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AGRICULTURAL DEVELOPMENT VERSUS INDUSTRIALIZATION IN A DUAL ECONOMY: A FIVE SECTOR, THEORETICAL ANALYSIS*

by

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I. INTRODUCTION

In recent years there has been a flood of literature pertaining to the problem of the development of dualistic economies.[1] The purpose of the present paper is to investigate the development of a dual economy in a more comprehensive framework. The generalizations are threefold. First, five sectors are considered as compared with two sectors in the models of Lewis (11,12), Fei and Ranis (2,3,4,15), Jorgenson (6,7), Dixit (1), Nicholls (14), and others. Increasing the number of sectors permits two additional generalizations which have considerable empirical relevance in many less-developed countries (LDC's) today. Specifically, in addition to inter-sectoral trade of consumer goods and labor considered by the above-mentioned authors, intersectoral trade in capital goods and nondurable manufactured inputs are explicitly incorporated into the model. Lastly, the role of the government is explicitly considered. This appears to be of tantamount importance in any investigation of economic development.

A five sector optimizing model involving three products (agricultural goods, manufactured goods, and capital goods) is used to examine intersectoral factor flows of labor, capital, and manufactured inputs. Agricultural goods (used only for consumption) are assumed to be produced by two sectors, a subsistence and a commercial sector. These goods are perfect substitutes in consumption and consequently there is only one price for the output from these two sectors.

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[1] The seminal article on the subject appears to be the classic article by Lewis (11).

Capital goods, produced by a third sector in the model, are used only as factors of production and are assumed to be infinitely durable.[1] A fourth sector produces the third product, manufactured goods, which may be used either for consumption or as nondurable factors of production.[2] Manufactured goods to be used as inputs in other sectors or as consumer goods are assumed to be perfect substitutes in production. In other words, manufactured consumer goods and manufactured inputs are produced by the same firms using the same production processes. These firms are assumed to be indifferent between producing consumer goods or manufactured inputs which leads to a common price for manufactured consumer goods and manufactured inputs.

The fifth sector is the government sector. The government collects taxes on all income earned and this tax revenue is used either to invest in social overhead capital (SOC) in the agricultural sectors or to augment "private" savings which are used to purchase capital goods from the capital goods sector. The allocation of private savings among the investment alternatives is under the direct control of the government.[3] The government is assumed to invest in these alternatives in a manner that tends to maximize

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[1]That is, depreciation is not included in the model. This is a simplifying assumption and is not necessary to the analysis. There is no reason to suspect any of the conclusions of this study would be appreciably altered by relaxing this assumption.

[2]A nondurable factor of production is one which is completely used up in production during the period of purchase.

[3]This can be interpreted as a centrally planned economy where the government owns all the fixed capital stock. Alternatively, it can be viewed as a privately owned economy with centrally guided investment.

social welfare over a finite horizon, where welfare is assumed to be a function of consumption over the horizon and the productive capacity of the system at the end of the planning period.

For simplicity, the supply of labor is assumed to be perfectly inelastic throughout the period.[1] Labor employed by the government, in the commercial agricultural sector, manufacturing sector, or the capital goods sector receives an exogenously fixed wage rate[2] which is assumed to be too high to allow all labor to be employed in the four advanced sectors.[3] Any labor which is not employed in the advanced sectors finds employment in the subsistence sector where an average productivity is earned. The subsistence wage rate is assumed to be lower than the wage rate in the advanced sectors which, in effect, makes the supply of labor to the advanced sectors perfectly elastic in the initial phases of development even though the entire labor supply is assumed to be perfectly inelastic.[4]

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[1] The implications of relaxing this assumption are investigated in Haessel (5).

[2] Various reasons for a rigid wage rate have been given. Perhaps the most plausible reason is that the laborers are organized in a union to maintain this wage rate. Other possible explanations include social legislation and unwillingness to work in other than traditional employment at a wage rate lower than the institutional wage rate.

[3] The commercial agricultural, manufacturing, capital goods and government sectors are collectively referred to as the advanced sectors.

[4] As explained in the following section, the marginal physical productivity of labor in the subsistence sector is never assumed to be zero. This appears to coincide with the evidence cited by Kao, Anschel and Eicher (9). Thus, withdrawing labor from the subsistence sector tends to reduce production in this sector and we are following Jorgenson (6) in this respect. However, a perfectly elastic labor supply curve to the advanced sector coincides with the assumptions of Lewis (11) and Fei and Ranis (4). Jorgenson (7,8) made

This model is used to investigate characteristics of dualistic economies which tend to make agricultural development relatively more or less desirable than development of industry. The formal model is introduced in the following section. The optimization of this model is discussed in Section III, which is followed by two sections discussing the merits of investing agriculture and industry, respectively. The paper concludes with a section discussing the relative advantages of investing in agricultural SOC or in private industry.

II. THE MODEL

In this section the formal model is presented. First we define the notation. This is followed by a presentation of the model and a discussion of the equations in the model.

Throughout this paper, the following convention on notation is used. All variables are denoted by upper case Latin letters. Lower case Latin letters and Arabic numerals are subscripts either on variables or parameters. Parameters are denoted by Greek letters. All parameters, indexes and variables are non-negative. Subscripts on variables refer to the following:

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an interesting attempt to test the appropriateness of the assumptions of zero versus positive marginal physical productivity of labor and concluded that, for the case of Japan, the data were consistent with a positive marginal physical productivity for labor. As Marglin (13) demonstrates, however, Jorgenson's test depends crucially on the assumption of unitary elasticity of substitution between labor and capital in the industrial sector.

s = subsistence agricultural sector.

1 = commercial agricultural sector.

2 = manufacturing sector.

3 = capital goods sector.

t = time period (discrete).

The variables are defined as follows:

Y_{it} = production of good i, (i = s, 1, 2, 3).

F_{it} = use of manufactured goods (originating from sector 2) as a factor of production in sector i, (i = s, 1, 2, 3).

C_{it} = consumption of good i, where i = 1 denotes agricultural goods and i = 2 denotes manufactured goods.

K_{it} = capital stock in sector i available for production during period t, (i = 1, 2, 3).

L_{it} = labor employed in sector i, (i = s, 1, 2, 3).

$\bar{L}_{it}$ = labor employed by the government in the accumulation of social overhead capital (SOC) in sector i, (i = s, 1).

P_{it} = price of good i, (i = 1, 2, 3); $P_s = P_1$; and $P_{2t} \equiv 1$.

I_{it} = private investment in sector i, (i = 1, 2, 3).

I_t = transfer of tax revenue from the government budget to the private savings budget.

G_{it} = level of SOC in sector i, (i = s, 1).

B_i = amount of land in sector i, (i = s,1).

X_{it} = Lagrangean multiplier corresponding to the i-th constraint in period t.

The parameters are defined as follows:

μ_1, μ_2, μ_{11}, μ_{12}, μ_{21}, and μ_{22} are parameters of the quadratic welfare function.

σ_i = "intercepts" of the Cobb-Douglas form of production function sector i, (i = s, 1, 2, 3).

λ = "elasticity of production" of SOC in the agricultural sectors.

ω = institutionally fixed wage rate in terms of manufactured goods.

α_j = elasticity of production of factor j, sector s, (j = 1, 2, 4).

β_j = elasticity of production of factor j, sector 1, (j = 1, 2, 3, 4).

γ_j = elasticity of production of factor j, sector 2, (j = 1, 2, 3).

δ_j = elasticity of production of factor j, sector 3, (j = 1, 2, 3). where j = 1 denotes manufactured inputs, j = 2 denotes labor, j = 3 denotes capital, and j = 4 denotes land.

τ = terminal period of the plan (i.e., $t = 0, 1, \ldots, \tau$).

ε = exogenous rate of technological change in the manufacturing and capital goods sectors.

ψ = marginal (= average) tax rate.

ρ = social discount rate on welfare.

θ = parameter indicating the weight the planning authorities place on the provision of post-plan productive capacity.

The mathematical model follows, beginning with the welfare function which is

$$W = \sum_{t=1}^{\tau} (\mu_1 C_{1t} + \mu_2 C_{2t} - \mu_{11} C_{1t}^2 + \mu_{12} C_{1t} C_{2t} - \mu_{22} C_{2t}^2)(1 + \rho)^{-t}$$

$$+ \theta\{P_{1\tau}\sigma_s G_{s\tau}^{\lambda} F_{s\tau}^{\alpha_1} L_{s\tau}^{\alpha_2} B_s^{\alpha_4} + P_{1\tau}\sigma_1 G_{1\tau}^{\lambda} F_{1\tau}^{\beta_1} L_{1\tau}^{\beta_2} K_{1\tau}^{\beta_3} B_1^{\beta_4} + (1+\epsilon)^{\tau}\sigma_2 F_{2\tau}^{\gamma_1} L_{2\tau}^{\gamma_2} K_{2\tau}^{\gamma_3}$$

$$- F_{s\tau} - F_{1\tau} - F_{2\tau} - F_{3\tau} + P_{3\tau}(1+\epsilon)^{\tau}\sigma_3 F_{3\tau}^{\delta_1} L_{3\tau}^{\delta_2} K_{3\tau}^{\delta_3}\} . \qquad (2.1)$$

This welfare function is to be maximized subject to the following constraints which apply to every period ($t=1,\ldots,\tau$) of the plan.

$$Y_{st} = \sigma_s G_{st}^{\lambda} F_{st}^{\alpha_1} L_{st}^{\alpha_2} B_s^{\alpha_4} \qquad (2.2)$$

$$Y_{1t} = \sigma_1 G_{1t}^{\lambda} F_{1t}^{\beta_1} L_{1t}^{\beta_2} K_{1t}^{\beta_3} B_1^{\beta_4} \qquad (2.3)$$

$$Y_{2t} = \sigma_2 (1+\epsilon)^t F_{2t}^{\gamma_1} L_{2t}^{\gamma_2} K_{2t}^{\gamma_3} \qquad (2.4)$$

$$Y_{3t} = \sigma_3 (1+\epsilon)^t F_{3t}^{\delta_1} L_{3t}^{\delta_2} K_{3t}^{\delta_3} \qquad (2.5)$$

$$L_o = \bar{L}_{st} + \bar{L}_{1t} + L_{st} + L_{1t} + L_{2t} + L_{3t} \qquad (2.6)$$

$$Y_{st} + Y_{1t} = C_{1t} \qquad (2.7)$$

$$Y_{2t} = F_{st} + F_{1t} + F_{2t} + F_{3t} + C_{2t} \qquad (2.8)$$

$$Y_{3t} = I_{1t} + I_{2t} + I_{3t} \qquad (2.9)$$

$$\psi\{P_{1t} Y_{st}(1-\alpha_1) + P_{1t} Y_{1t}(1-\beta_1) + Y_{2t}(1-\gamma_1) + P_{3t} Y_{3t}(1-\delta_1) \qquad (2.10)$$

$$+ \omega(\bar{L}_{st} + \bar{L}_{1t})\} = \omega(\bar{L}_{st} + \bar{L}_{1t}) + P_{3t} I_t \qquad (2.11)$$

$$\alpha_1 P_{1t} Y_{st} = F_{st} \qquad (2.12)$$

$$\beta_1 P_{1t} Y_{1t} = F_{1t}$$

$$\gamma_1 Y_{2t} = F_{2t} \quad (2.13)$$

$$\delta_1 P_{3t} Y_{3t} = F_{3t} \quad (2.14)$$

$$\beta_2 P_{1t} Y_{1t} = L_{1t}\omega \quad (2.15)$$

$$\gamma_2 Y_{2t} = L_{2t}\omega \quad (2.16)$$

$$\delta_2 P_{3t} Y_{3t} = L_{3t}\omega \quad (2.17)$$

$$(1-\psi)\{(\alpha_2+\alpha_4)Y_{st}P_{1t} + \beta_2 Y_{1t}P_{1t} + \gamma_2 Y_{2t} + \delta_2 Y_{3t}P_{3t} + \omega(\bar{L}_{st} + \bar{L}_{1t})\} = P_{1t}C_{1t} + C_{2t} \quad (2.18)$$

$$(1-\psi)\{(\beta_3+\beta_4)P_{1t}Y_{1t} + \gamma_3 Y_{2t} + \delta_3 P_{3t}Y_{3t}\} + P_{3t}I_t = P_{3t}(I_{1t}+I_{2t}+I_{3t}) \quad (2.19)$$

$$C_{1t} \geq 1 \quad (2.20)$$

$$C_{2t} \geq 1 \quad (2.21)$$

In addition, the following definitions apply to variables appearing in (2.2)-(2.5):

$$G_{st} = \bar{G}_{s1} + \sum_{i=1}^{t-1} \bar{L}_{si} \quad (2.22)$$

$$G_{1t} = \bar{G}_{11} + \sum_{i=1}^{t-1} \bar{L}_{1i} \quad (2.23)$$

$$K_{1t} = \bar{K}_{11} + \sum_{i=1}^{t-1} I_{1i} \quad (2.24)$$

$$K_{2t} = \bar{K}_{21} + \sum_{i=1}^{t-1} I_{2i} \quad (2.25)$$

$$K_{3t} = \bar{K}_{31} + \sum_{i=1}^{t-1} I_{3i} \quad (2.26)$$

Now we discuss each of the equations in turn. The welfare function (2.1) is assumed to be known by the planners. Welfare in every period is assumed to be a function of aggregate consumption[1] of agricultural goods (C_1) and manufactured goods (C_2) throughout the planning horizon. Welfare in future periods is discounted to the present at a constant rate, ρ. In addition, a positive value is imputed to the value of net productive capacity available at the end of the plan to be used in post-plan production.[2] Without loss of generality, consumption units can be chosen so that $C_{10}=C_{20}=1$. That is, in the period immediately preceding the initial period of the plan, one unit of agricultural and manufactured goods are consumed.

The parameters of (2.1) are assumed to be such that $\mu_1 > \mu_2$; $2\mu_{11} > 2\mu_{22} > \mu_{12}$; $\mu_1 - 2\mu_{11} + \mu_{12} > \mu_2 + \mu_{12} - 2\mu_{22} > 0$; and $(2\mu_{11}-\mu_{12})/(2\mu_{22}-\mu_{12}) > \mu_1/\mu_2$. These assumptions are sufficient to guarantee the following: 1) a marginal increment in agricultural goods consumption in the initial period will contribute more to welfare that a marginal increment in manufactured goods consumption. 2) The marginal welfare of consuming additional units of agricultural goods declines more rapidly than the marginal welfare of consuming additional units of manufactured goods. 3) The quadratic form in (2.1) is negative definite, which in turn implies a saturation or bliss point in

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[1] Since the labor force (and population) is assumed to be constant, by virtue of the nature of the product and income distribution assumptions, this is equivalent to maximizing a weighted average per capita consumption, where all subsistence employees consume at one rate and all advanced-sector employees consume at another (higher) rate. The weights in the average are the proportions of the labor force employed in the subsistence and advanced sectors.

[2] Net productive capacity is total capacity less requirements for intermediate inputs. The prices existing in the last period of the plan are used to aggregate net capacities after investment in the final period of the plan. The value of the net productive capacity is enclosed in { } in equation (2.1).

consumption exists. 4) This saturation point occurs at a consumption combination such that the ratio of manufactured goods/agricultural goods consumed is higher than in the initial period. In addition it is assumed that this saturation point cannot be attained within the finite horizon of τ periods.

Isowelfare lines corresponding to a quadratic form in which the parameters satisfy the foregoing assumptions would exhibit the general shape represented in Figure 1. The maximum occurs at the point denoted A. In the initial period, consumers would be consuming one unit of each good and the terms of trade (TT) implied by the isowelfare curve at that point would be

$$-\frac{dC_1}{dC_2} = \frac{\mu_2 - 2\mu_{22} + \mu_{12}}{\mu_1 - 2\mu_{11} + \mu_{12}} < 1.$$

Moving along the ray OR would tend to move the TT against the agricultural sector since $\partial^2 W/\partial C_{1t}^2 < \partial^2 W/\partial C_{2t}^2 < 0$. That is, if the consumers whose preferences are being represented by this welfare function were to be confronted by equiproportionately more of each good they would tend to bid the price of agricultural goods down relative to manufactured goods. This is consistent with Engel's law which states that consumers tend to spend a higher proportion of their income on nonfood (nonagricultural) items as their level of real income increases.

The production process in each sector is assumed to be described by a Cobb-Douglas production function. Output from the subsistence sector in period t is given by equation (2.2) Land (B_s) is assumed to be fixed throughout the period. Labour (L_s), purchased inputs (F_s), and SOC, G_s, are all variable. Purchased inputs include such items as fertilizers, insecticides, and any other items purchased from the industrial sector.

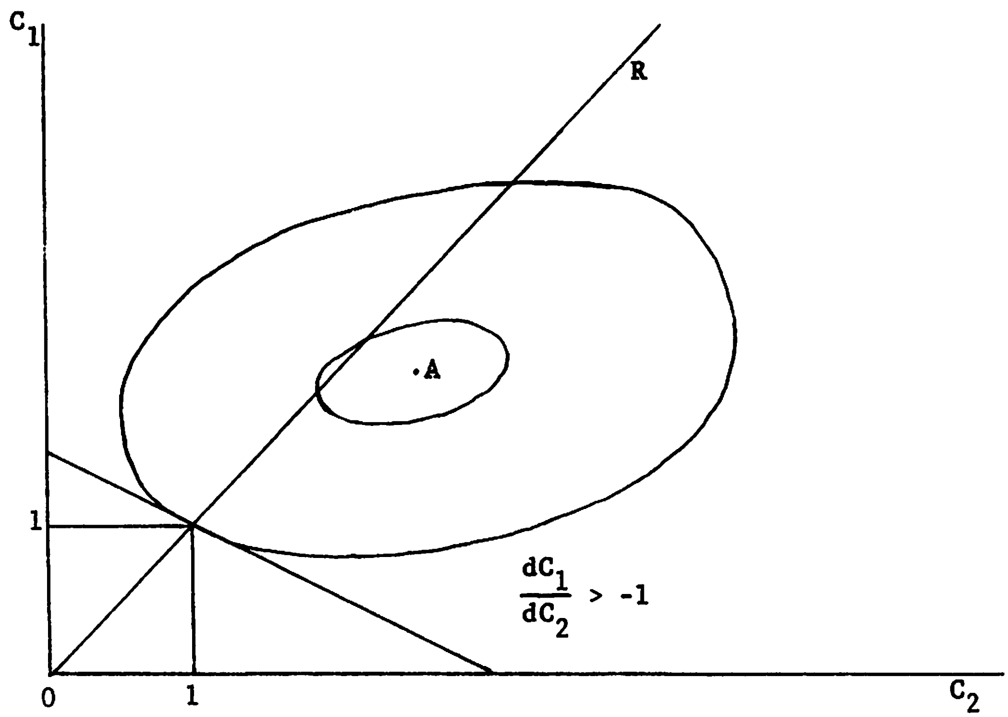

Figure 1. Isowelfare curves and implied terms of trade

Labor is measured in terms of man years and as such is productively employed in the sense that withdrawing labor from this sector would reduce output if all other factors remained at their previous levels.[1] The SOC variable is explained in detail below.

The production process in the commercial agricultural sector (2.3) differs from that in the subsistence sector since reproducible capital is used as a factor of production. Land is fixed while all other factors are variable.

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[1] In this respect we are following Jorgenson (6) rather than Lewis (11) or Ranis and Fei (15). This appears to be consistent with the empirical evidence cited by Kao, Anschel and Eicher (9).

We make the following specific assumptions about the production functions in the agricultural sectors: (A) $\Sigma\alpha_i = 1$; (B) $\Sigma\beta_i = 1$; (C) $\alpha_1 = \beta_1$; (D) $\alpha_2 < \beta_2$; (E) $\lambda < \beta_4$. Assumptions (A) and (B) imply constant returns to scale prevail if all the conventional factors (land, labor, capital and manufactured inputs) are varied proportionately. Assumption (C) indicates the elasticities of production with respect to manufactured inputs are equal between the two sectors, while (D) indicates the elasticity of production with respect to labor in the subsistence sector is less than in the commercial agricultural sector.[1] Assumptions (A)-(D) imply that $\alpha_4 > \beta_3 + \beta_4$, which suggests that the elasticity of production with respect to land in the s sector is greater than the combined elasticity of production of capital and land in the commercial sector.[2] Assumption (E) implies that since land is not variable diminishing marginal productivity of nonland resources are evident in agricultural production even if investment is made in SOC.[3]

The production processes in the manufacturing and capital goods sectors (2.4) and (2.5) are assumed to differ from production in the agricultural

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[1]Since labor is combined with capital in the commercial sector, a small change in labor input has a larger output response in sector 1 than a small change in labor input in sector s.

[2]This is because land is more intensively cultivated in the subsistence sector.

[3]Since $\alpha_4 > \beta_4$, this applies to sector s as well as sector 1. If $\lambda > \beta_4$ or if $\lambda > \alpha_4$, this would permit increasing returns to scale which would, in effect, lead to problems of nonconvexity. It is for this reason that land resources are kept fixed (that is, to guarantee convexity). Transferring land from one sector to the other would also lead to nonconvexity problems.

sectors since no primary or fixed factors are involved and technology improves at a constant exogenous rate of 100ϵ percent per year. Thus, production in these two sectors is assumed to be a function of manufactured inputs, labor, and capital inputs. Both of these sectors use their own output in further production.

Equation (2.6) indicates that employment in every period must equal the total fixed labor supply L_o. Equation (2.7) indicates that all agricultural output is consumed, while (2.8) indicates that manufactured output may be either consumed or used as a factor of production in all except the government sector. Output from the capital goods sector must be used for investment in fixed capital stock (2.9). The capital available in any sector in any period is defined in equations (2.24), (2.25) and (2.26). Once investment takes place in a particular sector, the capital goods cannot be transferred to other sectors.

The government is assumed to collect taxes on all income earned by labor, land and capital at a constant marginal (and average) tax rate.[1] This is equivalent to collecting taxes on all income received by the sectors less the cost of purchased manufactured inputs, plus collecting taxes from government employees. This tax revenue is given by the L.H.S. of (2.10), while government expenditure is given by the R.H.S. of (2.10). Government expenditure in the agricultural sectors can be used to accumulate SOC which is accomplished by hiring labor at a fixed wage rate, ω. This labor engages in various extension activities, educational programs, and other endeavors which have the effect of increasing productivity in the agricultural sectors. An alternative

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[1]The constant tax rate is not a necessary assumption and could be considered as an instrumental variable.

interpretation would be for this labor to engage in labor-intensive capital accumulation, such as building a road, dam, or irrigation system using labor as the only significant input.[1] The level of SOC available in the agricultural sectors in any period is defined by (2.22) and (2.23). The other alternative for utilizing government tax revenue is to transfer the revenue to the budget used to purchase capital goods (2.19). The amount of this transfer is $P_{3t}I_t$. This results in a one-way transfer possibility (i.e., resources can be transferred from the tax budget to the private savings budget but not in reverse). Other sources of private savings are the income earned on the existing capital stock in the advanced sectors and the rent on land in the commercial agricultural sector. (Rent in the subsistence sector is consumed). These private and public investment expenditures are allocated (by the government) in a manner that will maximize welfare over the planning horizon. Investment goods, either social overhead or productive, cannot be used in production until the following period.

Equations (2.11)-(2.14) describe the behavior of the private sector with respect to the purchase of manufactured inputs.[2] Equations (2.15)-(2.17) indicate that labor in the advanced sectors is paid its marginal value productivity. Since the wage rate (ω) is exogenous, (2.15)-(2.17) determine employment levels in the advanced sectors, and since government employment ($\bar{L}_s + \bar{L}_1$) is determined by other considerations, employment in

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[1]Labor intensive capital accumulation is also considered by Lewis (11 , p. 161) in his discussion of capital accumulation through monetary expansion.

[2]Note that the price of manufactured goods (inputs) is chosen as the <u>numeraire</u> and is arbitrarily set equal to one (i.e., $P_{2t} \equiv 1$).

subsistence agriculture (L_s) is a residual.

The inequalities in (2.20) and (2.21) are constraints imposed to prevent consumption from falling below initial levels in any period.

The intersectoral flows in this model are illustrated in Figure 2. Assume that in some particular period, I_t, I_{1t}, I_{2t}, I_{3t}, $\overline{L}_{st}$ and $\overline{L}_{1t}$ have been specified. In response to these specified levels of planned investment expenditures there will be certain flows among the sectors. The five sectors are represented as rectangles and the two ovals represent the two groups of income recipients, the capital owners and the laborers. Landowners are not included as a separate class of income recipients. The rent earned on land is simply attributed to the laborers in the subsistence sector and to the capitalists in the commercial agricultural sector. The flows above the diagonal line AA' represent expenditures and those below the line represent income receipts. Expenditure flows are discussed first.

The laborers spend all their income on consumption goods. This consumption expenditure is divided between agricultural goods (P_1C_1) and manufactured goods (P_2C_2). The expenditures on agricultural goods are divided between the commercial and the subsistence agricultural sectors. Consumption expenditures by labor are the only source of revenue for the agricultural sectors. The manufacturing goods sector, on the other hand, sells its product to the two agricultural sectors and the capital goods sector as well as to consumers. Hence the manufacturing goods sector receives revenue from all four of these sources.

The capital goods sector sells its output to the capitalists whose savings may be augmented by transfers from the government. The capitalists spend all their income on private investment goods. The government has two

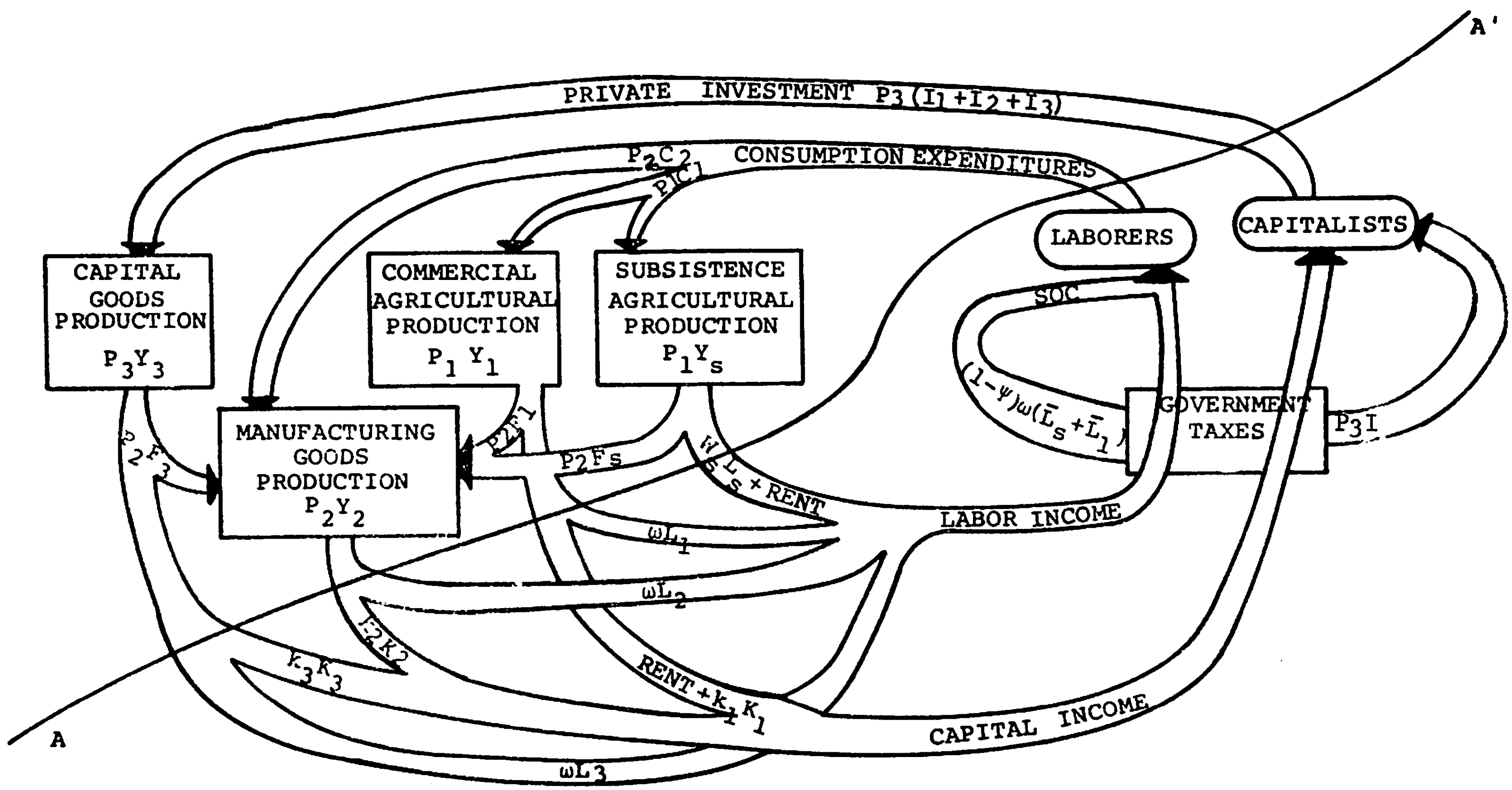

Figure 2. Income and expenditure flows

classes of expenditure alternatives. The tax revenue which the government collects may be spent on either SOC for the agricultural sectors or on investment goods for private capital accumulation in the manufacturing, commercial agricultural and capital goods sectors.

Turning now to the income flows, labor receives income from all five sectors. However, since capital is not used in the subsistence or the government sectors, the capital owners do not receive income from these two sectors. Net revenue[1] in the subsistence agricultural sector accrues to labor. Part of this net income is rent on the land which the laborers are presumed to own. The net revenue in the commercial agricultural sector is divided between the capitalists (who own the land in this sector) and the laborers. Since no primary factors are employed in the manufacturing and capital goods sectors, the net revenue in these sectors is divided between the laborers and capitalists as wages and return on capital stocks.

In the subsequent period the same process is repeated except that productive capacity in the various sectors will be augmented by the preceding periods investment.[2]

III. THE FIRST-ORDER CONDITIONS

The general nature of the optimizing problem for this model is to maximize a differentiable, concave function (2.1) subject to a number of differentiable, convex constraints (2.2)-(2.21). In addition, it is required

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[1] Net revenue in this section is defined as total revenue less the cost of purchased manufactured inputs and taxes.

[2] For a detailed discussion of some of the comparative statics properties of a very similar model, see Haessel (5).

that all variables must be non-negative. This type of problem can be maximized by application of the Kuhn-Turcker first-order conditions (10). This involves formulating a Lagrangean function which is presented in Appendix A. The Lagrangean multipliers are denoted as X_{it}, where the subscript i corresponds to the equation number of the associated constraint in Section II. The subscript t refers to the time period. The constraints in the function are formulated in a manner such that the associated dual variables (Lagrangean multipliers) are non-negative. The first-order conditions are derived by differentiating the Lagrangean function with respect to every time period for every variable. If the variable in question can be shown to be positive from a priori considerations, then the Kuhn-Tucker conditions become equivalent to the first-order conditions of classical calculus. Only six variables cannot be shown to assume positive values, specifically $\bar{L}_{st}$, $\bar{L}_{1t}$, I_{1t}, I_{2t}, I_{3t}, and I_t. Since these variables are the variables of principle concern, the first-order conditions relating to these variables are presented in the text, while the remaining first-order conditions are presented in Appendix B.

Subscripts on V denote partial derivatives on the Lagrangean function and $T = \tau + 1$. The same first order conditions apply to every period ($t=1,\ldots,\tau$). The Kuhn-Tucker first-order conditions for an optimum with respect to the six government instruments are as follows:

$$V_{\bar{L}_{st}} = \lambda \sum_{i=t+1}^{T} (X_{2i}Y_{si}/G_{si}) - X_{6t} + \omega(1-\psi)(X_{18t} - X_{10t}) \leq 0 ;$$

$$\bar{L}_{st}V_{\bar{L}_{st}} = 0; \quad \bar{L}_{st} \geq 0 ; \text{ where } X_{2T} \equiv P_{1\tau}\theta \text{ and } Y_{sT} \equiv \sigma_s G_{sT}^{\lambda} F_{s\tau}^{\alpha_1} L_{s\tau}^{\alpha_2} \qquad (3.1)$$

$$V_{\bar{L}_{1t}} = \lambda \sum_{i=t+1}^{T} (X_{3i}Y_{1i}/G_{1i}) - X_{6t} + \omega(1-\psi)(X_{18t} - X_{10t}) \leq 0 ;$$

$$\bar{L}_{1t}V_{\bar{L}_{1t}} = 0; \quad \bar{L}_{1t} \geq 0; \text{ where } X_{3T} \equiv P_{1\tau}\theta \text{ and } Y_{1T} \equiv \sigma_1 G_{1T}^{\lambda} F_{1\tau}^{\beta_1} L_{1\tau}^{\beta_2} K_{1T}^{\beta_3} \qquad (3.2)$$

$$V_{I_{1t}} = \beta_3 \sum_{i=t+1}^{T} (X_{3i}Y_{1i}/K_{1i}) - X_{9t} - X_{19t}P_{3t} \leq 0; \quad I_{1t}V_{I_{1t}} = 0;$$

$$I_{1t} \geq 0; \text{ where } X_{3T} \text{ and } Y_{1T} \text{ are as defined in (3.2)} \tag{3.3}$$

$$V_{I_{2t}} = \gamma_3 \sum_{i=t+1}^{T} (X_{4i}Y_{2i}/K_{2i}) - X_{9t} - X_{19t}P_{3t} \leq 0; \quad I_{2t}V_{I_{2t}} = 0; \; I_{2t} \geq 0;$$

$$\text{where } X_{4T} \equiv \theta \text{ and } Y_{2T} \equiv (1+\epsilon)^T \sigma_2 F_{2\tau}^{\gamma_1} L_{2\tau}^{\gamma_2} K_{2T}^{\gamma_3} \tag{3.4}$$

$$V_{I_{3t}} = \delta_3 \sum_{i=t+1}^{T} (X_{5i}Y_{3i}/K_{3i}) - X_{9t} - X_{19}P_{3t} \leq 0; \quad I_{3t}V_{I_{3t}} = 0; \quad I_{3t} \geq 0;$$

$$\text{where } X_{5T} = P_{3\tau}\theta \text{ and } Y_{3T} \equiv (1+\epsilon)^T \sigma_3 F_{3\tau}^{\delta_1} L_{3\tau}^{\delta_2} K_{3T}^{\delta_3} \tag{3.5}$$

$$V_{I_t} = P_{3t}(X_{19t} - X_{10t}) \leq 0; \quad I_tV_{I_t} = 0; \quad I_t \geq 0 \tag{3.6}$$

The first-order conditions (3.1)-(3.6) and the equations in Appendix B combined with the original equations in the model define optimal values for the variables over this planning period. Our principal concern will be with the analysis of the optimal allocation of government and private investment expenditures. Thus we will be concerned with (3.1)-(3.6).

The allocation of government SOC expenditures between the subsistence and commercial agricultural activities is discussed in the next section. This is followed by a section in which the factors affecting private investment decisions are analyzed. The third alternative considered for government expenditures is to transfer resources from the tax budget to the private savings budget. This choice involves a decision between agricultural development and industrialization and is discussed in a later section.

IV. INVESTMENT IN AGRICULTURAL SOC

From (3.1) we see that one of the conditions for optimality with respect to investment in subsistence sector SOC is that

$$\lambda \sum_{i=t+1}^{T} (X_{2i}Y_{si}/G_{si}) \leq X_{6t} + \omega(1-\psi)(X_{10t} - X_{18}t), \tag{4.1}$$

where X_{2i} represents the social marginal (present) value of additional subsistence agricultural production in period $i > t$, X_{18t} is the marginal social value of additional consumer income generated in period t by employing labor in SOC accumulation, and X_{6t} and X_{10t} represent the social opportunity costs of labor and government purchasing power, respectively. From the first-order conditions in (3.1) it is apparent that if it is optimum to invest in G_s in any period (i.e., $\bar{L}_{st} > 0$), then it is necessary that the L.H.S. equals the R.H.S. of (4.1). In other words, if $\bar{L}_{st}$ is to be positive, then the sum of the discounted marginal social value productivity in all subsequent plan periods and the social value imputed to post-terminal productive capacity of labor employed in G_s accumulation in period t plus the social marginal value of income paid to labor on the SOC project must be large enough to offset the social opportunity cost of the labor employed on the project plus the social opportunity cost of the government expenditure.

Similarly for the commercial agricultural sector from (3.2) it is required that

$$\lambda \sum_{i=t+1}^{T} (X_{3i} Y_{1i} / G_{1i}) \leq X_{6t} + \omega(1-\psi)(X_{10t} - X_{18t}). \tag{4.2}$$

Since the R.H.S. of (4.2) is identical with the R.H.S. of (4.1), the decision to invest in G_s versus G_1 depends on the relative magnitudes of the L.H.S's of these equations. Investment in SOC will be optimal only in that sector where the L.H.S. is the largest, but it does not follow that investment is desirable in that sector for which the L.H.S. is the largest. Hence in discussing the relative desirability of developing subsistence versus commercial agriculture, it is necessary to examine components of the sums on the L.H.S. of (4.1) and (4.2).

In discussing the relative magnitudes of these sums, it is advantageous to begin with the terminal period of the plan (τ) and working towards the start of the planning period. For convenience, let us define

$$A_t = \lambda \sum_{i=t+1}^{T} (X_{2i} Y_{3i} / G_{si}) \tag{4.3}$$

and

$$D_t = \lambda \sum_{i=t+1}^{T} (X_{3i} Y_{1i} / G_{1i}) \tag{4.4}$$

Thus for the final period of the plan we have $A_\tau = X_{2T} Y_{sT} / G_{sT}$ and $D_\tau = X_{3T} Y_{1T} / G_{1T}$. Recalling the definitions of X_{2T}, X_{3T}, Y_{sT} and Y_{1T} introduced in (3.1) and (3.2), we have

$$A_\tau = \lambda P_{1\tau} \theta \sigma_s G_{sT}^{\lambda-1} F_{s\tau}^{\alpha_1} L_{s\tau}^{\alpha_2} B_s^{\alpha_4} \tag{4.5}$$

and

$$D_\tau = \lambda P_{1\tau} \theta \sigma_1 G_{1T}^{\lambda-1} F_{1\tau}^{\beta_1} L_{1\tau}^{\beta_2} K_{1\tau}^{\beta_3} B_1^{\beta_4}. \tag{4.6}$$

Since we are interested in comparing the relative magnitudes of A_τ and D_τ, we can ignore the common terms $\lambda P_{1\tau}\theta$. Recall that B_s and B_1 denote the quantities of land in the subsistence and commercial agricultural sectors, respectively. The relative size of B_s and B_1 unquestionably varies greatly from country to country. However, the portion of the land that is farmed by mechanized means in many LDC's is small relative to that which is farmed by traditional means. Since the land in the traditional sector is frequently more intensively farmed than land on larger holdings (e.g., latifundia), it was assumed that the productivity of land in the subsistence sector was higher than in the commercial sector. In other words the share of the output attributable to land (α_4) is larger in the subsistence sector than the portion attributable to land in the commercial sector (β_4). Based on these assumptions, we have $B_s^{\alpha_4} > B_1^{\beta_4}$. To the extent that the commercial sector uses more modern and hence more productive techniques than the subsistence sector, the influence of land will be at least partly offset by the larger "index of technology." In other words, it is likely that $\sigma_s < \sigma_1$ because more modern and efficient practices are being used on the commercial farms. An additional offsetting factor is the influence of mechanization in the commercial sector. This influence is represented by $K_{1T}^{\beta_3}$. It has been assumed that the return to labor in the advanced sectors exceeds the net income per capita in the subsistence sector. Thus we have $\beta_2 Y_{1t}/L_{1t} > (\alpha_2+\alpha_4)Y_{st}/L_{st}$, and since $\beta_2 < (\alpha_2+\alpha_4)$ it follows that $Y_{1t}/L_{1t} > Y_{st}/L_{st}$. Even though it is assumed that $L_{st} > L_{1t}$, since $\alpha_2 < \beta_2$ it is impossible to determine on the basis of these assumptions whether $L_{st}^{\alpha_2}$ exceeds $L_{1t}^{\beta_2}$ in any particular period. Finally, from (2.11) and (2.12) and the assumption that $\alpha_1 = \beta_1$, it follows that $Y_{1t}/F_{1t} = Y_{st}/F_{st}$. Consequently the magnitudes of $F_{st}^{\alpha_1}$ and $F_{1t}^{\beta_1}$ are proportional to the relative magnitudes of Y_{st} and Y_{1t}.

Bringing all these considerations together, it follows that the larger the relative size of the subsistence labor force relative to the commercial agricultural labor force, the larger A_T will be relative to D_T. Similarly, the larger $B_s^{\alpha_4}$ relative to $B_1^{\beta_4}$, the larger A_T will tend to be relative to D_T. Counterbalancing these two items, the larger the capital stock in commercial agriculture (K_{1T}) and the greater the disparity between the productivity of subsistence and commercial techniques (σ_s versus σ_1), the larger D_T will tend to be relative to A_T. The influence of purchased inputs varies with the relative size (measured in terms of output) of the two sectors. Thus, the relative values of A_T and D_T vary directly with the relative sizes of all the foregoing factors. The only exception is the size of G_{sT} compared with G_{1T}. The relative sizes of A_T and D_T vary inversely with the relative quantities of SOC available in the two sectors.

The reason for the concern over A_τ and D_τ is that A_τ and D_τ form the base for all earlier values of A_t and D_t. For example, for period τ-1, we have from (4.3) that

$$A_{\tau-1} = \lambda X_{2\tau} Y_{s\tau}/G_{s\tau} + A_\tau \ , \tag{4.7}$$

and from (4.4) we have

$$D_{\tau-1} = \lambda X_{3\tau} Y_{1\tau}/G_{1\tau} + B_\tau \ . \tag{4.8}$$

Thus, the larger A_τ relative to D_τ, the larger $A_{\tau-1}$ will be relative to $D_{\tau-1}$. In comparing the two additional terms in (4.7) and (4.8), the same factors or components have the same influence as in A_τ and D_τ. This becomes obvious when these terms are written as

$$Y_{s\tau}/G_{s\tau} = \sigma_s G_{s\tau}^{\lambda-1} F_{s\tau}^{\alpha_1} L_{s\tau}^{\alpha_2} B_{s\tau}^{\alpha_4}$$

and

$$Y_{1\tau}/G_{1\tau} = \sigma_1 G_{1\tau}^{\lambda-1} F_{1\tau}^{\beta_1} L_{1\tau}^{\beta_2} K_{1\tau}^{\beta_3} B_1^{\beta_4} \ .$$

Since the outputs of these two sectors are perfect substitutes in consumption, $X_{2t} = X_{3t}$ and hence can be ignored. Finally, replacing τ by t in (4.7) and (4.8) it is obvious that the same variables and parameters have similar influences throughout the entire period.

Summary and Conclusions

In summary, the following conclusions appear to be relevant in consideration of the relative desirability of investing in subsistence or commercial sector SOC. 1) The larger the total labor force, L_o, relative to the resource base of the economy (land and fixed capital stock) the relatively more desirable it is to invest in SOC in general and in subsistence SOC in particular. 2) The larger the proportion of the total land base used for subsistence forms of production, the more attractive investment in subsistence SOC becomes relative to investment in commercial sector SOC. 3) There is a certain amount of complementarity between investing in private capital in the commercial sector and the desirability of investing in G_1. In other words, private investment in K_1 tends to make investment in G_1 more desirable. 4) To the extent that the commercial sector employs more modern and more productive techniques than the subsistence sector, it will be relatively more desirable to invest in G_1 rather than G_s. 5) Investment in either G_1 or G_s in any period tends to reduce the relative desirability of investing in SOC in that sector in subsequent periods. 6) Finally, it is impossible to determine a priori whether it is more desirable to develop subsistence or commercial agriculture or which should be developed first.

The discussion throughout this entire section has been conducted in terms of the relative desirability of choosing between two alternatives. At no point was investing in G_s rather than G_1 (or vice versa) advocated. This decision cannot be made in the absence of data on the magnitudes of the various parameters and variables. Furthermore, the discussion in this section almost completely abstracts from consideration of the social opportunity costs involved. As indicated above, the decision in any one period will depend on the magnitudes of A_t and D_t relative to the social opportunity cost of using government tax revenue and labor for SOC accumulation. One of the major factors influencing this social opportunity cost is the social desirability of transferring tax revenue to the private savings budget. This social desirability depends directly on the private investment opportunities available which are discussed in the following section.

V. PRIVATE CAPITAL ACCUMULATION

The allocation of private investment funds in this model is governed by the criterion of social desirability. The application of the social desirability criterion to the investment alternatives is summarized in the first-order conditions (3.3)-(3.5). The social desirability of transferring revenue from the tax budget to the private savings budget is summarized in condition (3.6). The problem of transferring these funds is deferred until the following section. This section contains a discussion of the allocation of private investment funds. The method of analysis is similar to that employed in the previous section on SOC accumulation.

From (3.3)-(3.5) we have

$$\beta_3 \sum_{i=t+1}^{T} (X_{3i} Y_{1i} / K_{1i}) \leq X_{9t} + X_{19t} P_{3t} \tag{5.1}$$

$$\gamma_3 \sum_{i=t+1}^{T} (X_{4i} Y_{2i} / K_{2i}) \leq X_{9t} + X_{19t} P_{3t} \tag{5.2}$$

and

$$\delta_3 \sum_{i=t+1}^{T} (X_{5i} Y_{3i} / K_{3i}) \leq X_{9t} + X_{19t} P_{3t} \tag{5.3}$$

As in the case of SOC, if investment in K_1 is to be desirable in period t, (i.e., it is optimal for I_{1t} to be positive) then the L.H.S. of (5.1) must be equal in magnitude to the R.H.S. of (5.1). In other words, if investment is socially desirable in period t, then the discounted present marginal social value productivity of private capital in commercial agriculture in all successive periods plus the social value of post-plan productive capacity must be equal to the social opportunity costs of using investment goods and private savings in this manner. Similar interpretations apply to (5.2) and (5.3).

Making detailed comparisons among the desirability of the three private investment alternatives is more difficult than analyzing the two alternatives available for investment in SOC. This enhanced difficulty results from the greater asymmetry involved in the choices among the private investment alternatives. One troublesome aspect of this asymmetry is that the products produced by the three sectors all have their own marginal social value. Thus, comparison among physical characteristics is no longer sufficient as in the decision between investing in either G_l or G_s. The relative values of X_{3t}, X_{4t}, and X_{5t} must be considered in comparing the relative magnitudes of the LHS of (5.1)-(5.3).

The allocation of the private savings among the three alternative sectors requires that investment must occur in at least one of these sectors in every period. This differs from the problem of deciding between G_l or G_s for SOC investment. In the allocation of government funds it was possible that investment might not occur in either G_l or G_s in a particular period since the entire tax budget could be transferred to the private savings fund and used to accumulate private capital. No similar transfer option exists for private savings. Consequently capital must be accumulated in at least one sector. Thus, the social opportunity cost of placing capital $(X_{9t} + X_{19t}P_{3t})$ cannot exceed the largest of the terms on the L.H.S. of (5.1)-(5.3). If investment occurs in more than one sector, the values of the L.H.S. of (5.1)-(5.3) corresponding to these sectors must be equal. Investment, however, will be socially desirable in only those sectors for which the value of the L.H.S. of the conditions equals the social opportunity cost. This equality will prevail only in those sectors with the larger values on the L.H.S. Thus, it becomes important to determine which economic factors contribute to increasing the value of the L.H.S.

The Share of Capital

One of the more obvious elements to be considered is the relative magnitudes of the three parameters β_3, γ_3, and δ_3. From (5.1)-(5.3) it is obvious that if any one of these parameters is large relative to the other two, it will be more likely that investment will be desirable in that sector for any given set of capital/output ratios. In other words, the larger the share of output attributable to capital in a particular sector, the higher the optimum capital/output ratio becomes relative to other sectors.

Social Value of Outputs

The desirability of investing in the various sectors is strongly influenced by the social values attached to the outputs of the three sectors X_3, X_4, and X_5. The social value of capital goods production (X_5) is an indirect or imputed social value since capital goods do not enter into the welfare function directly except in the evaluation of post-terminal productive capacity. Since capital goods are not consumed, the production of capital goods is desirable only from the standpoint of the increased production and consumption of agricultural and manufactured goods made possible in subsequent periods through the accumulation of capital. At the other extreme, agricultural output is used for consumption purposes only. Consequently, the social value of agricultural production is derived strictly from direct consumption benefits and no indirect value is imputed to agricultural production in this model. A positive social value on capital goods production expresses a concern for expanded future consumption, while a positive value for agricultural or manufacturing production expresses a concern for present welfare. Between the extremes exemplified by agricultural and capital goods is the social value of manufactured production. Since manufactured goods are used both for consumption and as a factor of production, X_4 contains elements of both direct and indirect social value.

Comparisons among the relative magnitudes of the three social values is difficult because of the nature of the considerations involved. The easier comparison is between X_3 and X_4 since intertemporal considerations are not explicitly involved within periods. During the initial periods of the plan the magnitude of X_3 might be expected to exceed the magnitude of X_4. Based on the assumptions about the welfare function discussed in section II, the marginal social utility of an additional unit of C_1 is assumed to exceed the marginal social utility of C_2 in the early periods of the plan. This implies that $X_2 = X_3 > X_4$. This is true even though manufactured goods are also used as factors of production. The marginal welfare derived from the consumption of additional units of agricultural goods declines more rapidly than the marginal welfare of additional manufactured goods consumption. The ratio X_{3t}/X_{4t} might decline over time if both agricultural and manufactured goods production increase. However, this need not be the case if the ratio C_{1t}/C_{2t} declines at a sufficiently rapid rate.

It is more difficult to make meaningful comparisons of X_{5t} with X_{3t} or X_{4t} than to make comparisons between X_{3t} and X_{4t}. Comparisons involving X_{5t} require consideration of the social value of present versus future consumption since the value of X_{5t} is an imputed value derived from increased consumption of manufactured and agricultural goods made possible. The intertemporal aspect of the problem arises because the social payoff from the production of capital goods in period t cannot be realized as expanded consumption before period t+1. Thus, if society places a high premium on present consumption relative to future consumption, the value of X_5 will be lower than if society was relatively less concerned with shorter term payoffs. The social rate of discount, ρ, is chosen by the policy-maker to reflect society's intertemporal preferences with respect to consumption. An increase in the social rate of discount will result in a decline in the social value of capital accumulation, X_5. The other parameter in the model which reflects society's intertemporal preferences is the weight given to post-plan productive capacity, θ. A greater concern by society to bequeath a large productive capacity to future generations is reflected in the model by an increase in θ. This terminal productive capacity must, to some extent, be acquired at the expense of reduced consumption during the plan. Consequently, an increase in the magnitude of θ leads to a concomitant increase in the social value of capital goods production. The value of X_5 is determined to a large extent by the social rate of discount and the relative emphasis given to terminal productive capacity. While the analysis of the consequences of choosing particular values for these parameters is an economic problem, the actual choice of the values of the parameters is essentially a political question involving the ethics of the well-being of current versus future generations as well as the problem of current versus delayed consumption within the present generation.

In summary, the problem of comparing the relative magnitudes of X_5 with X_3 and X_4 involves many diverse considerations such as levels of production of the three goods as well as the relative rates of expansion of C_{1t} and C_{2t}. The most difficult problem, however, arises from the inter-temporal aspects of current versus delayed consumption. In general, as relatively more emphasis is placed on current rather than future consumption, less emphasis will be placed on the accumulation of capital goods and the absolute and relative levels of C_1 and C_2 become proportionately more important in determining the allocation of investment. Concomitant with this is reduced emphasis on expansion of capital goods capacity as reflected by a lower value for X_5.

The rate of technical change and SOC accumulation

The only terms on the LHS of (5.1) - (5.3) remaining to be considered are the output/capital ratios. From (2.3) - (2.5) we have

$$Y_{1t}/K_{1t} = \sigma_1 G_{1t}^{\lambda} F_{1t}^{\beta_1} L_{1t}^{\beta_2} K_{1t}^{\beta_3} B_1^{\beta_4}/K_{1t}, \tag{5.4}$$

$$Y_{2t}/K_{2t} = \sigma_2 (1+\varepsilon)^t F_{2t}^{\gamma_1} L_{2t}^{\gamma_2} K_{2t}^{\gamma_3}/K_{2t}, \tag{5.5}$$

and

$$Y_{3t}/K_{3t} = \sigma_3 (1+\varepsilon)^t F_{3t}^{\delta_1} L_{3t}^{\delta_2} K_{3t}^{\delta_3}/K_{3t}. \tag{5.6}$$

Since the numerators of the ratios in (4.69)-(4.71) involve different units of account, the only meaningful comparisons among these ratios involve those factors which will tend to change the relative magnitudes of these ratios over time.

The most obvious factor is the rate of technical chance, ε, in the manufacturing and capital goods sector relative to the rate of SOC accumulation in commercial agriculture. The "effective" rate of SOC accumulation is

$$\frac{G_{1,t+1}^{\lambda} - G_{1t}^{\lambda}}{G_{1t}^{\lambda}} = \left[\frac{\bar{L}_{1t}}{\bar{G}_{11} + \sum_{i=1}^{t-1} \bar{L}_{1i}}\right]^{\lambda} \geq 0. \tag{5.7}$$

Since $\varepsilon > 0$, the productive influence of SOC accumulation in commercial agriculture may be greater than, equal to, or less than the exogenous rate of technical change in the manufacturing and capital goods sectors. Denote the LHS of (5.7) as $\Delta G/G$. If $\Delta G/G > \varepsilon$, then private capital accumulation in the agricultural sector would be relatively more desirable vis-à-vis the nonagricultural sectors than if $\Delta G/G < \varepsilon$. This is because, ceteris paribus, the larger the rate of increase of the output/capital ratio in a sector, the relatively more desirable it will be to expand the capital stock in that sector. While ε is a constant $\Delta G/G$ may vary over time. Consequently SOC accumulation will have a varied influence over time on the relative desirability of private investment in commercial agriculture.

Changes in the terms of trade

The remaining elements in (5.1)-(5.3) that can alter the output/capital ratios are the inputs of manufactured goods and labor. From (2.12)-(2.17) it is apparent that the influence of these factors is determined by the TT over time. Since $P_{2t} = 1$, the output/capital ratio in the manufacturing sector may be treated as a numéraire. If P_{1t} increases over time, it will become profitable to employ larger amounts of labor and manufactured inputs in this sector, which will tend to increase Y_{1t}/K_{1t} relative to Y_{2t}/K_{2t}. This increase in the output/capital ratio in commercial agriculture will tend to make investment in this sector relatively more desirable than investment in manufacturing. The opposite result ensues if P_{1t} declines over time. Similarly, changes in P_{3t} over time will have analogous implications for the relative desirability of investing in the capital goods sector. Thus, as the TT move in favor of a particular sector, this will tend to make investment in that sector socially more desirable since it becomes profitable to employ more variable factors of production in that sector.

Summary

In this section, the allocation of private investment funds has been analyzed. The following conclusions regarding certain economic and technical considerations appear to be relevant in deciding upon the relative social desirability of investing in one or more of the private investment alternatives. (1) In the initial periods of the plan the social desirability of marginal increments of agricultural goods may be expected to exceed the social desirability of marginal increments of manufactured goods. This tends to enhance the social desirability of investing in agriculture relative to manufacturing early in the plan. (2) Whether the relative social desirabilities of marginal increments of production of agricultural and manufactured goods remain unchanged over the planning horizon depends on the relative rates of increase of consumption of the two goods. If the ratio of agricultural goods/manufactured goods consumption declines over time at a sufficiently rapid rate, the social desirability of marginal increments of agricultural production may increase relative to the social value of an increment of manufactured goods production. (3) The social desirability of expanding productive capacity in the capital goods sector is enhanced relative to expansion of the productive capacity of the other sectors if relatively less weight is given to current rather than future consumption during the plan. In other words, the lower the social rate of discount the greater the social desirability of investing in expansion of the capital goods sector. (4) As society's concern to bequeath a large post-plan productive capacity to future generations increases, the social desirability of expanding the productive capacity of the capital goods sector during the plan will increase. (5) If the effective rate of SOC accumulation in commercial agriculture exceeds the exogenous rate of technical change in the manufacturing and capital goods sectors, this will tend to increase the social desirability of investing in agriculture relative to the investing in the manufacturing or capital goods sectors. This, however, may have adverse TT effects. (6) If the TT move in favor of a particular sector, this tends to increase the social desirability of expanding the productive capacity of that sector via investment. This conclusion depends crucially on the assumption of a closed economy.

The discussions of the allocation of private and public investment funds have largely abstracted from the opportunity costs of making these investments and the interrelationships between private and public investment. These problems are considered in the following section.

VI Private Investment Versus Agricultural SOC Accumulation

The marginal social desirability of investing in either private capital or SOC is determined by the potential payoffs involved from such investments and the total amount of investment funds available for these purposes. The potential payoffs have been extensively analyzed in the preceding sections. In this section the availability of funds is considered.

The total funds available for SOC accumulation are the tax revenue collected in the particular period. The government budget constraint is given in (2.10). The funds available for private capital accumulation are the income earned by the existing capital stock plus any funds transferred from the government budget. The private savings budget is given in (2.19).

The optimal transfer of funds from the government budget to the private savings budget must satisfy the first-order requirements in (3.6). Since $P_3 > 0$, it follows that

$$X_{19t} \leq X_{10t} \qquad (6.1)$$

This requires that the social value of a marginal increment of investment in private capital (X_{19t}) must not exceed the social value of a marginal increment in SOC accumulation (X_{10t}). This relationship is maintained by transferring government funds to the private savings budget if the social payoff to private investment exceeds the payoff to SOC accumulation. Furthermore, the social value of marginal increments in investment in these two alternatives must be equal if it is desirable for funds to be transferred from the government to the private budget.

Suppose that in period t, $\overline{L}_{st}$ and I_{1t} are positive. This implies that (4.1) and (5.1) will be satisfied as equalities. Substituting these equations into (6.1) we get

$$\frac{\beta_3 \sum_{i=t+1}^{T} (X_{3i}Y_{1i}/K_{1i}) - X_{9t}}{P_{3t}} \leq \frac{\lambda \sum_{i=t+1}^{T} (X_{2i}Y_{si}/G_{si}) - X_{6t}}{(1-\psi)\omega} + X_{18t}. \quad (6.2)$$

This equation indicates that the net social marginal benefit per unit of purchasing power in investment in private capital in the commercial agricultural sector must not exceed the net social marginal benefit per unit of purchasing power spent on SOC accumulation in the subsistence sector. This relationship is maintained by transferring resources.

Turning now to the interpretation of the individual terms in (6.2), the first term on the LHS represents the discounted present marginal social value productivity of private capital stocks in commercial agricultural production in subsequent periods of the plan, deflated by the price of investment goods in period t. The second term on the LHS of (6.2) indicates the social opportunity cost of using investment goods in this manner in period t, deflated by the cost of purchasing these goods. The first term on the RHS indicates the present social marginal value productivity of SOC in subsistence agricultural production in subsequent periods of the plan per unit of net government labor cost. The second term is the social opportunity cost (per unit of government purchasing power) of using labor for SOC accumulation in period t. Finally, the last term on the RHS is the marginal social benefit derived from the increased consumer income resulting from the employment of labor in SOC accumulation.

The relative importance of the social opportunity cost of using capital goods per unit of private savings expended (X_{9t}/P_{3t}) and the social opportunity cost of using labor per unit of government expenditure $(X_{6t}\{1-\psi\}\omega)$ will be influenced by the capacity of the capital goods industry and the size of the labor force. As the capacity of the capital goods industry increases relative to the size of the labor force, the social opportunity cost of using investment goods will decline relative to the social opportunity cost of using labor. This suggests the transfer of funds from the government budget to the private savings budget would be relatively more attractive in an economy which has a larger productive capacity in the capital goods industry. The opposite, of course, is true in an economy which has relatively more labor in proportion to capital goods capacity.

An additional influence tending to diminish the social desirability of transferring funds is the social benefit of the consumer income generated by the government employing labor for SOC accumulation. This results in a higher wage rate for any labor transferred from subsistence agriculture to the government payroll, and the magnitude of the increase in the wage rate would be expected to be larger as the ratio of labor to the resource base of the economy increases. Consequently, transfer of funds from the government budget to the private savings budget is less likely to occur in economies which have high ratios of labor to resources.

VII Summary and Conclusions

Previous research has demonstrated the necessity of developing the agricultural sector in less developed economies to provide the necessary resources (capital and labor) for the emergence of an industrial sector.[1] These investigations, however, have ignored the contribution made by the non agricultural sectors to agricultural development. In this study we have explicitly included the demands of the agricultural sector for manufactured inputs (e.g., fertilizers and pesticides) and services (e.g., marketing facilities).

According to Engel's law, the lower the level of development in an economy (per capita incomes) the greater will be the proportion of income spent on food. Thus, the greatest short run payoff in a country with very low levels of per capita incomes will be in the expansion of the agricultural sector. However, if the expansion of the agricultural sector increases its demands for goods and services provided by the manufacturing sector, agricultural expansion can occur only if the consumption of manufactured goods declines or output in the manufacturing sector also increases. This implies that even in the most underdeveloped economies it may be essential to expand the manufacturing sector as a corequisite to any significant expansion of the agricultural sector.

Early in the development of a "labor-surplus" type of economy, expansion of agricultural capacity via labor intensive SOC projects is more desirable than private investment in agricultural equipment and machinery for three reasons. Firstly, capacity in the capital equipment industry is usually very limited and can be more productively utilized to provide equipment for nonagricultural needs. Secondly, the social payoff to creating jobs in a labor abundant economy is a

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[1] See especially, Nicholls (14) and Jorgenson (6).

useful goal in itself. Finally, investment in SOC will tend to benefit a larger group of people (the agricultural workers) through increased productivity rather than simply allowing the gains to accrue to the few capital owners who have managed to accumulate some additional capital.

In later stages of development, the social payoff for further expansion of the agricultural sectors declines relative to expansion of the non-agricultural sectors. However, in the earlier stages the expansion of agriculture should be emphasized but not to the exclusion of industry.

APPENDIX A

The Lagrangean function is as follows, where all summations are over t from 1 to τ.

$$V = \Sigma(\mu_1 C_{1t} + \mu_2 C_{2t} - \mu_{11} C_{1t}^2 + \mu_{12} C_{1t} C_{2t} - \mu_{22} C_{2t}^2)(1+\rho)^{-t}$$

$$+ \theta\Big\{P_{1\tau}\sigma_s G_{sT}^{\lambda} F_{s\tau}^{\alpha_1} L_{s\tau}^{\alpha_2} B_s^{\alpha_4} + P_{1\tau}\sigma_1 G_{1T}^{\lambda} F_{1\tau}^{\beta_1} L_{1\tau}^{\beta_2} K_{1T}^{\beta_3} B_1^{\beta_4}$$

$$+ (1+\epsilon)^T \sigma_2 F_{2\tau}^{\gamma_1} L_{2\tau}^{\gamma_2} K_{2T}^{\gamma_3} - F_{s\tau} - F_{1\tau} - F_{2\tau} - F_{3\tau} + P_{3\tau}$$

$$(1+\epsilon)^T \sigma_3 F_{3\tau}^{\delta_1} L_{3\tau}^{\delta_2} K_{3T}^{\delta_3}\Big\}$$

$$+ \Sigma X_{2t}(\sigma_s G_{st}^{\lambda} F_{st}^{\alpha_1} L_{st}^{\alpha_2} B_s^{\alpha_4} - Y_{st})$$

$$+ \Sigma X_{3t}(\sigma_1 G_{1t}^{\lambda} F_{1t}^{\beta_1} L_{1t}^{\beta_2} K_{1t}^{\beta_3} B_1^{\beta_4} - Y_{1t})$$

$$+ \Sigma X_{4t}(\sigma_2(1+\epsilon)^t F_{2t}^{\gamma_1} L_{2t}^{\gamma_2} K_{2t}^{\gamma_3} - Y_{2t})$$

$$+ \Sigma X_{5t}(\sigma_3(1+\epsilon)^t F_{3t}^{\delta_1} L_{3t}^{\delta_2} K_{3t}^{\delta_3} - Y_{3t})$$

$$+ \Sigma X_{6t}(L_o - \bar{L}_{st} - \bar{L}_{1t} - L_{st} - L_{1t} - L_{2t} - L_{3t})$$

$$+ \Sigma X_{7t}(Y_{st} + Y_{1t} - C_{1t})$$

$$+ \Sigma X_{8t}(Y_{2t} - F_{st} - F_{1t} - F_{2t} - F_{3t} - C_{2t})$$

$$+ \Sigma X_{9t}(Y_{3t} - I_{1t} - I_{2t} - I_{3t})$$

$$+ \Sigma X_{10t}\{\psi[P_{1t}Y_{st}(1-\alpha_1) + P_{1t}Y_{1t}(1-\beta_1) + Y_{2t}(1-\gamma_1) + P_{3t}Y_{3t}(1-\delta_1)]$$

$$- \omega(1-\psi)(\bar{L}_{st} + \bar{L}_{1t}) - P_{3t}I_t\}$$

$$+ \Sigma X_{11t}(F_{st} - \alpha_1 P_{1t} Y_{st}) + \Sigma X_{12t}(F_{1t} - \beta_1 P_{1t} Y_{1t})$$

$$+ \Sigma X_{13t}(F_{2t} - \gamma_1 Y_{2t}) + \Sigma X_{14t}(F_{3t} - \delta_1 P_{3t} Y_{3t})$$

$$+ \Sigma X_{15t}(\omega L_{1t} - \beta_2 P_{1t} Y_{1t}) + \Sigma X_{16t}(\omega L_{2t} - \gamma_2 Y_{2t})$$

$$+ \Sigma X_{17t}(\omega L_{3t} - \delta_2 P_{3t} Y_{3t})$$

$$+ \Sigma X_{18t}\{(1-\psi)[(\alpha_2+\alpha_4)Y_{st} P_{1t} + \beta_2 Y_{1t} P_{1t} + \gamma_2 Y_{2t} + \delta_2 Y_{3t} P_{3t} + \omega(\bar{L}_{st}+\bar{L}_{1t})]$$
$$- P_{1t}C_{1t} - C_{2t}\}$$

$$+ \Sigma X_{19t}\{(1-\psi)[(\beta_3+\beta_4)P_{1t} Y_{1t} + \gamma_3 Y_{2t} + \delta_3 P_{3t} Y_{3t}] + P_{3t}(I_t - I_{1t} - I_{2t} - I_{3t})\}$$

$$+ \Sigma X_{20t}(C_{1t} - 1) + \Sigma X_{21t}(C_{2t} - 1).$$

APPENDIX B

In addition to the first-order conditions specified in equations (3.1)-(3.6), the following equations are necessary to define the optimum:

$$V_{C_1} = (\mu_1 - 2\mu_{11}C_1 + \mu_{12}C_2)(1+\rho)^{-t} - X_7 - X_{18} P_1 + X_{20} = 0 \qquad \text{(B.1)}$$

$$V_{C_2} = (\mu_2 + \mu_{12}C_1 - 2\mu_{22}C_2)(1+\rho)^{-t} - X_8 - X_{18} + X_{21} = 0 \qquad \text{(B.2)}$$

$$V_{Y_s} = -X_2 + X_7 + X_{10}P_1\psi(1-\alpha_1) - X_{11}P_1\alpha_1 + X_{18}P_1(1-\psi)(\alpha_2+\alpha_4) = 0 \qquad \text{(B.3)}$$

$$V_{Y_1} = -X_3 + X_7 + P_1\{X_{10}\psi(1-\beta_1) - X_{12}\beta_1 + X_{15}\beta_2 + X_{18}\beta_2(1-\psi)$$
$$+ X_{19}(1-\psi)(\beta_3+\beta_4)\} = 0 \qquad \text{(B.4)}$$

$$V_{Y_2} = -X_4 + X_8 + X_{10}\psi(1-\gamma_1) - X_{13}\gamma_1 + X_{16}\gamma_2 + X_{18}(1-\psi)\gamma_2$$
$$+ X_{19}(1-\psi)\gamma_3 = 0 \qquad \text{(B.5)}$$

$$V_{Y_3} = -X_5 + X_9 + P_3\{X_{10}\psi(1-\delta_1) - X_{14}\delta_1 + X_{17}\delta_2 + X_{18}(1-\psi)\delta_2$$
$$+ X_{19}(1-\psi)\delta_3\} = 0 \qquad \text{(B.6)}$$

$$V_{F_s} = X_2\alpha_1 Y_s/F_s - X_8 + X_{11} = 0 \quad (t=1, \ldots, \tau-1) \qquad \text{(B.7)}$$

$$V_{F_1} = X_3\beta_1 Y_1/F_1 - X_8 + X_{12} = 0 \quad (t=1, \ldots, \tau-1) \qquad \text{(B.8)}$$

$$V_{F_2} = X_4\gamma_1 Y_2/F_2 - X_8 + X_{13} = 0 \quad (t=1, \ldots, \tau-1) \qquad \text{(B.9)}$$

$$V_{F_3} = X_5\delta_1 Y_3/F_3 - X_8 + X_{14} = 0 \quad (t=1, \ldots, \tau-1) \qquad \text{(B.10)}$$

$$V_{L_s} = X_2\alpha_2 Y_s/L_s - X_6 = 0 \quad (t=1, \ldots, \tau-1) \tag{B.11}$$

$$V_{L_1} = X_3\beta_2 Y_1/L_1 - X_6 - X_{15}\omega = 0 \quad (t=1, \ldots, \tau-1) \tag{B.12}$$

$$V_{L_2} = X_4\gamma_2 Y_2/L_2 - X_6 - X_{16}\omega = 0 \quad (t=1, \ldots, \tau-1) \tag{B.13}$$

$$V_{L_3} = X_5\delta_2 Y_3/L_3 - X_6 - X_{17}\omega = 0 \quad (t=1, \ldots, \tau-1) \tag{B.14}$$

$$\begin{aligned} V_{P_1} &= X_{10}\psi\{Y_s(1-\alpha_1) + Y_1(1-\beta_1)\} - X_{11}\alpha_1 Y_s - X_{12}\beta_1 Y_1 \\ &+ X_{15}\beta_2 Y_1 + X_{18}[(1-\psi)\{(\alpha_2+\alpha_4)Y_s + \beta_2 Y_1\} - C_1] \\ &+ X_{19}(1-\psi)(\beta_3+\beta_4)Y_1 = 0 \quad (t=1, \ldots, \tau-1) \end{aligned} \tag{B.15}$$

$$\begin{aligned} V_{P_3} &= X_{10}\{\psi(1-\delta_1)Y_3 - I\} - X_{14}\delta_1 Y_3 + X_{17}\delta_2 Y_3 + X_{18}(1-\psi)\delta_2 Y_3 \\ &+ X_{19}\{(1-\psi)\delta_3 Y_3 + I - I_1 - I_2 - I_3\} = 0 \quad (t=1, \ldots, \tau-1) \end{aligned} \tag{B.16}$$

The following special first-order conditions apply to the final period of the plan.

$$V_{F_{3\tau}} = \theta P_{1\tau}\alpha_1\sigma_s G_{sT}^{\lambda} F_{s\tau}^{\alpha_1-1} L_{s\tau}^{\alpha_2} - \theta + X_{2\tau}\alpha_1 Y_{s\tau}/F_{s\tau} - X_{8\tau} + X_{11\tau} = 0 \tag{B.17}$$

$$\begin{aligned} V_{F_{1\tau}} &= \theta P_{1\tau}\beta_1\sigma_1 G_{1T}^{\lambda} F_{1\tau}^{\beta_1-1} L_{1\tau}^{\beta_2} K_{1T}^{\beta_3} - \theta + X_{3\tau}\beta_1 Y_{1\tau}/F_{1\tau} \\ &- X_{8\tau} + X_{12\tau} = 0 \end{aligned} \tag{B.18}$$

$$\begin{aligned} V_{F_{2\tau}} &= \theta\gamma_1(1+\varepsilon)^T\sigma_2 F_{2\tau}^{\gamma_1-1} L_{2\tau}^{\gamma_2} K_{2T}^{\gamma_3} - \theta + X_{4\tau}\gamma_1 Y_{2\tau}/F_{2\tau} - X_{8\tau} \\ &+ X_{13\tau} = 0 \end{aligned} \tag{B.19}$$

$$\begin{aligned} V_{F_{3\tau}} &= \theta P_{3\tau}\delta_1(1+\varepsilon)^T\sigma_3 F_{3\tau}^{\delta_1-1} L_{3\tau}^{\delta_2} K_{3T}^{\delta_3} - \theta + X_{5\tau}\delta_1 Y_{3\tau}/F_{2\tau} - X_{8\tau} \\ &+ X_{14\tau} = 0 \end{aligned} \tag{B.20}$$

$$V_{L_{s\tau}} = \theta P_{1\tau}\alpha_2\sigma_s G_{sT}^{\lambda}F_{s\tau}^{\alpha_1}L_{s\tau}^{\alpha_2-1} + X_{2\tau}\alpha_2 Y_{s\tau}/L_{s\tau} - X_{6\tau} = 0 \quad (B.21)$$

$$V_{L_{1\tau}} = \theta P_{1\tau}\beta_2\ \sigma_1 G_{1T}^{\lambda}F_{1\tau}^{\beta_1}L_{1\tau}^{\beta_2-1}K_{1T}^{\beta_3} + X_{3\tau}\beta_2 Y_{1\tau}/L_{1\tau} - X_{6\tau}$$

$$- X_{15\tau}\omega = 0 \quad (B.22)$$

$$V_{L_{2\tau}} = \theta\gamma_2(1+\epsilon)^T\sigma_2 F_{2\tau}^{\gamma_1}L_{2\tau}^{\gamma_2-1}K_{2T}^{\gamma_3} + X_{4\tau}\gamma_2 Y_{2\tau}/L_{2\tau} - X_{6\tau}$$

$$- X_{16\tau}\omega = 0 \quad (B.23)$$

$$V_{L_{3\tau}} = \theta\delta_2(1+\epsilon)^T\sigma_3 F_{3\tau}^{\delta_1}L_{3\tau}^{\delta_2-1}K_{3T}^{\delta_3} + X_5\delta_2 Y_{3\tau}/L_{3\tau} - X_{6\tau}$$

$$- X_{17\tau}\omega = 0 \quad (B.24)$$

$$V_{P_{1\tau}} = \theta(\sigma_s G_{sT}^{\lambda}F_{s\tau}^{\alpha_1}L_{s\tau}^{\alpha_2} + \sigma_1 G_{1T}^{\lambda}F_{1\tau}^{\beta_1}L_{1\tau}^{\beta_2}K_{1T}^{\beta_3}) + X_{10\tau}\psi\{Y_{st}(1-\alpha_1) + Y_{1t}(1-\beta_1)\}$$

$$- X_{11\tau}\alpha_1 Y_{s\tau} - X_{12\tau}\beta_1 Y_{1\tau} + X_{15\tau}\beta_2 Y_{1\tau} + X_{18\tau}\{(1-\psi)[(\alpha_2+\alpha_4)Y_{s\tau}$$

$$+ \beta_2 Y_{1\tau}] - C_{1\tau}\} + X_{19\tau}(1-\psi)(\beta_3+\beta_4)Y_{1\tau} = 0 \quad (B.25)$$

$$V_{P_{3\tau}} = \theta(1+\epsilon)^T\sigma_3 F_{3\tau}^{\delta_1}L_{3\tau}^{\delta_2}K_{3T}^{\delta_3} + X_{10\tau}\{\psi(1-\delta_1)Y_{3\tau} - I_\tau\} - X_{14\tau}\delta_1 Y_{3\tau}$$

$$+ X_{17\tau}\delta_2 Y_{3\tau} + X_{18\tau}(1-\psi)\delta_2 Y_{3\tau} + X_{19\tau}\{(1-\psi)\delta_3 Y_{3\tau} + I_\tau - I_{1\tau}$$

$$- I_{2\tau} - I_{3\tau}\} = 0 \quad (B.26)$$

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