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AN ECONOMIC APPROACH TO PRISONS,
PUNISHMENT AND REHABILITATION

by

Kenneth L. Avio*

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June, 1971

AN ECONOMIC APPROACH TO PRISONS, PUNISHMENT
AND REHABILITATION

Q: How about yourself? Do you think you are a good risk to be let back into society?

A: I'm not likely to invade Cambodia.
. . . Edgar Smith, death row's longest surviving inhabitant (14 years), in Esquire Magazine, June 1971.

Recently there has been a substantial renewal of interest by economists in the so-called "economics of crime."¹ The basic assumption is that the supply of criminal offenses of an economic nature depends largely upon the prospective criminal's evaluation of the expected net gains from criminal activity vis-a-vis the expected net gains from legitimate pursuits. The greater the positive difference between the two net expected gains, the larger is the probability that an offense will be committed.² In calculating the net expected gains from crime, it is assumed that individuals take into consideration the probability that they will be apprehended if they commit a crime, and the severity of punishment if found guilty. Insofar as society can control these variables, it has substantial influence over the number of offenses. The normative aspects of the economics of crime, then, deal with choosing optimal levels of law enforcement activity and severity of punishment, and hence, the optimal level of offenses.

More specifically, the Criminal Justice System (CJS), may be defined as the set of institutions which provide "protection services" for society. Included in the CJS are the police force, the courts, and the various correctional programs. The normative aspects of the economics of crime can then be listed as:

- 1) How much of society's scarce resources should be devoted to the CJS?
- 2) How should the resources devoted to the CJS be allocated among its member branches?
- 3) How should a particular branch of the CJS use its budgeted resources to provide the maximum in protection services to society?

Sections I and II of this paper analyze question (3) with respect to the correctional branch of the CJS. In I, an economic model of a prison is discussed, with attention focused on the trade-off among alternative uses of resources. The prison model is essentially an application of the standard theory of the multi-product firm. Section II presents a discussion of the optimal parole procedure. To my knowledge these sections contain the first discussion on the economic nature of correctional institutions in the literature. Once the prison model has been properly formulated, the problem of minimizing the social loss due to crime is discussed, with special reference to the problem of recidivism. This admits a framework within which society may address itself to questions (1) and (2) above. Section III should be considered an extension and correction of Becker's seminal paper [1].³

I. The Prison as an Economic Institution.

As noted above, the CJS may be defined as the set of institutions which provide protection services (however defined) to the public. As an integral part of the CJS, the prison system may be viewed as a non-profit firm which produces two services intermediate to protection services: incarceration services and rehabilitation ("training") services.⁴ Incarceration services are intermediate to protection services in that i) an offender cannot commit a crime against "legitimate" society while in custody, and ii) the mere existence of such a service provides a deterrent to prospective criminals who value their freedom.

If incarceration services were the only prison output, then clearly an increase in the average incarceration for each offense category would raise the level of protection of society. But by their very nature, prisons also produce training services for society, services which may be viewed as an investment in human capital. It is intermediate to protection services in that if it sufficiently raises the expected legitimate income of the criminal, he can be

expected to "go straight" upon release. However, training may also be "negative". That is, the prisoner may spend his time increasing his criminal skills via the flow of criminal technology from one inmate to another. Furthermore, the inmate may form associations with previously unknown offenders who may exert criminogenic influences upon release. If these latter two factors predominate in the sense that the net expected gains from criminal activity increase by more than the net expected gains from legitimate activity, then the prison will be processing an individual who has high probability of being a recidivist. Formally, the relation between positive training (t) and negative training (t^-) may be defined as

$$(1) \quad t^- = n(t), \quad n'(t) < 0.$$

An increase in positive training reduces the amount of time a prisoner can spend with other inmates in acquiring criminal skills. From (1), the specification of t determines t^- .

Ideally the problem for the correctional authorities is to determine the optimal mix of training and incarceration services subject to a budgeted resource constraint. In order to do so, the technological relationship

$$(2) \quad x = h(t, f)$$

is defined, where

$x \equiv$ a measure of the input cost of production

$t \equiv$ a measure of the quantity and quality of rehabilitative training per day

$f \equiv$ length of incarceration.

The function h identifies the bundles (t, f) that exactly exhaust the input cost x .

It is reasonable to assume that incarceration services and training services are technical complements (i.e., $\frac{\partial^2 x}{\partial t \partial f} = h_{12} < 0$), or at least

technically neutral ($h_{12} = 0$). Technical complementarity may be interpreted here as: an increase in the level of training reduces the marginal cost of incarceration. That is, the greater the intensity of rehabilitative training, the better will be the prisoner's attitudes toward confinement, resulting in a decrease on the margin in the need for guards, etc. The assumption of at least technical neutrality is sufficient to insure that the product transformation curves as depicted in Figure 1 ($x=x^0$, $x=x^1$, $x^0 < x^1$) are concave to the origin.⁵

In addition to the production possibility function (2), a production function for protection services, P , may be defined as $P = g(t, t^-, f)$, or in view of (1),

$$(3) \quad P = s(t, f).$$

Two such "equal protection" curves⁶ are drawn in Figure 1, with $P^1 > P^0$. The curves are drawn convex to the origin, indicating that as the intensity of training decreases, the length of incarceration needed to keep society at the same level of protection must be increased in increasing increments. This is necessary to negate the effects of increased "negative" training on society's protection level.⁷

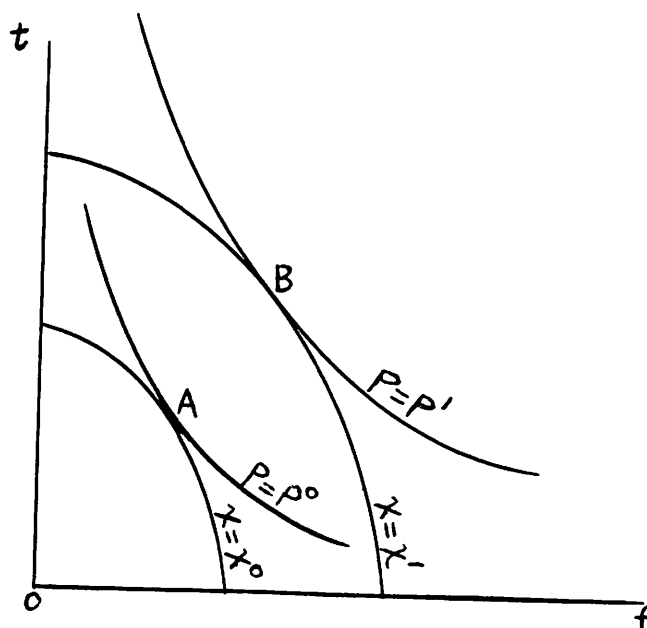


Figure 1

For the given budgeted level of inputs $x=x^0$, the correctional authorities should choose the bundle (t,f) represented by A in Figure 1. This bundle possesses the optimality property that the ratio of marginal benefit to marginal cost is the same for training services and incarceration services (i.e., $\frac{P_1}{h_1} = \frac{P_2}{h_2}$). The locus of all points possessing such a property is represented as

$$(4) \quad P^* = e(t^*, f^*),$$

the prison's efficient expansion locus.

While the above model may be an adequate representation of the economic nature of prisons, the bundles (t,f) are typically not subject to choice by correctional officials. The length of sentence is determined by the judicial section of the CJS subject to the prescriptions of the Criminal Code, and the level of inputs is determined by a budgeting procedure which may bear little reflection upon the optimal relationship between length of incarceration and training. Thus in practice, the problem facing prison officials is to choose only the level of training subject to given inputs and court determined sentences (except for parole considerations). It is clear from the above model that such decentralized decision-making would result in an optimal choice only by chance. This conclusion reached from economic considerations reinforces the view of the recent Royal Committee that greater harmony among the divisions of the CJS is needed.⁸ In particular, length of sentence and intensity of rehabilitative training should be determined simultaneously.⁹ This problem is discussed in detail in section III below.

II. A Digression on Optimal Parole

In the discussion above it was shown that decentralized decision-making in the CJS will usually result in a suboptimal choice of sentence length and training intensity. Furthermore, even if the chosen pair (t, f) is optimal "on the average", any given prisoner may respond differently to rehabilitation than any other prisoner. For these reasons correctional officials (the "parole board") can adjust the actual incarceration if they deem it is in the public interest to do so. In this section the problem facing the parole officer--the optimal parole--is discussed. Defining f^c as the court directed incarceration and f^a as the actual length of incarceration served at parole time, the problem facing the parole officer is to choose $f^a \leq f^c$, subject to given levels of f^c and t .

Ideally, the purpose of a correctional program while incarcerated is to lower the probability that a punished offender will be a recidivist.¹⁰ This probability is defined in an ex post sense as $r = r(t, f, w)$, where f represents the number of days of the sentence already served and w represents a host of variables such as attitudes and work habits, and including variables which influenced the decision to commit the original crime. Depending on w , the function r will differ among individuals. The form of $r = r(t, f, w)$ is specified as (for given w): $r_t < 0$, $r_f \begin{matrix} \cong \\ > \end{matrix} 0$ as $t \begin{matrix} \cong \\ < \end{matrix} k$. These characteristics can be explained with reference to Figure 2, where α is the probability that the offender will commit another crime if not apprehended for the original crime. For court determined f^c and a pre-determined value of t , the probability of recidivism falls or rises as the sentence is served, depending upon the value of t . In Figure 2, $t^1 > k > t^2$, where it is assumed that t^2 is so small, and hence (from equation 1), t^- is so large, that as the sentence is served, the probability of recidivism increases. If such were the case

the optimal procedure for the authorities would be either to i) incarcerate the prisoner for life (the "throw away the key" approach), or ii) put the offender on probation (i.e., choose $f^a=0$), or iii) choose some other form of punishment, e.g., fines or physical punishments such as whipping or removal of a limb. But $f^a=0$ could never be optimal, for as soon as prospective criminals perceive this parole procedure, the deterrent effect of incarceration would be close to zero. The alternative approach of a system of fines¹¹ is deficient in that it might raise the probability of recidivism as the offender turns to crime to pay off the fine. Also, fines may not serve as a deterrent to recidivism if the individual took the fine into consideration in figuring the net expected gains from criminal activity before committing the original offense. This leaves society with a choice of physical punishments¹² or incarceration for life, the latter of which would inflict a

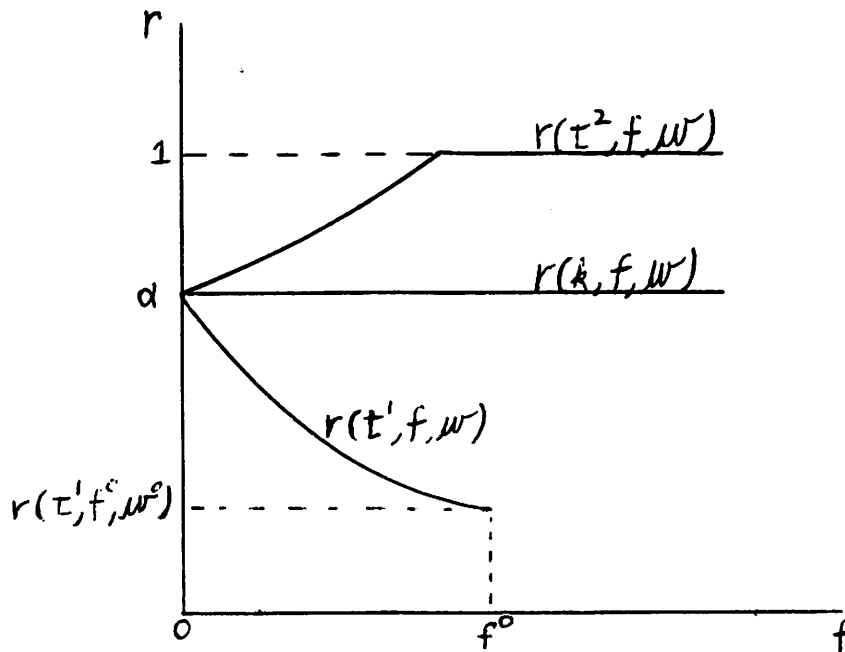


Figure 2

considerable cost on society. Since in the past fifty years society has relied primarily on neither of these possibilities, it will be assumed for the remainder of this paper that there does exist a value of $t > k$, and that such a t is chosen by the authorities.

As certain specified portions of the court directed incarceration are reached, the parole board must decide whether to parole the prisoner or to keep him incarcerated. In effect, the parole board weighs the expected "marginal" benefit to society from releasing a non-recidivist against the expected marginal loss to society from releasing a recidivist.¹³ Define $L^1 = L^1(f^c - f^o)$ as the marginal benefit to society if a non-recidivist is released after serving f^o of his sentence, and $L^2 = L^2(f^c - f^o)$ as the marginal loss to society if a recidivist is released at f^o . By definition $L^1(0) = L^2(0) = 0$, and it may be assumed that for any portion of the sentence served, $L^1 \leq L^2$. If the parole board reviews a case at $f = f^o$ (see Figure 2), the prisoner should be given a parole at that time if

$$(5) \quad E [B(f^o)] \geq E [L(f^o)]$$

where $E [B(f^o)] \equiv [1 - r(t, f^o, w^o)] L^1(f^c - f^o)$

and $E [L(f^o)] \equiv r(t, f^o, w^o) L^2(f^c - f^o)$.

If the inequality in (5) is reversed, then the prisoner should be denied parole and reconsidered at a later date. It may be the case that there does not exist an $f^o < f^c$ such that (5) holds, in which case the full sentence should be served.

As a parenthetical query it might be asked why the need for periodical review of the same prisoner? If the exogenous variable w were constant for a prisoner throughout his incarceration, then with equality holding in (5), the equation could be solved for f^a , the optimal incarceration. But with w

changing in an undetermined manner, the prisoner's case must be periodically reviewed.

This discussion of the optimal parole suggests that as far as rehabilitation is concerned, court ordered sentences should be indeterminate, and the prisoner should be released only when (5) holds (with f^c replaced by the expected longevity of the prisoner). Such legislation was in effect proposed by the Royal Committee¹⁴ in dealing with offenders who belong to the category "dangerous offender". In the terminology used here, a "dangerous offender" is a convicted offender who is deemed to have a value of $E[L]$ greater than some prescribed level. That is, the class of "dangerous offenders" encompasses those individuals who have committed a serious crime and who are judged likely to inflict significant cost on society if they are not remanded in custody. It might be argued that indeterminate sentences should apply to all convicted criminals. However, the practical difficulty of precisely measuring L^1 , L^2 and r , plus the misuse which such a system could easily fall prey (e.g., the indeterminate incarceration of political prisoners) make the suggestion less tenable. But even within present Canadian prisons there are policies which make sentences more determinate than an optimal policy would suggest. For example, the Penitentiaries Act of 1961 allows any fixed term prisoner remission of one-quarter of his sentence for good conduct.¹⁵ Another section allows for three days remission of sentence for each month of calendar time worked, and this remission is "not subject to forfeiture for any reason".¹⁶ Thus it is common for prisoners to feel that such "good time" is more in the nature of a right which they are entitled than a reward which they earn. Hence the incentive to good behavior and rehabilitation is less under the present system than it would be under a regime of indeterminate or semi-indeterminate sentences.

III. Social Welfare and Optimization Policies

The preceding discussion has established the need for a unified approach to the normative aspects of the economics of crime. Such an approach was taken by Becker [1], who attempted to find levels of police activity and severity of punishment that would minimize the social cost of crime. Becker defined this social cost as the sum of the net direct damage to society from crime, the cost of apprehending and convicting offenders, and the cost to society of implementing punishment. The position taken here is that Becker omitted an important consideration in failing to deal with the problem of recidivism. That is, he did not include in his analysis the cost borne by society for the failure of rehabilitation. That the recidivism rate is high is well known. In Canada, of 4,057 males admitted to Federal penitentiaries in 1969, only 868 (21%) had never experienced a previous commitment, whereas 1,196 (29%) had five or more previous commitments.¹⁷ Furthermore, it appears that the incidence of criminal careers may be more influenced by economic factors than other considerations. Glaser, for example, found that economic crimes without violence are the most recidivistic category, whereas non-economic crimes are the least recidivistic [7, pp. 41-49.] Hence the economic causes of recidivism are of particular interest. These considerations indicate that any neglect of the effects of rehabilitation will seriously bias the results of an economic study of optimality conditions.

The problem that society faces is to choose values of police activity, p (the probability that an offender will be apprehended,) the length of sentence, f , and the quality and intensity of rehabilitative training, t , in an attempt to minimize the social cost of crime, defined as:

$$(6) \quad L(O) \equiv D(O) + C(p,O) + b(t,f)O_p + r(t,f)O_p J$$

where $L(O) \equiv$ monetary loss to society due to level of offenses O .

$D(O) \equiv$ direct loss to society; the sum of foregone legitimate output due to the use of labor inputs in criminal activities, and to the loss of legitimate labor effectiveness due to the risk of loss of the fruits of one's legitimate labor. It is assumed that $D' > 0$ and $D'' > 0$.

$C(p,O) \equiv$ the cost of apprehending and convicting offenders, which depends on p , the probability of apprehension, and on O , the level of offenses. It is assumed that $C_p > 0$, $C_{pp} > 0$, $C_O > 0$, $C_{OO} > 0$ and $C_{Op} = 0$.

$bOp+rOpJ \equiv$ the social cost of punishment: the sum of the direct cost of physically implementing the punishment and the cost of ineffective punishment--the present value of the losses accruing to society from a life of crime.

The supply of offenses, $O = O(p,f,u)$, possesses the properties $O_p < 0$, $O_f < 0$, $O_{pp} > 0$ and $O_{ff} > 0$. The variable u contains all factors affecting criminality other than p and f . It is assumed that offenders do not consider the rehabilitative effects of incarceration when deciding whether or not to commit an offense.

The loss function (6) differs from Becker's in the terms incorporating the social cost of punishment.¹⁸ Here $b(t,f)$ is defined as the direct total cost of incarcerating an offender for f days with level of training t . It is assumed that $b_1 > 0$, $b_{11} > 0$, $b_2 > 0$, $b_{22} > 0$ and $b_{12} = b_{21} < 0$. That is, the cost of training increases at an increasing rate, the cost of incarceration increases at an increasing rate,¹⁹ and incarceration and training are technical complements (from the discussion in II). Furthermore, specify $b(0,f) = 0$; i.e., if the level of rehabilitative training is zero, prisoners can be forced to maintain the prison at negligible cost to society. Choosing $t = 0$, then, is analytically equivalent to choosing fines as a method of punishment in that the direct social cost of implementing punishment is approximately zero. bOp is the product of the cost of incarceration (b) and the number of convicted offenders (O_p).

The second social cost of punishment, $rOpJ$, represents the present value of social harm done by a punished offender who does not reform, where J represents the present value of the social loss of a criminal career after punishment is completed, and r represents the probability of recidivism.²⁰ It is assumed that $r_1 < 0$, $r_{11} > 0$, $r_2 < 0$, $r_{22} > 0$ and $r_{12} = r_{21} < 0$. That is, increased training reduces the probability of recidivism at a decreasing rate, increased incarceration reduces the probability of recidivism at a decreasing rate,²¹ and increased training causes marginal units of incarceration to be more effective in reducing the probability of recidivism.²² The specifications on r are consistent with the economic approach that an increase in expected legitimate earnings reduces the incentive for a would-be criminal to commit an offense.

The necessary conditions²³ for society to minimize the social cost of crime are:

$$(7) \quad D' + C_0 = - bp(1 - \frac{\phi_f}{\epsilon_f}) - rpJ(1 + \frac{\psi_f}{\epsilon_f})$$

$$(8) \quad D' + C_0 + C_p/O_p = - (bp + rpJ)(1 - \frac{1}{\epsilon_p})$$

$$(9) \quad b_1 = - r_1 J$$

where

$$\phi_f \equiv b_2 \frac{f}{b} \quad \epsilon_f \equiv - O_f \cdot \frac{f}{O}$$

$$\psi_f \equiv - r_2 \frac{f}{r} \quad \epsilon_p \equiv - O_p \cdot \frac{P}{O}$$

Assuming that the left-hand side of (8) is positive, then $\epsilon_p < 1$. From (7) and (8), it can be seen that $\epsilon_f/\epsilon_p < \phi_f$. It is probable that a percentage increase in sentence length results in a smaller relative increase in incarceration costs

(i.e., $\phi_f < 1$). Hence it follows that $\epsilon_f < \epsilon_p$, as in Becker's analysis. Taking $b > 0$ as a given constant, (7) and (8) can be compared with Becker's first order conditions:

$$(21-B) \quad D' + C_0 = - bpf(1 - \frac{1}{\epsilon_f})$$

$$(22-B) \quad D' + C_0 + C_p/O_p = - bpf(1 - \frac{1}{\epsilon_p}).$$

Assuming a given level of $p > 0$, a comparison of (7) and (21-B)²⁴ indicates that the direct consideration of the cost of a recidivist implies choosing a higher value of f . It pays society to lower the cost of recidivism by increasing sentence length. Assuming a given level of $f > 0$, a comparison of (8) and (22-B) indicates that a lower value of p will be chosen here. Interpreting the right-hand sides of (8) and (22-B) as the "marginal revenue" of an offense through a reduction in p , the (negative) "average revenue" is greater (in absolute terms) by the amount rpJ . That is, the (negative) "price" of an offense is understated by Becker through the neglect of the probabilistic cost of a recidivist.

From (7) it is seen that

$$(10) \quad b_2 > -r_2J.$$

That is, society should choose a value of f such that the marginal cost of incarceration exceeds the marginal benefit to society from keeping an offender in custody an extra day, where the "marginal benefit", $-r_2J$, is the product of the reduction in the probability of recidivism due to an extra day's incarceration and the cost to society of a criminal life. The reason why the usual optimality condition of marginal cost equal to marginal benefit is not observed here is because incarceration length has both a deterrent effect ($O_f < 0$) and a rehabilitation effect ($r_2 < 0$). Only if the deterrent effect were non-existent would society equate the marginal cost and marginal benefit of incarceration. Since $O_f < 0$, society should choose a greater value of f than otherwise. From (9) the marginal

cost of training is equated to the marginal benefit of training, where the "marginal benefit", $-r_1J$, is the product of the reduction in recidivism probability due to increased training and the cost of a criminal life. (9) is depicted in Figure 3.

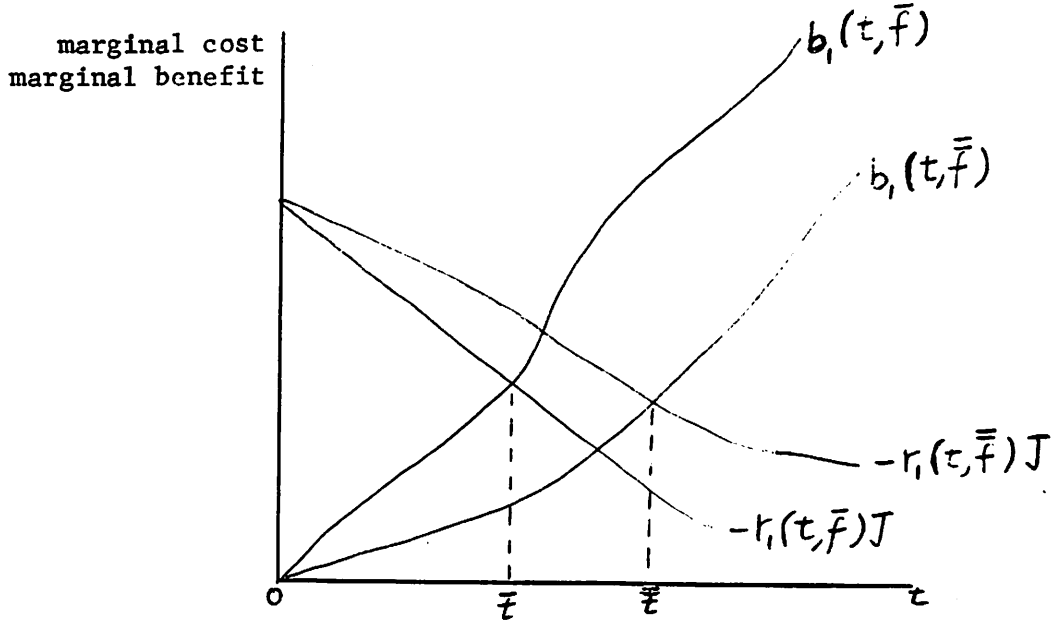


Figure 3

Since $b_{12} < 0$ and $r_{12} > 0$, an increase in f (in Figure 3, $\bar{f} < \bar{\bar{f}}$) increases both the marginal costs and marginal benefits of training. For every value of f , there exists an optimal value of t (e.g., the pairs (\bar{t}, \bar{f}) and $(\bar{\bar{t}}, \bar{\bar{f}})$ in Figure 3).

Hence the relationship

$$(11) \quad t^* = y(f^*), \quad y'(f^*) > 0$$

is specified. (11) is equivalent to (4), the prison's efficient expansion locus.

The concrete specification of the maximand of Section I (here presented as a minimand) has allowed the discovery that the expansion path is positively sloped.

The longer the prisoner is incarcerated, the more intensive should be his training program. Notice also that as f declines, so does t , and hence, b . Recalling

that a system of fines is analytically equivalent to choosing a value of $t = 0$,

(11) is consistent with the observed behavior that fines should only be considered

when the severity of punishment (interpreted here as f) is relatively small.

Finally, the analysis allows us to evaluate changes in the optimal levels of f , p and t due to exogenous parameter shifts. For example, the relative amounts of resources devoted to offenders of different ages can be evaluated. According to Glaser, "the younger a prisoner was when first arrested, convicted, or confined for any crime, the more likely he is to continue in crime."²⁵ Hence define the parameter β as the remaining expected longevity at age of first arrest. Then $J = J(\beta)$, $J'(\beta) > 0$. Implicitly differentiating (7), (8) and (9), and assuming the second order conditions hold, it is seen that

$\frac{d\hat{f}}{d\beta} > 0$, $\frac{d\hat{p}}{d\beta} > 0$ and $\frac{d\hat{t}}{d\beta} > 0$, where "hat" represents the optimal value of the variable. Thus the younger is an offender, the greater should be his incarceration length, training intensity and probability of apprehension. Also, if criminals could be classified according to their proclivity to commit certain types of crime ("normative typologies")²⁶ and these categories expressed in terms of a parameter δ , where a higher value of δ represents a greater social loss category of crime, then setting $J = J(\delta)$, $J'(\delta) > 0$, and performing the same operations as above, it can be seen that $\frac{d\hat{f}}{d\delta} > 0$, $\frac{d\hat{p}}{d\delta} > 0$ and $\frac{d\hat{t}}{d\delta} > 0$. Thus the more dangerous society considers any criminal "type," the more resources should be devoted to his apprehension and rehabilitation.

IV. Conclusion

Throughout this paper it was assumed that there exists a level of training such that prisons have a rehabilitative effect upon prisoners. In reviewing actual rates of recidivism it is not at all clear that the correctional authorities have chosen such training programs, or even that such programs exist. In the interests of efficiency, the evaluation of different programs as they affect the recidivism rate is important. The paucity of data and research on such

programs (and penal institutions in general) is shocking, especially in view of the economic burden they impose upon society.

It should also be remarked that nothing in this paper should be construed as minimizing the importance of destroying the roots of crime. Just as preventive medicine is attaining its well-deserved position of prominence in the medical hierarchy, so should "preventive criminology" be regarded by social scientists. It is well known that the intelligence level of prisoners is distributed in approximately the same fashion as the population at large, but that educational achievements and job-skill levels are much lower for the prison population. Hence anti drop-out and vocational training programs should be given high priority and administered at an early age.

Finally, it should be noted that the purpose of this paper is to stimulate the interest of economists in the problems of prisons and rehabilitation. Economists have devoted a considerable amount of resources to the study of such non-profit organizations as hospitals and postal organizations, but have not yet scratched the surface of the complexities of the penal system. One might argue that prisons are not susceptible to economic analysis because the people who consume the services are the offenders, who do not desire them. However it is really society at large that is consuming prison services, and since resources are devoted to their output, it is a legitimate economic question to ask if such resources are being used in an efficient manner.

Mathematical Appendix

An attempt will be made here to establish the conditions for a local (interior) minimum of the function L with respect to the variables f, p and t. In several cases the mathematical conditions do not unambiguously hold, but in no case does the assumption of the condition violate intuition. The assumptions on the functions are:

$$(A-1) : O_p < 0 \quad O_f < 0 \quad (A-4) : b_1 > 0 \quad b_2 > 0$$

$$O_{pp} > 0 \quad O_{ff} > 0 \quad b_{11} > 0 \quad b_{22} > 0$$

$$(A-2) : C_p > 0 \quad C_0 > 0 \quad b_{12} = b_{21} < 0$$

$$C_{pp} > 0 \quad C_{00} > 0 \quad (A-5) : r_1 < 0 \quad r_2 < 0$$

$$C_{p0} = C_{0p} = 0 \quad r_{11} > 0 \quad r_{22} > 0$$

$$(A-3) : D' > 0, D'' > 0 \quad r_{12} = r_{21} < 0$$

$$(A-6) : \epsilon_p = \alpha, \epsilon_f = \beta$$

$$\psi_f = \gamma, \phi_f = \delta$$

a, β , γ , δ all constants

(A-1) through (A-5) are explained in the text except for $C_{0p} = 0$, which is assumed following Becker. For the sake of simplification, all elasticities are taken to be constant (A-6).

The problem is to choose values of f, p and t that

$$(M-1) \text{ minimize } L(O) = D(O) + C(p, O) + b(t, f)Op + r(t, f)OpJ. \\ f, p, t$$

The necessary and sufficient conditions for a local (interior) minimum are:

- 1) $L_f = 0, L_p = 0, L_t = 0$
- 2) $L_{ff} > 0, L_{pp} > 0, L_{tt} > 0$
- 3) $L_{pp}L_{tt} > L_{tp}^2, L_{ff}L_{tt} > L_{tf}^2, L_{pp}L_{ff} > L_{pf}^2$
- 4)
$$\begin{vmatrix} L_{ff} & L_{fp} & L_{ft} \\ L_{pf} & L_{pp} & L_{pt} \\ L_{tf} & L_{tp} & L_{tt} \end{vmatrix} > 0$$

From (M-1),

$$(M-2) \quad L_f = (D' + C_0)O_f + bp(1 - \phi_f/\epsilon_f)O_f + rpJ(1 + \psi_f/\epsilon_f)O_f,$$

from which the first order condition

$$(M-3) \quad D' + C_0 = - bp(1 - \phi_f/\epsilon_f) - rpJ(1 + \psi_f/\epsilon_f)$$

is derived. From (M-3) it is seen that

$$bp(1 - \phi_f/\epsilon_f) + rpJ(1 + \psi_f/\epsilon_f) < 0,$$

which can be reduced to $b_2 > -r_2J$, a condition discussed in the text.

From (M-2),

$$(M-4) \quad L_{ff} = (D'' + C_{00})O_f^2 + [D' + C_0 + bp(1 - \phi_f/\epsilon_f) + rpJ(1 + \psi_f/\epsilon_f)]O_{ff}.$$

If (M-3) holds, then (M-4) may be written as

$$(M-5) \quad L_{ff} = (D'' + C_{00})O_f^2 > 0$$

Hence L_{ff} is unambiguously positive.

Also from (M-1),

$$(M-6) \quad L_p = (D' + C_0)O_p + C_p + (bp + rpJ)(1 - 1/\epsilon_p)O_p,$$

from which the first order condition

$$(M-7) \quad D' + C_0 + C_p/O_p = - (bp + rpJ)(1 - 1/\epsilon_p)$$

is derived. Following Becker in assuming that $D' + C_0 + C_p/O_p > 0$, then

(M-7) implies $\epsilon_p < 1$. From (M-6),

$$(M-8) \quad L_{pp} = (D'' + C_{00})O_p^2 + C_{pp} + (b + rJ)(1 - 1/\epsilon_p)O_p \\ + O_{pp}[D' + C_0 + (bp + rpJ)(1 - 1/\epsilon_p)] ,$$

which in view of (M-7) may be rewritten as

$$(M-9) \quad L_{pp} = (D'' + C_{00})O_p^2 + C_{pp} + (b + rJ)(1 - 1/\epsilon_p)O_p - \frac{O_{pp}C_p}{O_p} .$$

Each of the four terms in (M-9) is positive, hence $L_{pp} > 0$. In comparing

(M-3) with (M-7), it is seen that

$$(M-10) \quad \frac{b}{rJ} = \frac{rJO_p(-1/\epsilon_p - \psi_f/\epsilon_f) + C_p}{rJO_p(1/\epsilon_p - \phi_f/\epsilon_f)} > 0 .$$

The numerator of (M-10) is positive, so for (M-10) to hold, $(1/\epsilon_p - \phi_f/\epsilon_f) < 0$, or $\epsilon_f/\epsilon_p < \phi_f$, a condition which is discussed in the text.

Differentiating (M-1) with respect to t ,

$$(M-11) \quad L_t = b_1 O_p + r_1 O_p J ,$$

from which the first order condition

$$(M-12) \quad b_1 = - r_1 J$$

is derived. Again from (M-11),

$$(M-13) \quad L_{tt} = b_{11} O_p + r_{11} O_p J .$$

Both of the terms in (M-13) are positive, so that L_{tt} is unambiguously positive.

Next consider the inequality $L_{pp} L_{tt} > L_{tp}^2$. L_{tp} may be written as

$$L_{tp} = b_1 O_p + b_1 O + r_1 O_p J + r_1 O J ,$$

or

$$(M-14) \quad L_{tp} = (O + pO_p)(b_1 + r_1 J) .$$

From (M-12), $L_{tp} = 0$. Since $L_{pp} > 0$ and $L_{tt} > 0$, $L_{pp}L_{tt} > L_{tp}^2$ unambiguously.

The inequality $L_{ff}L_{tt} > L_{tf}^2$ may be written as

$$(M-15) \quad (D'' + C_{00})O_f^2(b_{11} + r_{11}J) > (b_{12} + r_{12}J)^2 O_p,$$

which does not unambiguously hold, but does not appear to violate intuition.

Differentiating (M-6) with respect to f ,

$$L_{pf} = (D'' + C_{00})O_f O_p + [D' + C_0 + (bp + rpJ)(1 - 1/\epsilon_p)]O_{pf},$$

or recalling (M-7)

$$(M-16) \quad L_{pf} = (D'' + C_{00})O_f O_p - \frac{O_{pf}C_p}{O_p}.$$

Assuming that the first term in (M-16) dominates so that L_{pf} is positive, the inequality $L_{pp}L_{ff} > L_{pf}^2$ holds if

$$(M-17) \quad (D'' + C_{00})O_p^2 + C_{pp} + (b + rJ)(1 - 1/\epsilon_p)O_p - \frac{O_{pp}C_p}{O_p} \\ > (D'' + C_{00})O_f O_p - \frac{O_{pf}C_p}{O_p}$$

and

$$(M-18) \quad (D'' + C_{00})O_f^2 > (D'' + C_{00})O_f O_p - \frac{O_{pf}C_p}{O_p}.$$

(M-17) may be rewritten as

$$(M-19) \quad (D'' + C_{00})(O_p - O_f)O_p + C_{pp} + (b + rJ)(1 - 1/\epsilon_p)O_p \\ + C_p/O_p(O_{pf} - O_{pp}) > 0.$$

Since the second and third terms of (M-19) are positive, a sufficient condition for (M-19) to hold is

$$(M-20) \quad (D'' + C_{00})(O_p - O_f)O_p + C_p/O_p(O_{pf} - O_{pp}) > 0.$$

Sufficient conditions for (M-20) are $O_p - O_f < 0$ and $O_{pf} - O_{pp} < 0$. These conditions do not appear to violate intuition. (M-18) may be rewritten as

$$(M-21) \quad (D'' + C_{OO})(O_f - O_p)O_f + \frac{O_{pf}C_p}{O_p} > 0 .$$

For (M-20) and (M-21) to hold simultaneously with $O_p - O_f < 0$, it must be that $O_{pf} < 0$, which agrees with intuition. Hence it can be said that although $L_{pp}L_{ff} > L_{pf}^2$ does not unambiguously hold, its assumption does not appear unreasonable.

The final condition to be checked is the sign of the determinant

$$\begin{vmatrix} L_{ff} & L_{fp} & L_{ft} \\ L_{pf} & L_{pp} & L_{pt} \\ L_{tf} & L_{tp} & L_{tt} \end{vmatrix} .$$

Since $L_{tp} = 0$, the conditions may be written as

$$(M-22) \quad L_{tt}(L_{ff}L_{pp} - L_{fp}^2) - L_{tf}^2L_{pp} > 0 .$$

Both terms in (M-22) are positive, hence the inequality sign is not unambiguously positive. However, an examination of (M-22) does not indicate any reason why the inequality should not be as presented.

Footnotes

¹ See the references at the end of this paper and in Becker [1].

² The economic approach to crime does not deny sociological and psychological factors, but it holds that for a given individual and "other things equal," changes in economic factors significantly affect the decision to commit crime.

³ For an extension of Becker's paper in another direction, see Harris [8]. Harris discusses society's optimal strategies when the social loss from punishing an innocent person is directly considered.

⁴ Rehabilitation may be achieved through various different types of programs, including formal education, vocational training and on-the-job training. Glaser [7, chapter 11] notes that: 1) regular work during incarceration would be the longest continuous employment that most prisoners have ever had; 2) regularity of prior employment is statistically a better indicator of recidivism than is type of work previously performed, and 3) the major contributions of work in prison to inmate rehabilitation are the positive influence of work supervisors on inmates, and habituation of inmates to regularity in employment. Hence the form of rehabilitative training may be less important than the length of the training period. In Becker's [2] terminology, it is general and not specific training that is the major rehabilitative influence of prison employment. This is significant because in the non-institutional part of the economy, general training must be paid for by the laborer (Becker [2], page 13). This suggests that one characteristic common to offenders is that they refuse to pay the cost of general training when given the choice.

⁵ The second derivative of the product transformation curve $x^0 = h(t, f)$ is

$$\left. \frac{d^2 t}{df^2} \right|_{x=x^0} = -\frac{1}{h_1} [h_1 h_{22} - 2h_2 h_{12} + \frac{h_2^2}{h_1} h_{11}]. \quad \text{Assuming that } h_1 > 0, h_2 > 0,$$

$h_{11} > 0, h_{22} > 0$, then $h_{12} < 0$ is sufficient for $\left. \frac{d^2 t}{df^2} \right|_{x=x^0} < 0$. That is, technical

complementarity ensures concave to the origin product transformation curves.

⁶ The concept of "protection" has been left purposefully vague here. In III, "maximizing the level of protection" is construed to mean "minimizing the social cost of crime".

⁷It might be argued that for low values of t and high values of t^- , the incremental changes in incarceration needed to keep society at the same level of protection decrease as t^- increases. At the extreme, perhaps society is better off not incarcerating an offender if he is going to spend his time learning criminal skills. However, less severe punishment will increase the number of original offenses because the deterrent effect is lessened, hence acting to decrease the level of protection. The equal protection curves in Figure 1 are drawn under the assumption that this latter effect is the stronger.

⁸Indeed, the Royal Committee [19] did not stress the importance of harmony among the CJS branches nearly as strongly as is suggested in this analysis. However, the need for such an approach is clearly indicated [e.g., p. 16 and pp. 274-275].

⁹Gingeroff [6, p. 20] suggests a practical way of achieving coordination of sentence length and training by a review of sentence by the sentencing court after the offender has spent some time in custody.

¹⁰Here "recidivist" refers to a criminal who commits a crime after release from prison, whether or not he is apprehended and convicted for the repeat offense.

¹¹Becker [1] argues the case for fines. However, his analysis is remiss in that he does not consider the possibility of recidivism.

¹²The Royal Committee recommended the abolition of corporal punishment [19, p. 208]. At the time of this writing, whipping is a legal form of punishment in Canada, although it has been rarely evoked by the courts in the past ten years.

¹³The term "marginal" is not being used in its usual sense. The parole board periodically decides whether to release the prisoner or to hold him. If they decide not to grant parole, they typically do not mention when the prisoner should be granted parole. Thus, in calculating "marginal" benefits, for example, they compare the benefit to society from releasing a prisoner at the time of review with the benefit from holding him until the court appointed incarceration has expired. The difference is the "marginal" benefit. They do not compare the benefit to society from releasing the prisoner at review time with the benefit from releasing him at the next parole review.

¹⁴See the Royal Committee [19, Chapter 13].

¹⁵Section 22(1) of the Penitentiaries Act of 1961.

¹⁶Section 24 of the Penitentiaries Act of 1961.

¹⁷[17, Table 5, p. 20]. This is not to suggest that the overall recidivism rate is as high as indicated above. Because of data limitations, the actual rate of recidivism is almost impossible to calculate. An unpublished Canadian study of the St.-Vincent-de-Paul (Quebec) complex carried out by Justin Ciale and reported in the Royal Committee exhibited a 56% recidivism rate within five years of release [19, pp. 335-336]. In a review of recent U.S. studies, Glaser puts the rate at about 33% [7, chapter 2].

¹⁸There is also a slight difference in the explication of $D(0)$, direct social losses, but the difference is of no consequence here.

¹⁹Justification for $b_{22} > 0$ is that the longer a prisoner's sentence, other things equal, the less²² is the incentive for him to behave in conformity to prison rules. For example, a prisoner serving a one year sentence is less likely to attempt an escape than a prisoner serving a twenty year term.

²⁰It is obvious that a one time recidivist will not necessarily spend his entire life in the pursuit of crime, as is implied in the explication of $rOpJ$. However, if society held constant its values of t and f , the same calculations which led an offender to be a first-time recidivist would do so again. Of course, f does increase for multiple offenders, so the statement is only an approximation of reality, and is adopted here primarily for analytical convenience.

²¹See section II above.

²²"...prison education is statistically associated with above average post-release success only when the education is extensive and occurs in the course of prolonged confinement." Glaser [7, p. 508].

²³The second order conditions appear in the mathematical appendix.

²⁴Becker's bf corresponds to $b(t, f)$ in the notation used here.

²⁵Glaser [7, pp. 36-37].

²⁶See, for example, Schafer [11, esp. chapter VII].

REFERENCES

1. Becker, G., "Crime and Punishment: An Economic Approach," Journal of Political Economy, March/April 1968, pp. 169-217.
2. _____, "Investment in Human Capital: A Theoretical Analysis," Journal of Political Economy, Oct. 1962 (Sup.), pp. 9-49.
3. Blumstein, A. and R. Larson, "Models of a Total Criminal Justice System," Operations Research, March-April 1969 (17), pp. 199-232.
4. Evans, R. Jr., "The Labor Market and Parole Success," Journal of Human Resources, III, 2, pp. 201-212.
5. Firey, W., "Limits to Economy in Crime and Punishment," Social Science Quarterly, June 1969, Vol. 50, No. 1, pp. 72-77.
6. Gingeroff, A., "Crime, Criminal Law and the Concept of Treatment," Canadian Journal of Corrections, Vol. 10, 1968, pp. 12-24.
7. Glaser, D. The Effectiveness of a Prison and Parole System, Bobbs-Merrill Co., 1964.
8. Harris, J., "On the Economics of Law and Order," Journal of Political Economy, Jan./Feb. 1970, pp. 165-174.
9. Katzman, M., "The Economics of Defense against Crime in the Streets," Land Economics, 44, 1968, pp. 431-440.
10. Reynolds, M., Economics of Crime, unpublished Ph.D. Dissertation, University of Wisconsin, 1971.
11. Schafer, S., Theories in Criminology, Random House, New York, 1969.
12. Schelling, T., "What is the Business of Organized Crime," Harvard Discussion Paper No. 182, April 1971.
13. Stigler, G., "The Optimum Enforcement of Laws," Journal of Political Economy, May/June 1970, pp. 526-536.
14. Sutherland, E. and D. Cressey, Criminology, 8th edition, 1970, J. B. Lippincott Co., Philadelphia.
15. Thurow, L. and C. Rappaport, "Law Enforcement and Cost Benefit Analysis," Public Finance, 1969 (24) pp. 48-63.
16. Tullock, G., "An Economic Approach to Crime," Social Science Quarterly, June 1969, Vol. 50, No. 1, pp. 59-71.
17. Correctional Institution Statistics 1969-1970, Dominion Bureau of Statistics, Queen's Printer, Ottawa.

18. President's Commission on Law Enforcement and Administration of Justice, Task Force on Corrections, U.S. Government Printing Office, Washington, 1967.
19. Report of the Canadian Committee on Corrections, (R. Ouimet, Chairman), The Queen's Printer, Ottawa, March 1969.
20. "Round Table on Allocation of Resources in Law Enforcement," (T. Schelling, Chairman), discussants A. Downs, S. Rottenberg and R. Hoffman, American Economic Review, May 1969, pp. 504-512.