1985

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ISSN 0228-4235
ISBN 0-7714-0682-7

THE CENTRE FOR THE STUDY OF INTERNATIONAL ECONOMIC RELATIONS

WORKING PAPER NO. 8530C

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This paper contains preliminary findings from research work still in progress and should not be quoted without prior approval of the authors.

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A DYNAMIC EQUILIBRIUM MODEL OF INFLATION AND UNEMPLOYMENT

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October, 1985

I. INTRODUCTION

There has been a long-standing debate within the economics profession concerning the effects of inflation and its impact on unemployment. Following the statistical work of Phillips (1958) a growing belief developed that unemployment and inflation had a negative influence on each other. At the time it seemed a short step to conclude, as did many in the profession, that the new-found Phillips curve was a structural relationship, and consequently offered a 'menu of choice' between the levels of unemployment and inflation, which could be exploited by the policymaker. After some years of experimentation with this relationship, disillusionment began to set in as both unemployment and inflation rose. The theoretical underpinnings of the Phillips curve tradeoff began to be questioned by Friedman (1968) and Phelps (1970), and received their ultimate blow with the work of Lucas (1972). Friedman (1977) eyeing emerging data at the time, in fact, suggested that the operational Phillips curve was upward sloping. He conjectured that inflation has disruptive effects on economic activity with high rates of inflation going hand-in-hand with high unemployment. To date, the theoretical links between inflation and unemployment seem to have been difficult to ascertain.

Perhaps as a consequence of the perceived failure of existing models, which emphasized the dominant role of monetary factors as the driving force behind cyclical fluctuations, recently a literature has developed which suggests that business cycles are primarily a real phenomenon (see, for example, Kydland and Prescott (1982) and Long and Plosser (1983)). This line of research has tended to de-emphasize the role that monetary factors play in causing economic fluctuations. As a result, it has not been oriented toward seeking an explanation of the observed anomalous correlations between inflation, and unemployment or output.

The present paper attempts to capture part of the spirit of the real business cycle literature in a model in which money is introduced. A stochastic general equilibrium model is constructed which is capable of examining the covariance properties between unemployment and inflation, both conditioned and unconditioned upon exogenous factors such as the current growth rate of the money supply, the level of productivity, etc. The model exhibits some interesting features. First, it is shown that natural forces in the economy may operate to generate an endogenous negative relationship between unemployment and inflation. This is interesting since it has been argued that real business cycle models are incapable of explaining the procyclical nature of prices (see Lucas (1977)). Second, despite this apparent negative association between unemployment and inflation the actual tradeoff which the policymaker confronts is one in which these two variables are positively related. Also, in contrast to the work of Lucas (1972), the model employed does not rely on agents' misperceptions about the current rate of monetary expansion.

The current work borrows ingredients from several sources. First Aschauer (1980), Aschauer and Greenwood (1983), and Carmichael (1985 a,b) have analyzed the deleterious impact that inflation can have an equilibrium employment. Their papers build on Stockman (1981) who investigates the adverse effect that inflation can have on an economy's steady-state capital stock. The current study models the disruptive effects of inflation in a similar fashion. Second, a role for real shocks as a driving force behind aggregate fluctuations is introduced in a manner similar to that of Kydland and Prescott (1982), and Long and Plosser (1983). Third, King and Plosser (1984) have suggested that some components of the money supply may react endogenously to real disturbances. This observation is incorporated in the model of this paper. Fourth, drawing the link in equilibrium models between shifts in unemployment (rather than employment) on the one hand, and monetary and nonmonetary disturbances on the other is tentative at best. The present study attempts to overcome this obstacle by introducing a nonconvexity into agents' labor supply decisions along the lines proposed by Rogerson (1985). Such nonconvexities are capable of generating unemployment in equilibrium models, and Rogerson (1985) shows how they can often be easily handled by a simple extension to the standard competitive equilibrium construct.

The remainder of the paper is organzied as follows. In Section II the underlying physical environment of the economy is described. The representative agent/firm's optimization problem and decision rules are presented in Section III. Next, Section IV characterizes the model's general equilibrium. The joint behavior of unemployment and inflation in response to monetary and real shocks is investigated in Section V. A discussion of the results and their implications is contained in Section VI.

II. THE PHYSICAL ENVIORNMENT

Consider the following model of a closed economy inhabited by a continuum of identical agents distributed uniformly over the interval [0,1]. An agent's goal is to maximize the expected value of his lifetime utility as given by

$$E_0\{\sum_{t=0}^{\infty} \beta^t [U(c_t) + V(\ell_t)]\} \qquad \beta \in (0,1)$$

where β is the agent's (constant) subjective discount factor, and c_t and ℓ_t denote his period-t consumption and work effect, respectively.[1] Both $U(\cdot)$ and $(-V(\cdot))$ are increasing, strictly concave continuously differentiable functions. Each agent is additionally endowed with a single unit of capital in every period, which he chooses to supply inelastically.

The aggregate output of a given market in period t is given by the constant-returns-to-scale production process

$$y_t = f(\phi_t \ell, K, \lambda_t)$$

with $\phi_t \ell$ and K representing the aggregate amount of labor and capital employed respectively, and λ_t representing a technology shock which is known at the beginning of period t. The value of λ_t, which is realized at the beginning of period t is generated by a stochastic process whose distribution is denoted by $G(\lambda_t)$. Restrictions on the function $f(\cdot)$ will be listed below. Given the behavior of agents with respect to capital, note that $K=\int_0^1 1ds = 1$.

Each agent has two sources of income in each period t. First, at the beginning of each period he earns a wage-cum-dividend payment associated with the operation of a firm running the production process $f(\cdot)$. Second, he receives a transfer payment in the real amount of τ_t from the government. The individual can use this period-t income either to undertake consumption, c_t, purchase real bonds, b_t, or acquire real cash balances, m_t. A real bond purchased for one unit of consumption in period t pays back $(1 + r_t)$ units of consumption in period t + 1, so that r_t is the period-t (possibly state-contingent) real interest rate. All transactions in the model must be effected using currency.[2] Thus, for instance, if in period t the individual buys c_t units of the consumption good this must be bought using currency from his holdings of real cash balances, m_t.

Each agent is assumed to face a nonconvexity in his labor supply decision. Specifically, he either works the fixed amount $\ell > 0$ or does not work at all.[3] The efficiency property of the standard competitive equilibrium concept is destroyed by such a nonconvexity in the agent's choice set. As discussed by Rogerson (1985), this apparent deficiency in the notion of the competitive equilibrium can be removed by extending the agent's choice set to include the possibility of a lottery over his consumption, labor, and asset holding decisions so as to convexify the choice set.[4] Specifically, imagine the production process as being owned by a firm who offers the agent an income-employment contract of the following form. In each period t the firm and the agent agree on a probability ϕ_t^w that the agent will be called to work ℓ units. From an agent's point of view ϕ_t^w is then a choice variable. An individual who is chosen to be employed receives a wage-cum-dividend payment from the firm designed to allow the agent to undertake c_t^w units of consumption, buy b_t^w units of bonds, and acquire m_t^w units of real currency. The probability of being unemployed is $\phi_t^u = (1 - \phi_t^w)$, and in this state the agent receives a wage-cum-dividend to provide for c_t^u units of consumption, b_t^u units of real bonds, and m_t^u units of real currency.

There is a government in this economy whose purpose is to provide transfer payments, τ_t, in each period t to its citizens via money creation. Its period-t budget constraint is

$$P_t \tau_t = M_{t-1}^s (\mu_t - 1), \qquad \text{(with } \mu_t \equiv M_t^s / M_{t-1}^s\text{)}$$

where P_t is the period-t nominal price level, M_t^s is the nominal supply of money in this period, and μ_t is the gross growth rate of the nominal money supply between periods t and t-1. It is assumed that the value of μ_t is generated by a stochastic process whose transition distribution function is $F(\mu_t | \mu_{t-1})$.

Before proceeding to an analysis of private sector decision-making, a brief statement about the timing of transactions will be given. An individual enters a period with a certain amount of currency left over from the previous period. At the beginning of this period the individual receives in cash an earnings-cum-dividend payment from the firm arising from its sales during the previous period. At this time the agent knows the value of all period-t exogenous variables including the current growth of the money supply and his state of employment for the period. He then enters the asset market, redeems the bonds he purchased during the previous period for money, and acquires new holdings of bonds with cash. During the remainder of the period the agent uses his holdings of currency to buy his consumption quantities of output and either works or stays at home depending on his state of employment.

Finally, it will be convenient to state now some assumptions to be used in the next section.

<u>Assumption (A1)</u> $U(c)$ is twice differentiable and strictly concave, and $\lim_{c\to 0} U'(c) = \infty$. Also, let $\lim_{c\to 0} [cU'(c)] > 0$ and $[cU'(c)]$ be nondecreasing in c. Last, assume $-\infty < V(\ell) < V(0) < \infty$.

<u>Assumption (A2)</u> Let λ_t be determined by a stochastic process whose distribution function is $G(\lambda_t)$. Let $Z = [\underline{\lambda}, \bar{\lambda}]$ so then $G(\cdot)$: $Z \to [0,1]$. For all continuous functions $w(\lambda)$ the integral $\int_Z w(\lambda)\, dG(\lambda)$ is assumed to exist.

<u>Assumption (A3)</u> $f(\cdot, \cdot, \cdot)$ is increasing, twice continuously differentiable, strictly concave in all its arguments. Assume $\lim_{\phi\to 0} [\frac{\partial f(\phi\ell,K,\lambda)}{\partial\phi}] = \infty$, and $f(0,K,\lambda) = 0 \quad \forall\ \lambda\in Z$.

<u>Assumption (A4)</u> $F(\mu_t|\mu_{t-1})$ is assumed to be the transition distribution function for μ_t. Let $Q = [\underline{\mu}, \bar{u}]$ and hence for all $\mu_t \in Q$, $F(\cdot|\mu_t)$: $Q \to [0,1]$. It is assumed that for all $\mu_t\in Q$, $\lambda\in Z$

$$\max_{\phi\in[0,1]} \left[\frac{\phi\,\frac{\partial f(\phi\ell,K,\lambda)}{\partial\phi}}{f(\phi\ell,K,\lambda)}\right]\left[\frac{\beta(\bar{y}\,U'(\bar{y}))}{-[V(\ell) - V(0)]}\int_Q \frac{1}{\mu_{t+1}}\, dF(\mu_{t+1}|\mu_t)\right] \le \gamma < 1$$

where $\bar{y} \equiv f(1\cdot\ell,K,\bar{\lambda})$, and γ will be specified below.

Also, for all continuous functions $w(\mu)$, the function

$\int_Q w(\mu)\, dF(\mu|\mu')$ is assumed to be continuous.

Finally, all integrals should be interpreted as being Lebesgue integrals with the qualification "almost everywhere" being omitted.

III. PRIVATE SECTOR DECISION-MAKING

The decision-making of consumer-workers and firms in competitive equilibrium can be summarized by the outcome of the following "representative" agent/firm's programming problem with the choice variables being c_t^j, m_t^j, b_t^j for $j=w,u$, and $\phi_t^w = (1-\phi_t^u)$

$$W(a_t, s_t) = \max\{\sum_j \phi_t^j [U(c_t^j)+V(\ell^j)]+\beta \int_Q \int_Z W(a_{t+1}, s_{t+1})dG(\lambda_{t+1})dF(\mu_{t+1}|\mu_t)\}$$

s.t.

$$\sum_j \phi_t^j [m_t^j + b_t^j] = \frac{P_{t-1}(s_{t-1})}{P_t(s_t)} f(\phi_{t-1}^w \ell,K,\lambda_{t-1}) + \tau_t + \sum_j \phi_{t-1}^j \Big[\frac{P_{t-1}(s_{t-1})}{P_t(s_t)}$$

$$\cdot (m_{t-1}^j - c_{t-1}^j) + (1+r_{t-1}(s_t,s_{t-1}))b_{t-1}^j\Big] \equiv a_t \qquad (1)$$

$$c_t^j \le m_t^j \qquad \forall j = w,u \qquad (2)$$

where $s_t \equiv (\mu_t,\lambda_t)$ denotes the economy's state vector.[5] The first constraint is the representative agent/firm's budget constraint. The left-hand side of this equation illustrates his uses of financial wealth at the beginning of period t, which take the form of acquisitions of money and bonds for the two states of nature, while the right-hand portrays his sources of funds at this time, which in total value are defined to be a_t. The second equation represents the agent/firm's cash-in-advance constraints and states that period-t consumption in state j cannot exceed his period-t money holdings in this state. The first-order conditions associated with the above problem are shown where φ_t and α_t^j are the Lagrange multipliers associated with the constraints (1) and (2).

$$\phi_t^j U'(c_t^j) = \phi_t^j \beta \int_Q \int_Z W_a(a_{t+1}, s_{t+1}) \frac{P_t(s_t)}{P_{t+1}(s_{t+1})} dG(\lambda_{t+1}) dF(\mu_{t+1}|\mu_t) + \alpha_t^j \qquad j=w,u \qquad (3)$$

$$\alpha_t^j + \phi_t^j \beta \int_Q \int_Z W_a(a_{t+1}, s_{t+1}) \frac{P_t(s_t)}{P_{t+1}(s_{t+1})} dG(\lambda_{t+1}) dF(\mu_{t+1}|\mu_t) = \varphi_t \phi_t^j \qquad j=w,u \qquad (4)$$

$$\phi_t^j \beta \int_Q \int_Z (1+r_t(s_t, s_{t+1})) W_a(a_{t+1}, s_{t+1}) \frac{P_t(s_t)}{P_{t+1}(s_{t+1})} dG(\lambda_{t+1}) dF(\mu_{t+1}|\mu_t)$$
$$= \varphi_t \phi_t^j \qquad j=w,u \qquad (5)$$

$$U(c_t^w) - U(c_t^u) + V(\ell) - V(0) - \beta \int_Q \int_Z W(a_{t+1}, s_{t+1}) [\frac{P_t(s_t)}{P_{t+1}(s_{t+1})}] \{ \frac{\partial f(\phi_t \ell, K, \lambda_t)}{\partial \phi_t}$$
$$+ (m_t^w - c_t^w) - (m_t^u - c_t^u) + [1+r_t(s_{t+1}, s_t)](b_t^u - b_t^w) \} dG(\lambda_{t+1}) dF(\mu_{t+1}|\mu_t)$$
$$= \varphi_t [b_t^w - b_t^u - m_t^w + m_t^u] \qquad (6)$$

Several interesting implications are obtained from the above first-order conditions. First, combining (3) and (4) yields

$$U'(c_t^j) = \varphi_t, \qquad j=w,u \qquad (7)$$

implying the lottery provides equal consumption across the employed and unemployed states, i.e., $c_t^w = c_t^u = c_t = U'^{-1}(\varphi_t)$. Thus, the income-employment contract offered by the firm to consumer-workers effectively provides for unemployment insurance benefits that allow individuals to maintain their consumption in the unemployed state.[6]

Next, so long as the implicit nominal interest rate, i_t, in the economy is positive the cash-in-advance constraints (2) will always hold with equality. To see this note that if $\alpha_t^j > 0$ then $m_t^j = c_t^j$. Now from (4) and (7), for α_t^j to be strictly positive the following must be true

$$\frac{U'(c_t)/P_t(s_t)}{\beta \int_Q \int_Z (U'(c_{t+1})/P_{t+1}(s_{t+1}))dG(\lambda_{t+1})dF(\mu_{t+1}|\mu_t)} > 1 \qquad (8)$$

where use has been made of the fact that $W_a(\cdot t) = \varphi_t$. The expression on the left-hand side of (8) is merely the formula for the <u>gross</u> nominal interest rate, $1+i_t$. The numerator of this expression is merely the utility worth of a dollar today while the denominator is the utility worth of one tomorrow. The ratio therefore measures the relative price of a current dollar in terms of future dollars which is of course the gross nominal interest rate. So long as $i_t > 0$, which as in Lucas (1982) will be assumed from here on, bonds dominate money as an abode of purchasing power and consequently individuals will not hold money across adjacent periods. This "corner" could be incorporated into the model in a manner similar to that of Lucas and Stokey (1984). Since this problem is not germane to any of the issues being addressed here, the assumption that $i_t > 0$ for all t does not seem particularly severe.

Finally, through the use of equations (5) and (7), and the fact that equation (2) holds with equality, equation (6) can be simplified to yield

$$-[V(\ell)-V(0)] = \beta \int_Q \int_Z U'(c_{t+1})\frac{P_t(s_t)}{P_{t+1}(s_{t+1})} \frac{\partial f(\phi_t^w \ell, K, \lambda_t)}{\partial \phi_t^w} dG(\lambda_{t+1})dF(\mu_{t+1}|\mu_t)$$

This is the central equation of the model. The left-hand side illustrates the expected utility gain agents realize when the probability of each agent working, ϕ_t^w, increases. The right-hand side shows the expected utility gain, through the rise in consumption, associated with a rise in this probability. Specifically, as ϕ_t^w becomes bigger a greater fraction of the population is working. As a consequence output in period t rises by $\partial f(\phi_t^w \ell, K, \lambda_t)/\partial \phi_t^w$ and the firm's period-t nominal sales by $P_t(s_t)\partial f(\phi_t^w \ell, K, \lambda_t)/\partial \phi_t^w$. As has been mentioned, the income derived from these sales is not distributed by the firm to agents until the beginning of period t+1 and at that time will be worth $[P_t(s_t)/P_{t+1}(s_{t+1})]\partial f(\phi_t^w \ell, K, \lambda_t)/\partial \phi_t^w$. The expected discounted utility value of these extra earnings to individuals is given by the right side of equation (9).

This equation also displays another notable feature. The right-hand side displays the uncertainty associated with technology, λ_t, and money growth, μ_t, shocks. <u>This is aggregate non-diversifiable uncertainty</u>. The left-hand side displays the uncertainty associated with the employment lottery. This is <u>contrived uncertainty</u> which agents themselves manufacture. Because of the nonconvexity in labor supply it is optimal for agents to construct uncertainty which in the aggregate is diversifiable.

IV. <u>THE MODEL'S GENERAL EQUILIBRIUM</u>

In the model's general equilibrium the goods and money markets must clear each period. Thus, $\sum_j \phi_t^j c_t^j = f(\phi_t^w \ell, K, \lambda_t)$ and $\sum_j \phi_t^j m_t^j = M_t^s/P_t(s_t)$ for all t.[7] Noting that consumption is equalized across the employed and unemployed states and that equation (2) holds with equality, the following expression is obtained: $M_t^s/P_t(s_t) = f(\phi_t^w \ell, K, \lambda_t)$. By utilizing these facts equation (9) can be rewritten as

$$\left[\frac{f(\phi_t^w \ell, K, \lambda_t)}{\partial f(\phi_t^w \ell, K, \lambda_t)/\partial \phi_t^w}\right]$$

$$= A \int_Q \int_Z \left[\frac{U'(f(\phi_{t+1}^w \ell, K, \lambda_{t+1}))f(\phi_{t+1}^w \ell, K, \lambda_{t+1})}{\mu_{t+1}}\right] dG(\lambda_{t+1})dF(\mu_{t+1}|\mu_t) \quad (10)$$

where $\mu_{t+1} \equiv M_{t+1}^s/M_t^s$, and $A \equiv -\beta/[V(\ell) - V(0)]$. Equation (10) implicitly defines a solution for the current equilibrium employment rate ϕ_t^w. It will now be shown that there exists a stationary function $\phi(\cdot)$ which maps period-t money growth rates and technology parameters $(\mu_t, \lambda_t) \equiv s_t$ into equilibrium employment rates, ϕ_t^w. (Henceforth ϕ_t^w will be written more simply as ϕ_t.) In the following proposition, a restriction on the value of γ, described in assumption (A4), will also be developed.

<u>Proposition 1</u>: Under assumption (A1)-(A4) there exists a unique bounded continuous function ϕ: Q X Z → [0,1] satisfying equation (10).

<u>Proof</u>: Define the function

$$H(\phi_t, \lambda_t) \equiv \frac{f(\phi_t \ell, K, \lambda_t)}{\partial f(\phi_t \ell, K, \lambda_t)/\partial \phi_t}$$

with $H(0,\lambda_t) = 0$ for all $\lambda_t \in Z$. Note

$$H: [0,1] \times Z \to \left[0, \sup_{\lambda} \left[\frac{f(1\ell, K, \lambda)}{\partial f(\phi\ell, K, \lambda)/\partial \phi\Big|_{\phi = 1}}\right]\right]$$

and $H(\cdot,\cdot)$ is increasing in its first argument. Since $f(\cdot,\cdot,\cdot)$ is twice differentiable, by the Inverse Function Theorem there exists a function $H_\lambda^{-1}(\cdot)$ such that $H_\lambda^{-1}(H(\phi, \lambda)) = \phi$. Also $H_\lambda^{-1}(\cdot)$ is both differentiable and increasing. Furthermore, since $\frac{\partial H(\phi, \lambda)}{\partial \phi} \geq 1$, the derivative of $H_\lambda^{-1}(\cdot)$ is in the interval [0,1]. Equation (10) can now be rewritten as

$$\phi(s_t)=H_{\lambda_t}^{-1}\left(A\int_Q\int_Z \frac{U'(f(\phi(s_{t+1})\ell,K,\lambda_{t+1}))f(\phi(s_{t+1})\ell,K,\lambda_{t+1})}{\mu_{t+1}}\, dG(\lambda_{t+1})dF(\mu_{t+1}|\mu_t)\right) \tag{11}$$

Assumption (A4) will guarantee that the range of H_λ^{-1} is $\subset$ [0,1], given the restriction on γ below.

Let $\mathfrak{F}$ be the space of bounded continuous functions h: $Q \times Z \to [\phi^*,1]$ with norm $\|h\| = \sup_{\substack{\mu\in Q\\ \lambda\in Z}} |h(\mu,\lambda)|$, and where the constant ϕ^* is defined as

$$\phi^* \equiv \min_{\substack{\mu\in Q\\ \lambda\in Z}} \{H_\lambda^{-1}(A(\lim_{c\to 0} U'(c)c) \int \frac{1}{\mu'}\, dF(\mu'|\mu))\}.$$

Equation (11) describes a mapping $\hat{\phi} = T(\phi)$ from $\mathfrak{F}$ into $\mathfrak{F}$. It will now be shown that T is a contraction operator.

To prove this let

$$G(\phi;\mu) = \int_Q\int_Z \frac{AU'(f(\phi\ell,K,\lambda))f(\phi\ell,K,\lambda)}{\mu'}\, dG(\lambda)dF(\mu'|\mu).$$

Now

$$\frac{\partial G(\phi;\mu)}{\partial \phi} \leq \max_{\substack{\lambda\in Z\\ \phi\in[\phi^*,1]}} \left.\left\{\left[\frac{\phi}{c}\frac{\partial c}{\partial \phi}\right]\frac{A\bar{y}U'(\bar{y})}{\phi}\left[1 + \frac{cU''(c)}{U'(c)}\right]\int_Q (\frac{1}{\mu'})dF(\mu'|\mu)\right\}\right|_{c=f(\phi\ell,K,\lambda)}$$

$$\leq (\frac{\gamma}{\phi^*}),$$

by assumptions (A1) and (A4). Note that assumption (A1) implies

$\left| 1 + \frac{cU''(c)}{U'(c)} \right| \leq 1$ for all $c \in [0,\bar{y}]$. Now, choose $\gamma < \phi^*$ in assumption (A4) and let $\phi^1, \phi^2 \in \mathfrak{F}$. Finally by the Mean Value Theorem, equation (11) implies

$$\|T(\phi^1) - T(\phi^2)\| \leq \max \left| \frac{\partial H_\lambda^{-1}(G(\phi;\mu))}{\partial G} \cdot \frac{\partial G(\phi;\mu)}{\partial \phi} \right| \| \phi^1 - \phi^2 \|$$

$$\leq \left(\frac{\gamma}{\phi^*}\right) \| \phi^1 - \phi^2 \| ,$$

where the maximum is taken over all ϕ in the convex hull spanned by ϕ^1, ϕ^2. The second in equality derives from the fact that $H_\lambda^{-1}(\cdot)$ has a derivative between 0 and 1. Therefore the operator T has a unique fixed point on (by the Banach Fixed Point Theorem).

<u>Corollary</u>: The fixed point described in Proposition 1 does not have $\phi(\cdot) = 0$ or $\phi(\cdot) = 1$ as a solution for any realizations of the sample space (except, of course, on a set of measure zero).

<u>Proof</u>: That $\phi(\cdot) = 0$ is not a solution was shown in the proposition and is a consequence of assumption (A1). That $\phi(\cdot) = 1$ cannot be a solution can be seen from equation (10). Assumption (A4) implies that the right-hand side of equation (10), evaluated at its maximal possible value is strictly less than the left side evaluated at $\phi_t^w = 1$. Since the left-hand side is increasing in ϕ, it follows that $\phi(\cdot) = 1$ cannot be a solution.

The corollary states that the employment rate, and therefore the unemployment rate, is always between zero and unity. This obtains because in effect assumptions (A1) and (A4) imply that the model's (necessary) efficiency condition (9) will never be satisfied at either zero or full employment. Bounding the employment rate away from unity is done by making sure that the marginal product of labor, as well as the marginal utility of consumption becomes sufficiently small as $\phi \to 1$. Last, the corollary shows that there is no need formally to add a restriction to the representative agent/firm's dynamic programming problem that $\phi \in [0,1]$.

V. THE STOCHASTIC PROPERTIES OF UNEMPLOYMENT AND INFLATION

Of particular interest in dynamic models of this sort are the stochastic properties of aggregate endogenous variables. For this model in particular it is of interest to study the covariation between unemployment and inflation both conditioned and unconditioned upon the monetary and real shocks. Common Keynesian folklore dictates that unemployment and inflation should display negative correlation. This could indeed be true, but it would still not imply the existence of an exploitable tradeoff between these variables. It will be demonstrated later in this section that indigenous forces in a competitive economy can result in a negative association between unemployment and inflation in the model. In spite of this, policy-engineered increases in the rate of monetary expansion, if they have an impact on the real side of the economy, cause the unemployment rate to rise. That is the operational Phillips curve the policymaker faces is <u>always</u> <u>non</u>-negatively sloped. To illustrate this latter point the following assumption will be made.

<u>Assumption (A5)</u> Suppose the distribution function $F(\cdot|\cdot)$ satisfies

$$0 < -F_2(\cdot|\cdot) < F_1(\cdot|\cdot).$$

This amounts to requiring that the growth rate of the currency supply follows a stable stochastic process with positive autocorrelation.

Proposition 2: Under the hypothesis of assumptions (A1)-(A5), there exists an equilibrium in which $\phi(\mu,\lambda)$ is nonincreasing in its first argument.

Proof: Let $\tilde{\phi}(\mu_{t+1},\lambda_{t+1})$ be nonincreasing in μ_{t+1}. Then

$$\left[\frac{U'(f(\tilde{\phi}(s_{t+1})\ell,K,\lambda_{t+1}))f(\tilde{\phi}(s_{t+1})\ell,K,\lambda_{t+1})}{\mu_{t+1}}\right]$$

is nonincreasing in μ_{t+1} since $cU'(c)$ is nondecreasing in c. By an argument similar to that employed in Lucas (1978), Lemma 1, this implies that

$$\int_Q \int_Z \frac{U'(f(\tilde{\phi}(s_{t+1})\ell,K,\lambda_{t+1}))f(\tilde{\phi}(s_{t+1})\ell,K,\lambda_{t+1})}{\mu_{t+1}} dG(\lambda_{t+1})dF(\mu_{t+1}|\mu_t)$$

is also nonincreasing in μ_t.[8] Finally, since $H_\lambda^{-1}(\cdot)$ is increasing, the mapping T, described in the previous proposition, given by equation (11) maps nonincreasing functions into nonincreasing functions. Thus if

$\approx{\phi} = \lim_{n\to\infty} T^n \tilde{\phi}$ there exists an equilibrium in which $\approx{\phi}$: Q X Z → [0,,1]

and where $\approx{\phi}$ is nonincreasing in its first argument. Further, $\approx{\phi}$ is a solution to equation (11) and hence is an equilibrium employment rate function.

It is straightforward to see that the increase in the current growth rate of the money supply is also associated with a rise in the contemporaneous inflation rate. Note that the current rate of inflation, π_t, is determined by the equation

$$(1+\pi_t) \equiv P_t/P_{t-1} = \mu_t f(\phi(\mu_{t-1},\lambda_{t-1})\ell,K,\lambda_{t-1})/f(\phi(\mu_t,\lambda_t)\ell,K,\lambda_t). \quad (12)$$

Clearly, π_t is increasing in μ_t since $\phi(\cdot)$ is nonincreasing in μ while $f(\cdot)$ is increasing in ϕ.[9]

The intuition underlying the above proposition is straightforward. A shift upward in the current money supply growth rate <u>signals</u> an increased probability of higher future money supply growth rates. This portends greater inflation. Now recall that the firm holds agents' nominal earnings for one period before distributing them. Thus, the expected purchasing power of these earnings will be reduced by the higher expected inflation. The expected marginal return to working consequently is eroded, causing a drop in employment, output and consumption. Market activity--here, the operation of a firm--requires the use of currency and is taxed by inflation while non-market activity--leisure--does not require the use of currency and hence escapes the inflation levy. As a result when the rate of inflation rises individuals <u>on average</u> move out of market activity (production and consumption) into non-market activity (leisure).[10] The role greater current money growth plays in signalling the higher probability of increased future money growth should be emphasized. If current and future monetary growth rates were independently distributed, then it is easy to see from equation (11) that a large realization of the currency supply growth rate would have no implications for this period's equilibrium employment rate--the only effect would be a once-and-for-all increase in the price level.[11]

Recent work in macroeconomics has stressed how business cycle fluctuations can arise from purely real phenomenon--for example, Kydland and

Prescott (1982) and Long and Plosser (1983). The technology parameter λ in the model can be thought of as being the driving force behind the real side of the economy. It is of interest to consider the effects that such shocks can have upon inflation and unemployment in the model. King and Plosser (1984) have found that there is a positive correlation between output and a measure of inside money. The importance of this finding for the association between inflation and unemployment cannot be overemphasized. It implies that some of the cyclical behavior of monetary aggregates is an endogenous component of the business cycle--as opposed to being wholly determined by outside forces such as policymakers. To incorporate this finding into the analysis suppose that the period-t money supply growth rate, μ_t, can be written as

$$\mu_t = \theta_t + \eta(\lambda_t)$$

The term $\eta(\lambda)$ reflects that part of the growth rate in the money supply which is determined in association with real factors in the economy, and is a continuous increasing function of λ.[12] The term θ represents the component of the growth rate of money which is determined by outside forces. It is assumed that the stochastic process governing θ_t is determined by the distribution function $F(\cdot|\cdot)$ described in assumptions (A4) and (A5). A needed consistency requirement is that $F(\cdot|\cdot)$ be defined over the domain

$Q'XQ'$ where $Q' \equiv [\underline{\theta},\bar{\theta}]$, and $\underline{\mu} - \eta(\underline{\lambda}) = \underline{\theta} < \bar{\theta} = \bar{\mu} - \eta(\bar{\lambda})$.

This slight extension to the model does not involve introducing any new technical considerations into the earlier analysis. In particular, the existence proof of an equilibrium for the economy proceeds along lines identical to those outlined in Proposition 1, with Proposition 2 implying that $\phi(\cdot)$ is nonincreasing in θ_t. It is easy to show that the equilibrium employment can rise in response to a positive technology shock. Specifically, if assumption (A6) shown below is made, the equilibrium employment rate, ϕ, increases in response to an upward movement in the technology parameter, λ.

Assumption (A6) For $\phi \in (\phi,1)$ and $\lambda \in Z$ let

$$\frac{\partial^2 f(\phi\ell,K,\lambda)}{\partial\phi\partial\lambda} > [\frac{\partial f(\phi\ell,K,\lambda)}{\partial\phi} \frac{\partial f(\phi\ell,K,\lambda)}{\partial\lambda}]/f(\phi\ell,K,\lambda).$$

To see that this assumption guarantees a positive relationship between ϕ and λ, let D: $Q' \rightarrow R_{++}$ be continuous, and $\hat{\phi}(\theta,\lambda)$ be the implicit solution to

$$\frac{f(\hat{\phi}\ell,K,\lambda)}{\partial f(\hat{\phi}\ell,K,\lambda)/\partial\hat{\phi}} = D(\theta). \tag{13}$$

Since f(•) is twice differentiable, an application of the Implicit Function Theorem guarantees that $\hat{\phi}(\cdot)$ is differentiable in λ and together with (A6) implies $\hat{\phi}(\cdot)$ is increasing in this variable. Finally, by letting $D(\theta)$ represent the right side of equation (10), with μ replaced by θ, it is clear that the equilibrium employment rate function $\phi(\theta,\lambda)$ falls within the class of functions satisfying (13). Note that assumption (A6) ensures that a positive technological shock increases the marginal product of labor by an extent sufficient to induce a rise in the equilibrium employment rate.

The last unresolved question is the relationship between the technology shock and inflation in the model. As groundwork for addressing this issue suppose that

$$\frac{\partial\eta(\lambda)}{\partial\lambda} > \left[\frac{\eta(\lambda)+\bar{\theta}}{\lambda}\right] \sup_{\theta \in Q'} \left[\xi(y,\lambda) + \xi(y,\phi)\ \xi(\phi,\lambda)\right] \quad \text{for all } \lambda \in Z$$

with $y \equiv f(\phi\ell,K,\lambda)$, $\phi = \phi(\theta,\lambda)$, and where, for example, $\xi(y,\lambda)$ represents the elasticity of y with respect to λ,

i.e. $\xi(y,\lambda) \equiv \left[\frac{\lambda}{f(\phi\ell,K,\lambda)}\right]\left[\frac{\partial f(\phi\ell,K,\lambda)}{\partial\lambda}\right]$. In the above condition it is assumed that the production function, $f(\cdot)$, is restricted in a manner such that the elasticity expressions, $\xi(\cdot)$, are always bounded in value. To determine the impact of the period-t technology shock, λ_t, on the inflation rate, π_t, differentiate equation (12) to get

$$\frac{d\pi_t}{d\lambda_t} = \left[\frac{y_{t-1}}{y_t}\right]\left\{\frac{\partial\eta(\lambda_t)}{\partial\lambda_t} - \left[\frac{\eta(\lambda_t)+\theta_t}{\lambda_t}\right][\xi(y_t,\lambda_t) + \xi(y_t,\theta_t)\xi(\theta_t,\lambda_t)] > 0\right\} \tag{14}$$

As equation (14) illustrates, a stimulative productivity shock may be linked with a contemporaneous rise in inflation rate if it is associated with a sufficiently large increase in the quantity of inside money. Specifically in the current setting all that is required is for the money supply to rise proportionately more than income when a high productivity shock is realized, a fact guaranteed by the previous assumption.

The model's import will now be discussed. Imagine a policymaker in a heretofore non-interventionist economy. By eyeing the relationship between money and employment he may be tempted to conclude that an expansionary monetary policy can stimulate (mitigate) employment (unemployment) since

$$\mathrm{Cov}(\phi(\theta_t,\lambda_t),\eta(\lambda_t)|\theta_t) = \int_Z \phi(\theta_t,\lambda_t)[\eta(\lambda_t)-\bar{\eta}]dG(\lambda_t) > 0,$$

for $\theta \in Q'$ and where $\bar{\eta} \equiv \int_Z \eta(\lambda_t)dG(\lambda_t)$, with the sign of the above expression following from the fact that the covariance between the two increasing functions of a variable must be positive.[13] Yet in reality there is no tradeoff between inflation and unemployment facing the policymaker in the assumed environment. An activist monetary policy, represented by the distribution function $F(\cdot)$ in the model, in fact has a detrimental impact on employment in the economy as

$$\mathrm{Cov}(\phi(\theta_t,\lambda_t),\theta_t|\lambda_t,\theta_{t-1}) = \int_{Q'} \phi(\theta_t,\lambda_t)[\theta_t-\bar{\theta}_t]dF(\theta_t|\theta_{t-1}) \leq 0,$$

$$\text{for } \lambda_t \in Z_t \text{ and where } \bar{\theta} \equiv \int_{Q'} \theta_t\, dF(\theta_t|\theta_{t-1}),$$

with it being noted that the covariance between a variable and a nonincreasing function of itself is nonpositive. Finally, with an activist policymaker in the economic environment the observed relationship between employment and money growth--and consequently the Phillips curve relationship--may appear to be either weak or noisy since it will depend on the relative strengths of the endogenous and exogenous components of the money supply. Specifically,

$$\mathrm{Cov}(\phi(\theta_t,\lambda_t),\ \mu_t|\theta_{t-1}) = \int_Z \mathrm{Cov}(\phi(\theta_t,\lambda_t),\ \theta_t|\lambda_t)dG(\lambda_t)$$

$$+ \int_{Q'} \mathrm{Cov}(\phi(\theta_t,\lambda_t),\ \eta(\lambda_t)|\theta_t)dF(\theta_t|\theta_{t-1}) \gtrless 0$$

$$[\text{recall } \mu_t = \theta_t + \eta(\lambda_t)]$$

where the integrals of the conditional covariances have the same signs as the conditional covariances themselves.

VI. CONCLUSIONS

A stochastic dynamic equilibrium model was presented to examine the relationship between unemployment and inflation. The introduction of a nonconvexity into agents' labor supply decisions resulted in a certain fraction of the population being unemployed at any particular time. This permitted the study of the determinants of unemployment within the context of a representative agent model. It was argued that indigenous forces in a competitive economy could result in the traditional negative relationship between inflation and unemployment. In particular, shocks which result in increased productivity could stimulate the return to working, thereby reducing unemployment and boosting output, and given sufficient endogeneity in the money supply, result in increased inflation. This conditional negative covariation between unemployment and inflation did not imply an exploitable tradeoff between the two variables. In fact any engineered inflation by authorities in the model had an adverse impact on unemployment by reducing the return to market activity. The policymaker, while perhaps observing a negative sloped Phillips curve, in actuality faced Friedman's positively sloped one.

In the model of this paper, changes in the amount of outside money are relevant only insofar as they yield information about future money supply growth rates. As noted in Huffman (1985), this is merely a consequence of the representative agent paradigm which is employed. Within the context of a model with heterogeneous agents with finite planning horizons, Huffman (1985) shows that single period purely transitory increases in the money supply, or wealth shocks, can have effects on the price level which persist over many periods. More work is needed along these lines to determine the lagged effect that changes in the money supply have upon unemployment-inflation correlations.

It may be argued that there are other policies which the government can undertake in order to lower the level of unemployment. One might ask, for example, what might happen if the increase in fiat money was introduced through increased government consumption rather than transfers to individuals. Even in this case, however, unemployment will not necessarily decline. This is because such a policy has two offsetting effects. The increased government consumption lowers private consumption and encourages agents to work more. The inflation, however, lowers the return to work effort and thereby encourages agents to work less. Hence inflation-financed increases in government expenditures will not necessarily lower unemployment.

In the model presented here, increases in the quantity of inside money were assumed to be correlated with productivity shocks in a manner consistent with the work of King and Plosser (1984). The model, however, is not constructed in such a manner as to lend insight into the underlying process which determines the level of inside money. Much more work needs to be done on this. One approach to this issue is explored in Freeman (1985).

APPENDIX

This appendix will be devoted to an examination of the effect that a nonconvexity in agent's labor-leisure choice set has on the economy's general equilibrium. Again there is a continuum of agents on the unit interval. Each agent has preferences

$$U = \frac{1}{\alpha} c^{\alpha} - \frac{1}{\rho} \ell^{\rho} \qquad \text{(1A)}$$

where c the consumption of an agent, and ℓ his labor effort. There is a firm in this economy which has access to a constant return to scale production function

$$f(K,\tilde{\ell}) = \lambda K^{\beta}(\tilde{\ell})^{1-\beta}$$

where K is aggregate quantity of capital employed and $\tilde{\ell}$ equals the aggregate quantity of labor employed. Hence the firm maximizes

$$\lambda K^{\beta}(\tilde{\ell})^{1-\beta} - w\tilde{\ell} - rK$$

where w is the wage rate of labor and r represents the rental rate on capital.

Each agent is assumed to supply K units of capital inelastically. In a competitive equilibrium the agent faces the budget constraint

$$c = w\ell + rK \qquad \text{(2A)}$$

The economy is now parameterized as follows:

$$\alpha = .75, \quad \rho = 1.25, \quad \beta = .25, \quad \lambda = 15.0, \quad K = 100.$$

<u>Economy 1</u>: Each agent maximizes (1A) subject to (2A) and $c,\ell \geq 0$. The resulting equilibrium consumption, labor effect, and utility levels are respectively,

$$c^* = 814.97, \quad \ell^* = 44.33, \quad U^* = 111.86.$$

The prices associated with these choices are

$$w^* = 13.79 \qquad r^* = 2.04.$$

This equilibrium is illustrated by point F in Figure 1.

<u>Economy 2</u>: Each agent maximizes (1A), subject to (2A) and the added constraint, $\ell \in \{0,\ell^N\}$. That is, each agent has a dichotomous choice problem with respect to his labor effort. For $\ell^N = 73.14$, the equilibrium quantities and prices are

$$c^N = 1186.47, \qquad \ell^N = 73.14 \qquad U^N = 98.40$$
$$w^N = 12.17, \qquad r^N = 2.97.$$

This equilibrium is illustrated by point N in Figure 1. To have <u>all</u> agents employed the utility return for all agents must exceed or equal that of being employed.

<u>Economy 3</u>: Now consider introducing a lottery into the environment of Economy 2. Each agent receives a constant level of consumption c^L and each agent faces probability ϕ of being called to work ℓ^N units. Aggregate employment is then

$\tilde{\ell} = \phi\ell^N$ and in equilibrium $c^L = \lambda(k)^{\beta}\,(\phi\ell^N)^{1-\beta}$. The equilibrium values for this economy are

$$\phi = .76, \qquad c^L = 963.98, \qquad \tilde{\ell} = 55.46$$
$$U^u = 230.67 \qquad U^w = 59.52, \qquad U^L = 100.9$$
$$w^L = 1304, \qquad r^L = 2.41$$

There are two types of agents ex post: those that work and those that don't. Those agents who work receive an ex post utility level of U^w while those agents who do not work receive an ex post utility level of U^u. This is shown in Figure 1. Each agent then attains an ex ante utility level of

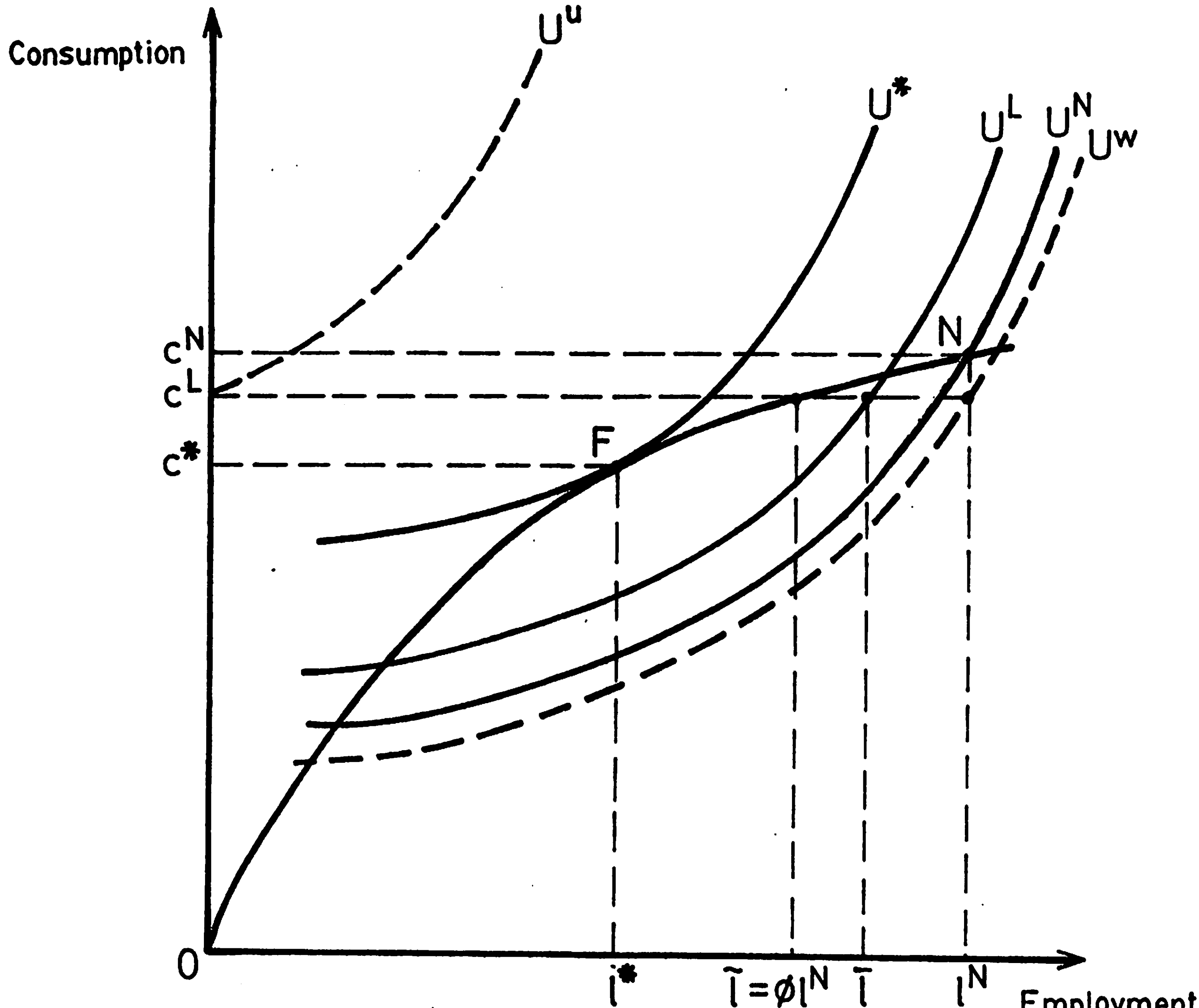
Consumption
U^u
U^*
U^L
U^N
U^w
c^N
c^L
c^*
F
N
0
l^*
$\tilde{l}=\phi l^N$
$\bar{l}$
l^N
Employment

Figure 1

$U^L = \phi U^W + (1-\phi)U^u$. Note that the utility level with the lottery, U^L, exceeds the utility level with the lottery, U^N. Also, note that due to risk aversion an agent would be indifferent between working the amount $\bar{\ell} > \phi\ell^N$ and taking his chances with the lottery.

FOOTNOTES

[1]Each agent's consumption and labor effort could also be indexed by his (random) period-t position, say σ, on the interval [0,1], so for instance $c_t(\sigma)$ and $\ell_t(\sigma)$ would represent agent σ's period-t quantities of these variables. It turns out that there is really no need to do this--c.f. footnote 5 below.

[2]A precise physical environment giving rise to a cash-in-advance economy has not been specified. A modification of the above economy along the lines of Townsend (1980, p. 284) could produce a cash-in-advance economy as an <u>outcome</u> of the assumed structure of the environment. Such a modification, while making the model more cumbersome, would not seem to change any of the results obtained here.

[3]This particular formulation is used because of its tractibility. Agents could be allowed to choose their labor supply over some more general discrete (nonconvex) set. Also, it could be assumed that agents who don't work the maximal number of hours in the market place, instead engage in some amount of home production. These extensions would not affect the main conclusions obtained.

[4]The reader is referred to the Appendix for a more detailed explanation.

[5]The fact that the decentralized decision making by a continuum of agents in competitive equilibriums with nonconvexities and lotteries can be summarized by a simple representative agent's programming problem is discussed formally by Rogerson (1985).

[6]This result obtains because of the separability of the momentary utility function in consumption and work effort. With a non-separable momentary utility function consumption would not be equalized across the employed and unemployed states.

[7]Utilizing the notation of footnote 1 these conditions can be perhaps more transparently written as $\int_0^1 c_t(\sigma)d\sigma = f(\phi_t^w \ell, K, \lambda_t)$ and $\int_0^1 m_t(\sigma)d\sigma = M_t^s/P_t(s)$, where $(c_t(\sigma), m_t(\sigma)) = (c_t^w, m_t^w)$ for $\sigma \in [0, \phi_t^w]$, and with $(c_t(\sigma), m_t(\sigma)) = (c_t^u, m_t^u)$ for $\sigma \in (\phi_t^w, 1]$.

[8]Specifically, make the following definition: $h(s_{t+1}) \equiv U'(f(\tilde{\phi}(s_{t+1})\ell, K, \lambda_{t+1}))f(\tilde{\phi}(s_{t+1})\ell^w, K, \lambda_{t+1})/\mu_{t+1}$. Now let $\pi = F(\mu_{t+1}|\mu_t)$ and invert this function to get $\mu_{t+1} = J(\pi, \mu_t)$. Note $J_2(\cdot) = -F_2(\cdot)/F_1(\cdot)$. Hence by the standard change in variable transform $D(\mu) \equiv \int_Q \int_Z h(\mu_{t+1}, \lambda_{t+1}) \cdot dG(\lambda_t)dF(\mu_{t+1}|\mu_t) = \int_0^1 \int_Z h(J(\pi, \mu_t), \lambda_{t+1})dG(\lambda_{t+1})d\pi$. Consequently, it follows that $D'(\mu_t) = \int_0^1 \int_Z h_1(J(\pi, \mu_t), \lambda_t)J_2(\pi, \mu_t)dG(\lambda_{t+1})d\pi \leq \int_Q \int_Z h_1(\mu_{t+1}, \lambda_{t+1})dG(\lambda_t)dF(\mu_{t+1}|\mu_t) \leq 0$ since it was assumed that $0 < -F_2(\cdot)/F_1(\cdot) < 1$, and $h_1(\cdot) \leq 0$ for $\mu \in Q$ and $\lambda \in Z$.

[9]By conditioning equation (12) on the value of μ_{t-1}, it can be seen that the <u>expected</u> rate of inflation is also an increasing function of the present money supply growth rate.

[10]Note that a low realization of μ will signal a more deflationary trend for the economy. This will increase the expected return to working and consequently cause the unemployment rate to fall. Similarly, distribution functions for μ having negative serial correlation properties could <u>occasionally</u> result in a high money shock being associated with a drop in unemployment, again because of the increased probability of a more deflationary price path for the economy. Among the class of deflationary monetary policies the best is to follow the optimum quantity of money rule, which results in the gross (net) interest rate being set equal to unity (zero)

--see Aschauer and Greenwood (1983). This class of deflationary monetary policies, while potentially lowering unemployment, is certainly not what most people have in mind when thinking about expansionary monetary policies.

[11]One might be tempted to conclude that if the money supply growth rates exhibited negative serial correlation, than the employment rate must necessarily be increasing in the current money growth rate. This is false. Such correlations, in general, turn out to be of indeterminate sign.

[12]This particular representation of the money growth shock may connote that the technology is somehow causing an upward shift in the money supply growth rate. In fact no inference about the direction of causality between $\eta_t = \eta(\lambda_t)$ and λ_t need be made. One could write $\lambda_t = \eta^{-1}(\eta_t)$ and replace the distribution function $G(\lambda_t)$ with the one $G(\eta^{-1}(\eta_t))$ defined on the domain $N = [\eta(\underline{\lambda}), \eta(\bar{\lambda})]$.

[13]Trivially, the conditional covariance between period-t the unemployment rate, $(1 - \phi(s_t))$, and the endogenous money supply growth rate shock, $\eta(\lambda_t)$, can be written as $\mathrm{Cov}((1-\phi(\theta_t,\lambda_t)),\eta(\lambda_t)|\theta_t) = -\mathrm{Cov}(\phi(\theta_t,\lambda_t), \eta(\lambda_t)|\theta_t) < 0$.

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