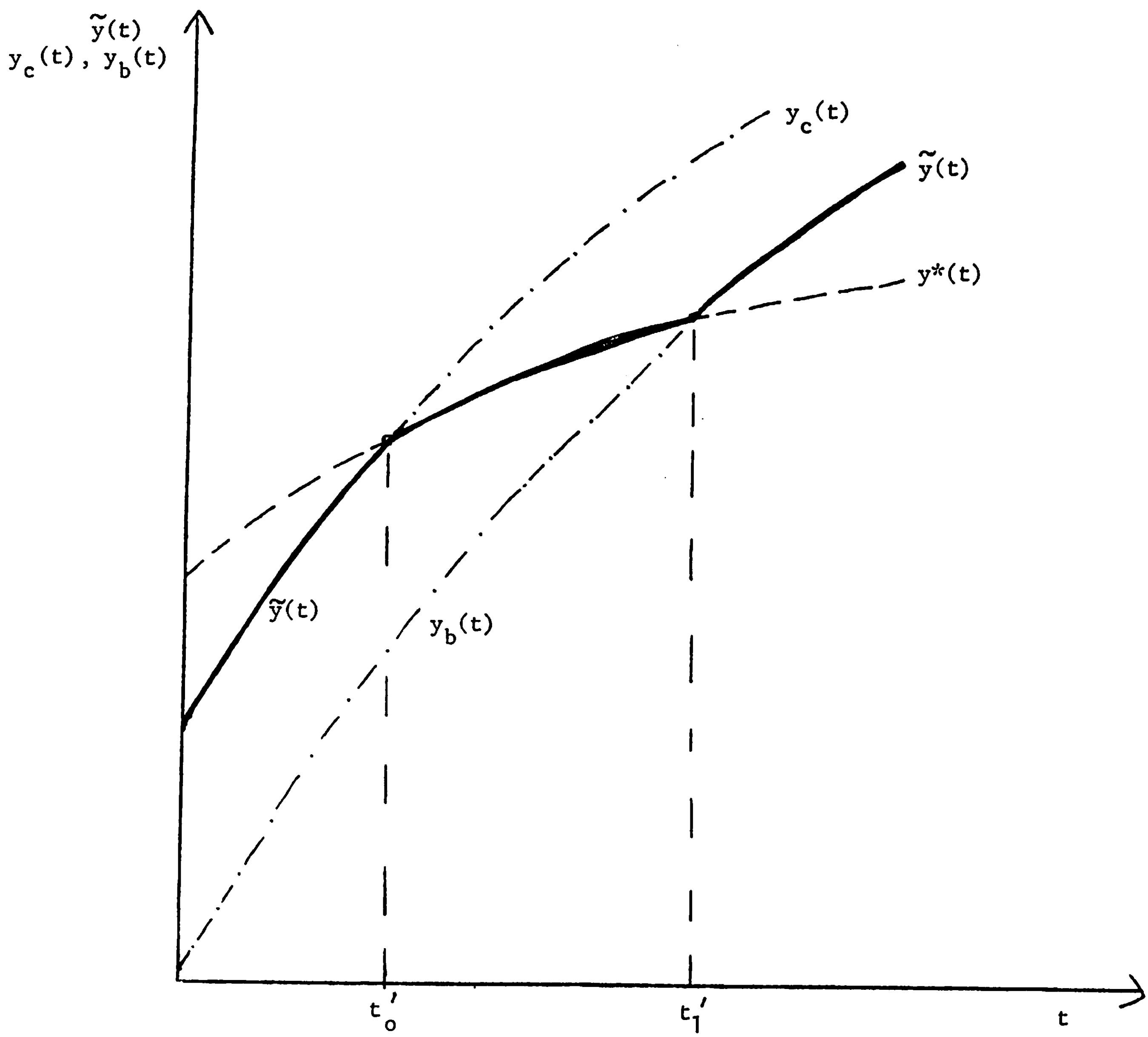
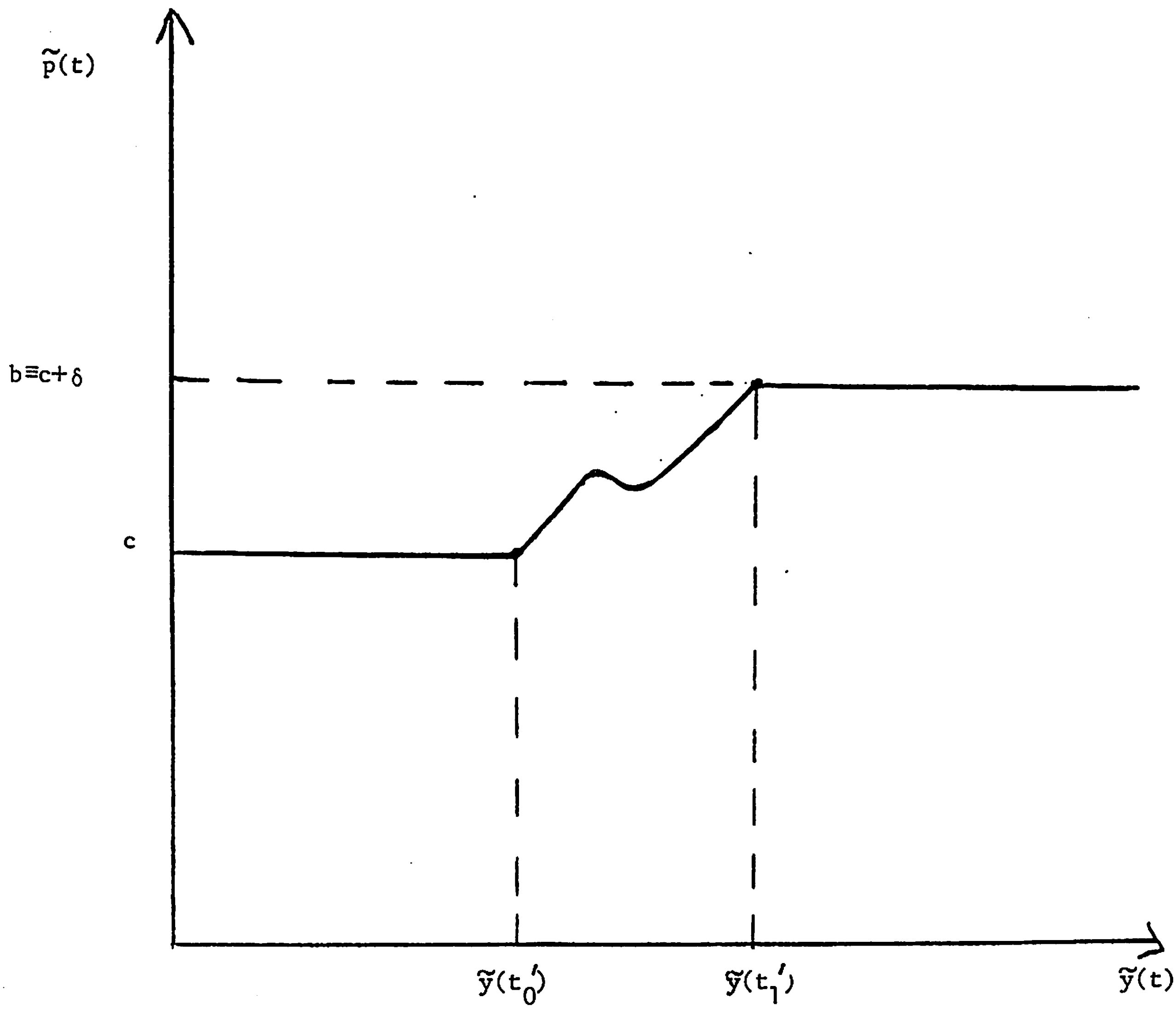


Figure 3



extent by the choice of an enforcement mechanism. An increase in δ can partially relax the constraint (24), since $dy_b(t)/d\delta < 0$, and consequently increase ex ante welfare, W. Given that enforcement mechanisms are costly, the marginal benefit from doing this should be equated to the marginal cost of stricter enforcement.

Second, consider the case where $S_{yt}(y^*(t),t) < 0$ for all t. The solution for y(t) is now described by the following conditions:

$$\tilde{y}(t) = y_c(t) \qquad \text{for } t \in [0,\tilde{t}_0)$$

$$\tilde{y}(t) = y_c(\tilde{t}_0) \text{ for } t \in [\tilde{t}_0,\tilde{t}_1] \supset [t'_0,t'_1] \text{ such that } \int_{\tilde{t}_0}^{\tilde{t}_1} S_y(y_c(\tilde{t}_0),t)f(t) = 0$$

$$\tilde{y}(t) = y_b(t) \quad \text{for } t \in (\tilde{t}_1,1] \text{ where } y_b(\tilde{t}_1) = y_c(\tilde{t}_0)$$

The solution is characterized by the imposition of a quantity limit, $y_c(\tilde{t}_0)$, on the consumption of y over the range of agent types $[\tilde{t}_0,\tilde{t}_1] \supset [t'_0,t'_1]$, since the incentive compatibility condition (7) is a binding constraint here. [See Figure 4.] The pricing scheme, $\tilde{p}(t)$, corresponding to this allocation system is

$$\tilde{p}(t) = c \qquad \text{for } t \in [0,\tilde{t}_0)$$

$$\tilde{p}(t) = V_y(y_c(\tilde{t}_0),t) \quad \text{for } t \in [\tilde{t}_0,\tilde{t}_1]$$

$$\tilde{p}(t) = b \qquad \text{for } t \in (\tilde{t}_1,1] \quad \text{[Note } \tilde{p}(\tilde{t}_0) = c, \text{ and } \tilde{p}(\tilde{t}_1) = b.\text{]}$$

This pricing schedule is graphed (again in price-quantity space) by Figure 5.

Figure 4

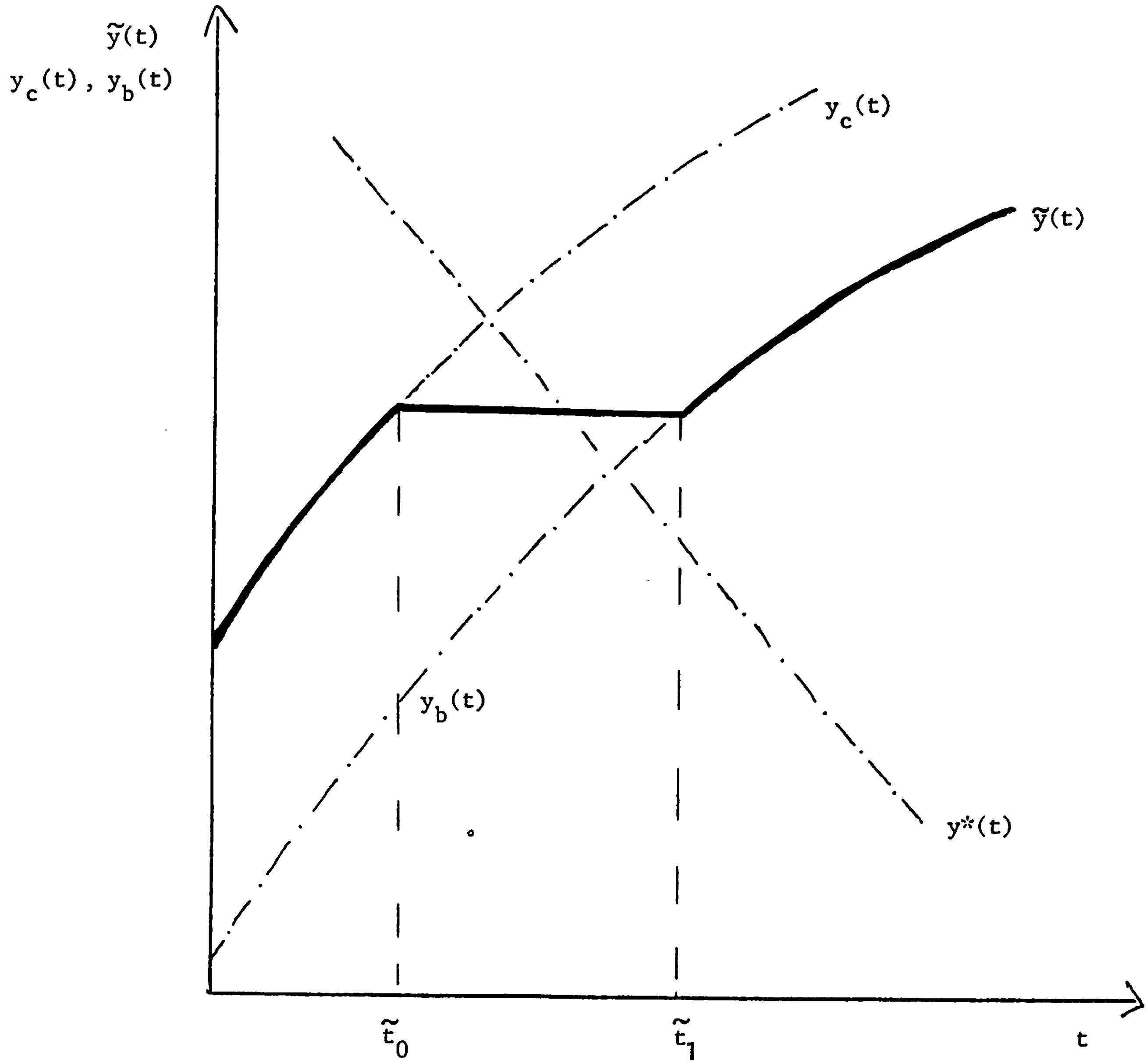
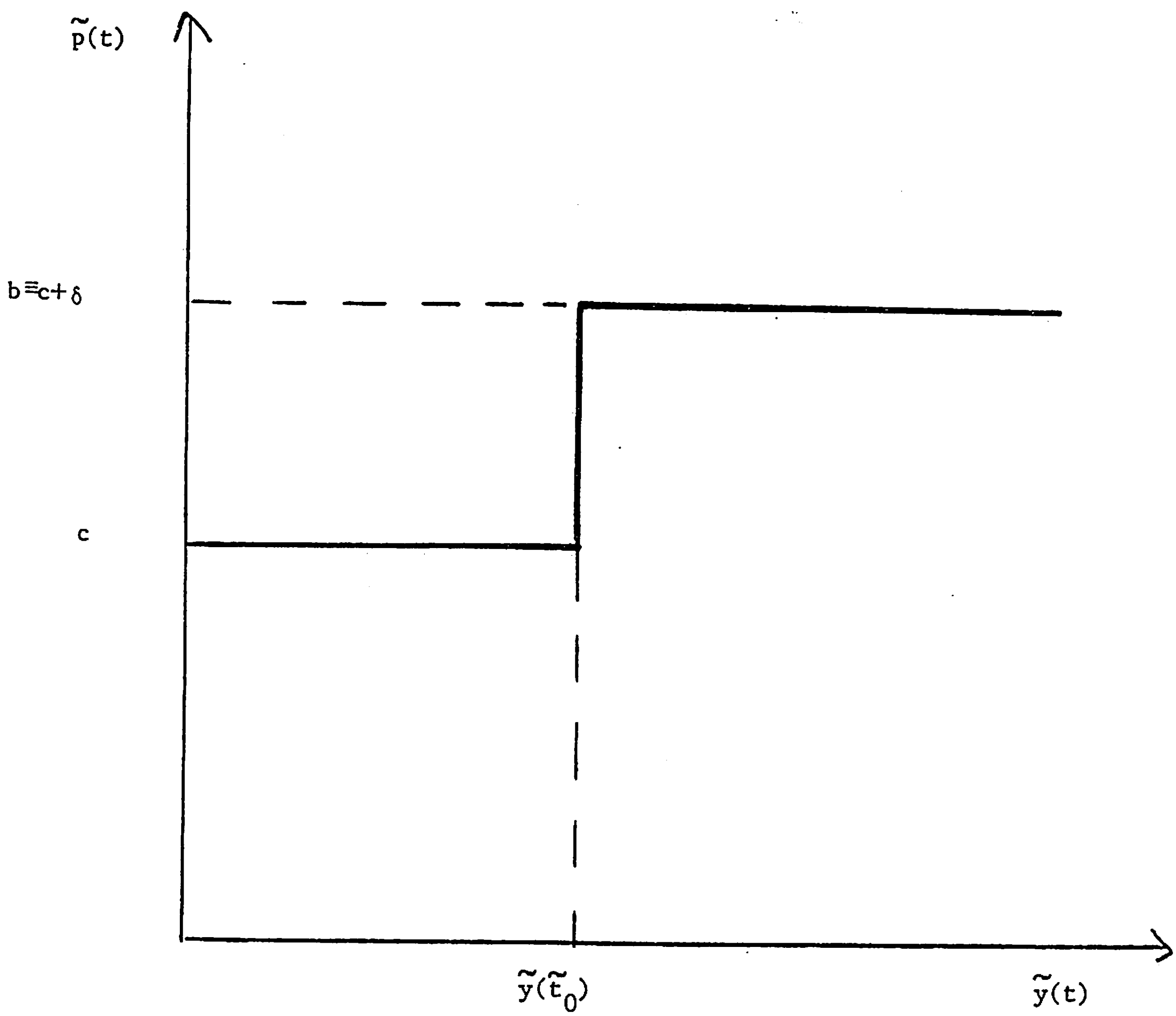


Figure 5



It is interesting since it shows that the allocation mechanism generates what appears to be a two-part pricing scheme. Those agents purchasing a quantity less than $\tilde{y}_c(t_0)$ pay a fixed per unit price of c, while those buying more than $\tilde{y}_c(t_0)$ pay a fixed per unit price of $b = c+\delta$. Individuals acquiring exactly the amount $\tilde{y}_c(t_0)$ pay some price $p(t) \in [c,b]$ contingent on their type, t.

Finally, to conclude this section Table 1 is presented which summarizes the main features of the model developed so far.

VI. Externalities in Production

The case of production externalities will now be analyzed. Let each agent be endowed with a certain amount of time, $\bar{y}$, which can be divided between leisure and work effort. Once again suppose that agent's utility is separable and linear in x so that $\underline{U}(t) = x(t) + V(y(t),t)$ where now y(t) represents agent t's consumption of leisure. So as to highlight better the issues being focussed on in this section, it has been assumed that there are no externalities in consumption. Agent t's production of good x, denoted by $x^s(t)$, is governed by the technology described by (3). Thus, $x^s(t) = \underline{c}(t)[\bar{y}-y(t)]$ where the technology coefficient, $\underline{c}(t)$, is given by $\underline{c}(t) = \int_0^1 \gamma(y(s),s,t)f(s)ds$, and $\gamma(\cdot)$ is specified as $\gamma(y(s),s,t) = e(y(s),s) + c(t)$. Society's optimal allocation system for this environment is described by the solution to the following programming problem

$$\max_{x(t),y(t)} W = \int_0^1 [x(t) + V(y(t),t)]f(t)dt \tag{25}$$

s.t.

$$\int_0^1 x(t)f(t)dt = \int_0^1 [c(t) + \int_0^1 e(y(s),s)f(s)ds][\bar{y}-y(t)]f(t)dt \tag{26}$$

Table 1

	$S_y(y^*(t),t) \geq 0$	$S_y(y^*(t),t) < 0$
No Black Market	-Socially Efficient Quantities -Taxed According to Type	-Quantity Limits Imposed -Taxed According to Type
Black Market	-Limited Ability to Vary Optimally Quantities According to Type -Limited Ability to Tax According to Type	-Quantity Limits Imposed -Two Part Pricing Scheme

$$y'(t) \geq 0 \quad \text{for all } t \tag{7}$$

The above programming problem can be simplified by substituting the goods market clearing condition (26) into the objective function (25) so as to eliminate the x(t) terms, and by defining a new choice variable, $\bar{e} \equiv \int_0^1 e(y(s),s)f(s)ds$. Specifically, one obtains

$$\max_{y(t),\bar{e}} \quad W = \int_0^1 \{[c(t) + \bar{e}][\bar{y}-y(t)] + V(y(t),t)\}f(t)dt$$

s.t.

$$y'(t) \geq 0 \qquad \text{for all } t$$

$$\bar{e} = \int_0^1 e(y(t),t)f(t)dt$$

Proceeding in a parallel manner with the preceding discussion, the solution to the above programming problem for two cases will be analyzed. First, consider the solution in the benchmark case where the incentive compatibility constraint (7) isn't binding. The efficiency condition governing the optimal path for y, or y*(t), is given by the following integral equation[11]

$$V_y(y^*(t),t)-\{c(t)+\int_0^1 e(y^*(t),t)f(t)dt-e_y(y^*(t),t)[\bar{y}-y^*(t)]\} \equiv S_y(y^*(t),t) = 0 \tag{27}$$

for $t \in [0,1]$

This expression simply states that each agent's marginal disutility from working measured in terms of x--the first term in the equation--should be set equal to his marginal <u>social</u> product again in terms of x--the term in braces.

The pricing mechanism supporting such an allocation specifies that each agent t should be paid a real wage, $w^*(t)$, set equal to the marginal disutility from working so that

$$w^*(t) = V_y(y^*(t),t) \qquad \text{for } t \in [0,1].$$

Following the earlier analysis it is straightforward to see that may increase or decrease in agent type over intervals depending on whether $-V_{yt}(\cdot)/V_{yy}(\cdot) \gtrless -S_{yt}(\cdot)/S_{yy}(\cdot)$, [c.f. (20)].

Next consider the case where the above "first-best" solution isn't feasible because it violates the incentive compatibility constraint (7) over some range of agent types $[t_0,t_1] \subset [0,1]$ implying $y^{*\prime}(t) < 0$ for $t \in [t_0,t_1]$. Recall from (16) that an equivalent characterization of this violation is $S_{yt}(y^*(t),t) < 0$ for $t \in [t_0,t_1]$. Now, from (27) it is easy to check that

$$S_{yt}(y^*(t),t) = V_{yt}(y^*(t),t) - c'(t) + [\bar{y}-y^*(t)]e_{yt}(y^*(t),t)$$

Thus, for the incentive compatibility constraint to be violated when $t \in [t_0,t_1]$ it must be the case that either (or both) $c'(t) > 0$ or $e_{yt}(y^*(t),t) > 0$ for each $t \in [t_0,t_1]$. This would correspond to the situation where those agents who don't like to work (the high t's) are in fact society's most productive workers.

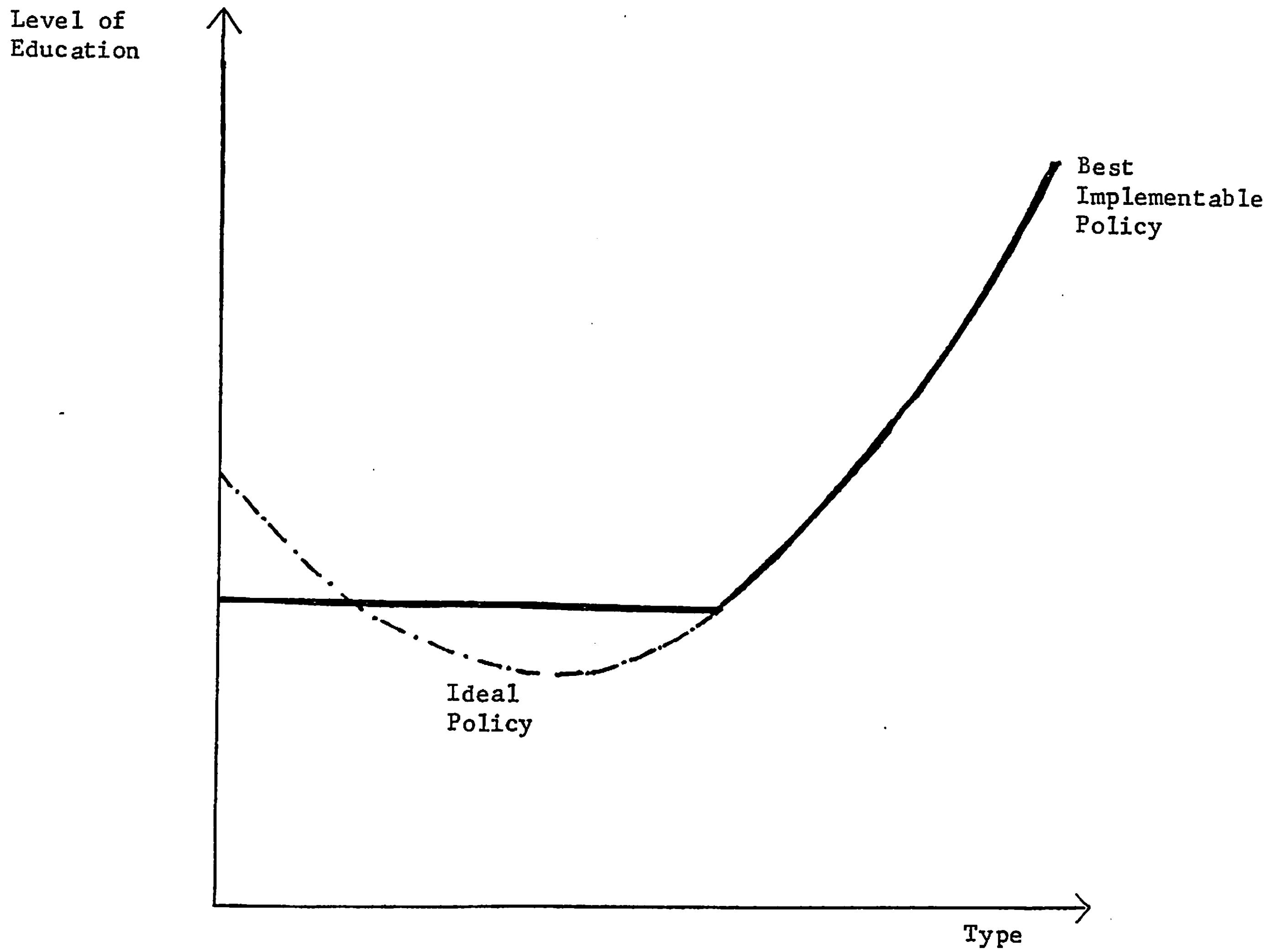
The solution for the optimal y path in this situation, denoted by $\hat{y}(t)$, will again be given by equations (21) and (22). A rather socialistic attitude toward hours of work arises: everybody (over the interval $[\hat{t}_0,\hat{t}_1]$) works exactly the same amount, $\bar{y}-\hat{y}^*(t_0)$. A rather capitalistic attitude, however, characterizes the compensation scheme. The wage, $\hat{w}(t)$, paid to each agent t is an increasing function in agent type t (over the interval $[\hat{t}_0,\hat{t}_1]$), a fact easily deducted by replacing $\hat{p}'(t)$ by $\hat{w}'(t)$ in (23). Hence the more productive agents (over this range) are in fact the ones who are being paid the most.

VII. Conclusions

An analysis of the externalities problem in environments with asymmetric information was undertaken. In the full information world of Pigou, externalities are regulated via tax-cum-subsidy schemes. By contrast in settings with asymmetric information, it was shown that externalities may have to be controlled by the imposition of quantity limits on economic activity. This conclusion emerges as a direct consequence of the incentive compatibility constraint placed on the design of the allocation mechanism. Loosely speaking, quantity limits should be imposed on an economic activity in the circumstance where the individuals who most (least) desire to engage in it are the one's who's participation is least (most) socially desirable. Finally, when consumption is unobservable, incentives are present for black markets to emerge outside of the arranged allocation system. It is clear that an allocation scheme, when being conceived, should take into account the potential for black markets to develop. This possibility can severely limit the amount of price discrimination, or variation in consumption allocations, across agents that mechanisms can support.

To conclude, an example will be presented which generates a policy prescription which conforms with a policy action actually observed in the "real world". It is often argued that there are significant externalities associated with education. A legislated minimum level of education improves societal welfare, it is commonly stated, since it strengthens the social fabric by reducing poverty and instilling notions of mutual respect and cooperation among individuals which work to eliminate breeding grounds of social discontent that aversely affect every member of society. Certainly, a prime motivating force behind many social reforms was the fear of the consequences arising from social discontent among segments of society. Also, at the other end of the spectrum the availability of higher eduction may foster greater research and development, or cultural activities, which raise the general standard of living in society. The underlying factors influencing individuals' decisions to acquire an education are not well understood, but it may be the case that those social types [the word is used loosely] who society most wants to have some minimal level of education have the least private demand for it. The theory developed predicts a quantity limit specifying a minimum level of education should then be imposed. Additionally, it could be the case that those types who generate the greatest social return from higher education are the ones who most desire it. Higher education should then be subsidized. Thus, an optimal education policy--see Figure 6--would consist of a minimum standard of education at the low end and subsidized levels of education at the high end, something in fact observed.

Figure 6



Level of Education
Best Implementable Policy
Ideal Policy
Type

Footnotes

[1] See Chatterjee (1984) for a discussion of the relationship between market participation and economic efficiency.

[2] Weitzman (1974) is one of the few exceptions to this generalization. This work discusses the relative benefits of regulating economic activity by prices versus quantity limits. In the analysis the cost and benefits of producing some good are of uncertain magnitude. It is assumed that the regulator must pick the price or quantity of production before the resolution of this uncertainty. The conditions under which the control of the production activity is better on average via price or quantity regulation are examined. This turns out to depend upon the relative curvature of the marginal benefit and cost schedules. A different set of issues from these is being addressed in the current study.

[3] This restriction on preferences is often refered to as the "single-crossing" property, and implies that the indifference curves for different types of agents can only intersect once. Cooper (1984) provides a discussion of the importance of the single-crossing property for self-selection models.

[4] Spence (1980) derives a similar result in a model where there is a finite number of types of agents who have preferences which are additively separable in the two goods. Cooper (1984) proves the necessity part of the above theorem, without establishing sufficiency.

[5] The derivation of the efficiency conditions presented in this paper are straightforward exercises in optimal control theory.

[6]The slope of the pricing schedule in price/quantity space can easily be deduced from (15), (16), (17), and (20) to be given by $dp^*(t)/dy^*(t) = p^{*\prime}(t)/y^{*\prime}(t)$. From (16) and (20) it can be seen that this schedule also rises or falls depending on whether

$$\left.\frac{dy(t)}{dt}\right|_{p=\bar{p}} \gtrless \frac{dy^*(t)}{dt}.$$

[7]Clearly, as can be seen from (15), $S_{yt}(y^*(t),t) < 0$ if and only if $-\int_0^1 \varepsilon_{yt}(y^*(t),t,s)f(s)ds > V_{yt}(y^*(t),t)$.

[8]This point is perhaps easier seen by noting that in the current example the above programming problem can be written as,

$$\max_{y(t),\bar{y},\hat{t}_0,\hat{t}_1} \int_0^{\hat{t}_0} S(y(t),t)f(t)dt + \int_{\hat{t}_0}^{\hat{t}_1} S(\bar{y},t)f(t)dt + \int_{\hat{t}_1}^{1} S(y(t),t)f(t)dt$$

$$\text{s.t.} \quad \hat{t}_0 \le t_0, \; t_1 \le \hat{t}_1$$

which also generates the efficiency conditions (21) and (22).

[9]In this discussion it is being presumed that there is an "aggregate" desire to transform x into y. Contrarily, one could instead assume that in aggregate agents desire to transform y into x. But this story turns out to be symmetric to the one provided in the text. Here one could let y be transformed into x according to the black market production process $x = (c-\delta)y$ implying that y will exchange for x at the price $c-\delta$. Finally, there is the borderline case to consider where in aggregate just the right amount of x and y exist and agents just reallocated these totals among themselves. Here the black market price for y, or b, lies somewhere in the interval $[c-\delta, c+\delta]$,

being determined by the condition

$$\int_S [y(t)-y_d(b,t)]f(t)dt = \int_B [y_d(b,t) - y(t)]f(t)dt$$

with $x_d(\cdot)$, $y_d(\cdot) \equiv \text{argmax}\{x_d + V(y_d,t)$ subject to $x_d + by_d \leq x(t) + by(t)$, and $x_d + (b-\delta)y_d \leq x(t) + (b-\delta)y(t)\}$, and where $S = \{y_d(\cdot) | y_d(\cdot) < y(\cdot)\}$ and $B = \{y_d(\cdot) | y_d(\cdot) \geq y(\cdot)\}$. This borderline case has been abstracted from in the text.

[10] Over the interval $[\tilde{y}(t_0), \tilde{y}(t_1)]$ the slope of the pricing schedule in quantity space is given by $dp^*(t)/dy^*(t) = p^{*\prime}(t)/y^{*\prime}(t)$--see footnote 6. Thus the pricing schedule can fall over portions of this interval [c.f.(20)].

[11] By the Banach Fixed Point Theorem a sufficient condition for the intergral equation (27) to have a unique solution for $y^*(t)$ is

$$\max_{\substack{y^*(t)\in[0,\bar{y}] \\ t\in[0,1]}} \left| \frac{\int_0^1 e_y(y^*(t),t)f(t)dt}{V_{yy}(y^*(t),t)+e_{yy}(y^*(t),t)[\bar{y}-y^*(t)]-e_y(y^*(t),t)} \right| < 1.$$

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