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I. INTRODUCTION

This paper will present a stochastic equilibrium model in which the optimizing behavior of a heterogeneous population will determine the price of a real asset. The characteristics of the equilibrium asset price, and the determinants of its variability will be investigated. The heterogeneity of the population may provide further insights into the determination of excess stock price variability.

That stock prices appear to exhibit a high degree of variability is well documented (see Grossman and Shiller [6], LeRoy and Porter [17], and Shiller [22]). In addition there have been several attempted explanations of this anomaly, such as: (i) a high degree of risk aversion exhibited by agents (LeRoy and LaCivita [16], Grossman and Shiller [6]), (ii) fluctuations in non-capital income or changes in the cost of producing new capital (Huffman [11]), (iii) or flaws in the tests which (ostensibly) implied excessive price variability (Flavin [5], and Kleidon [13]). This paper will present yet another factor which may enable us to better understand the determinants of asset prices. This paper is, to some extent, viewed as a contribution to the theoretical studies of asset price determination as in Huberman [9] and Lucas [18]. In both of these papers there are many capital assets which yield dividends each period whose distribution is governed by a stochastic process. The prices of the assets are determined by the behavior of optimizing agents, and these prices may vary, depending upon present and expected future realizations of factors in the environment. Lucas employs the paradigm of an infinitely-lived representative agent, whereas Huberman uses a model of two-period lived overlapping generations who have a non-capital endowment in the first period of their life.

As mentioned elsewhere (Huffman [11]), the infinitely-lived representative agent paradigm has not met with startling success when confronted with the data (see Grossman and Shiller [6], Hansen and Singleton [7], Prescott and Mehra [20]). This has then led to an analysis of the determinants of asset pricing with heterogeneous agents. Because, in general, all agents who participate in the asset market are not faced with the same planning horizon or budget constraints, it would seem to be of interest to inquire as to whether these factors could deliver a pattern for the behavior of asset prices substantially different from those described in the existing literature. This paper is then a contribution to the growing literature which seeks to develop environments in which the conventional representative agent asset-pricing formulas do not obtain.

In the model presented below agents ~~who~~ live for two periods. Each period, there are born some agents who have different endowment streams, and potentially different preferences, than other agents born at the same date. A special case of this would be an environment in which some agents are endowed only in the first period of their life while their cohorts are endowed only in the second period of their life. The latter agents may appropriately be referred to as borrowers while the former agents may be referred to as lenders.[1] Hence, there are, at any date, four different types of agents. In addition, there exists shares of a capital asset which yields an exogenous stochastic dividend each period.

For convenience, it is assumed that each period a signal is given to the population. These signals will be distributed according to a Markov process, and will completely characterize the state of the economy. The signal will completely determine the dividend payments and the composition of the population. The price of capital is then determined to be a function of this signal or state vector.

The structure of this environment is important because it will permit the study of an environment in which there is borrowing and lending among agents at the same time as capital assets are held. Those agents who are heavily endowed in the second period of their life may wish to borrow some of the consumption good in the first period of their life, and repay the loan in the second period. Agents heavily endowed in the first period of their life may be induced to participate in such a scheme only if the expected rate of return on these "loans" is at least as great as the prospective return on holding the capital asset.

Scheinkman and Weiss [21] have indicated that restrictions on borrowing and lending opportunities may contribute to excess asset-price variability. In contrast, the environment employed in this paper is of interest because it may permit the analysis of whether heterogeneous participation in the asset market together with private borrowing and lending will produce high asset price variability. Furthermore, the finite-lived overlapping generations construct seems appropriate when one considers that titles to durable capital goods typically last beyond the time period when any single agent participates in the market for capital.

The remainder of this paper is as follows. Section II presents a detailed description of the physical environment, including preferences and technology. Section III contains a description of the optimization problems faced by agents and a characterization of their solution. Detailed proofs are relegated to the appendices. Section IV contains a discussion of the effects on asset pricing of intragenerational borrowing and lending. This is illustrated with a series of examples. The summary is contained in Section V.

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II. THE PHYSICAL ENVIRONMENT

This section describes an economy in which at each date $t \geq 1$, there is born a generation of two-period lived agents. In each period t, the agents receive a signal z_t simultaneously as the members of generation (t) appear in the economy. The sequence $\{z_t\}$ is distributed according to a Markov process with time-autonomous density function $f(z_t | z_{t-1})$ over the space Z. For convenience, it is assumed Z is a compact subset of $\mathbb{R}^n$. Henceforth, z_t will be referred to as the state of the economy at time t, and variables which change over time will be written as functions of this state variable.

In each period t there are born N identical two-period lived agents of type "ℓ". These agents are endowed with $y_1^{\ell}(z_t)$ units of the homogeneous consumption good in the first period of their life, and $y_2^{\ell}(z_t, z_{t+1})$ units in the second period of their life. Each of these agents maximizes the separate strictly concave, twice continuously differentiable utility function $U_{\ell}(c_{1t}^{\ell}) + V_{\ell}(c_{2t}^{\ell})$, which represents utility derived from consumption in the first and second periods of the agent's life. In addition, at each date (t), there are born $M^b(z_t)$ identical two-period lived agents, of type "b", who are endowed with $y_1^b(z_t)$ of the consumption good in the first period of their life. In addition, these agents also have an endowment of $y_2^b(z_t, z_{t+1})$ in the second period of their life. Each of these agents maximizes a separable strictly concave twice continuously differentiable utility function $U_b(c_{1t}^b) + V_b(c_{2t}^b)$ which, again, represents utility derived from consumption in both first and second periods of the agent's lifetime.

Define the function $\theta(z_t) \equiv \frac{M^b(z_t)}{N}$. The functions $y_1^j(\cdot)$, $y_2^j(\cdot,\cdot)$, $j = \ell, b$, $M^b(\ )$, and hence $\theta(\ )$ are restricted to be continuous. For convenience, the endowments are further restricted so that

$$y_1^{\ell}(z_t) + y_1^b(z_t)\theta(z_t) = 1$$

and

$$y_2^{\ell}(z_t, z_{t+1}) + y_2^b(z_t, z_{t+1})\theta(z_t) = \theta(z_t).^2$$

The notation is used to indicate several points. First, agents in the first period of their life do not know with certainty what their endowments will be in the second period. These endowments will depend upon the realizations of the state variables in both the first and second periods of the agent's life. Alternatively, second period endowments could have been written as functions of only that period's state vector. Instead, this notation was not used because, as will be shown in Appendix B, the competitive equilibrium will be shown to exist only for those economies in which second period endowments are dependent upon both the current and previous state vector. Secondly, as will be described below, it may be convenient to suppose that $\frac{1}{2} < y_1^{\ell}(z_t) < 1$ and $\frac{1}{2} < y_2^b(z_t, z_{t+1})\theta(z_t) < 1$ for all z_t, $z_{t+1} \in Z$. Then, the notation "ℓ" is meant to signify that agents endowed with $y_1^{\ell}(z_t)$ units of the consumption good in the first period of their life will seek to lend some of the consumption good to those agents endowed with only $y_1^b(z_t)$ units. These latter agents may be thought of as borrowers, and hence the superscript "b". These agents have a larger endowment in the second period of their life than in the first, and will therefore, at certain rates of interest, attempt to borrow from their cohorts endowed heavily in the first period of their life. Lastly, as the notation indicates, the utility functions of the two types of agents need not be identical. That is, they may have different risk aversion properties.

At time $t=1$, there also exist N one-period lived agents who seek to maximize the utility of their period 1 consumption. These agents also hold, in aggregate, K units of non-augmentable, non-depreciatble capital. Members of this generation can, with their endowment, purchase this capital from the members of generation zero, at a price P_t per unit of capital. In general, members of generation (t), in aggregate, will purchase this capital from members of generation (t-1) at the price P_t. Each period this capital yields a dividend of $d(z_t)$, per unit of capital, in units of the consumption good. The function $d(\cdot)$ is also assumed to be continuous.

At each date (t) the members of generation (t) purchase the K units of capital from the members of generation (t-1). Trade will take place after the dividends have been paid. That is, members of generation (t-1) receive the dividend as well as the proceeds from the sale of the capital.

Henceforth, the following assumptions are made.

(i) The utility functions satisfy the following conditions:

$$\lim_{x\to 0} U_b'(x) = \lim_{x\to 0} U_\ell'(x) = \infty$$

$$\lim_{x\to 0} V_b'(x) = \lim_{x\to 0} V_\ell'(x) = \infty.$$

As well, define

$$\gamma \equiv \max_{j=\ell,b} \{ \sup_{x \in R_{++}} |\frac{V_j''(x)}{V_j'(x)}| \} .$$[3]

(ii) Assume $\underline{d} \equiv \inf_{z \in Z} (d(z)) > 0$

and $[\frac{2}{1+\underline{d}} + \gamma] < 1$. This is a restriction on second period risk aversion.

(iii) The transition density function $f(z'|z)$ is such that the integral

$\int \Psi(z',z) f(z'|z) dz'$

exists and is a continuous function of z if $\Psi: Z \times Z \to \mathbb{R}$ is continuous.

(iv) For convenience, it is assumed that $K = N$.

III. EXISTENCE OF AN EQUILIBRIUM

Define $R_t(z_t, z_{t+1})^{-1}$ to be the value, in time (t) consumption units, of a unit of the consumption good at time (t+1), when the state at (t+1) is z_{t+1}, and the state at time t is z_t. Then the problem to be solved by the N lenders who are members of generation (t) is to

$$\underset{c^{\ell}_{1t}, c^{\ell}_{2t}}{\text{maximize}} \{U_{\ell}(c^{\ell}_{1t}) + E_t V_{\ell}(c^{\ell}_{2t})\} \qquad c^{\ell}_{1t}, c^{\ell}_{2t} \geqq 0$$

subject to

$$c^{\ell}_{1t} \leqq y^{\ell}_1(z_t) - q^{\ell}(z_t) \tag{1}$$

$$c^{\ell}_{2t} \leqq y^{\ell}_2(z_t, z_{t+1}) + R_t(z_t, z_{t+1}) q^{\ell}(z_t) \tag{2}$$

Similarly, the problem to be solved by the $M^b_t(z_t)$ borrowers who are members of generation (t) is to

$$\underset{c^{b}_{1t}, c^{b}_{2t}}{\text{maximize}} \{U_b(c^b_{1t}) + E_t V_b(c^b_{2t})\} \qquad c^b_{1t}, c^b_{2t} \geqq 0$$

subject to

$$c_{1t}^{b} \leqq y_1^b(z_t) + q^b(z_t) \tag{3}$$

$$c_{2t}^{b} \leqq y_2^b(z_t,z_{t+1}) + R_t(z_t,z_{t+1})q^b(z_t) \tag{4}$$

Here $q^{\ell}(z_t)$ represents the net lending by a member of generation (t), in state z_t, who is endowed with $y_1^{\ell}(z_t)$ in the first period of his or her life. Similarly, $q^b(z_t)$ represents net lending (which may be negative) by a member of generation (t), in state z_t, who is endowed with $y_1^b(z_t)$ units of the consumption good in the first period.[4] Of course, the solution of these optimization problems implies

$$U_{\ell}'(c_{1t}^{\ell}) = E_t\{R_t(z_t,z_{t+1})V_{\ell}'(c_{2t}^{\ell})\} \tag{5}$$

$$U_b'(c_{1t}^{b}) = E_t\{R_t(z_t,z_{t+1})V_b'(c_{2t}^{b})\} \tag{6}$$

Equilibrium in the capital market implies that the aggregate saving of members of generation (t) must equal their purchases of capital from generation (t-1). That is,

$$Nq^{\ell}(z_t) + M^b(z_t)q^b(z_t) = P_tK. \tag{7}$$

Furthermore, members of generation (t-1) receive the dividend yielded by the capital as well as the value of the capital sold to members of generation (t). Hence, the following equilibrium condition must hold

$$R(z_{t-1},z_t)[Nq^{\ell}(z_{t-1}) + M^b(z_{t-1})q^b(z_{t-1})] = [P_t + d(z_t)]K. \tag{8}$$

Equations (7) and (8) imply the simultaneous clearing of the markets for both private loans and capital. For example, if $q^b(z_t) < 0$, then the saving of the type "ℓ" agents must be sufficient to make the desired quantity of loans to type "b" agents <u>and</u> buy the capital from members of generation (t-1). Consequently, the left side of equation (7) represents the demand for capital. The right side of equation (7) then represents the supply of capital. The quantity of capital supplied (K) is independent of the price because members of generation t-1 supply this quantity inelastically. This is a consequence of the assumption that agents live for only two periods. Similarly, equation (8) states that second period consumption of a given generation equals the proceeds of the sale of all capital, ex dividend, Clearly, agents who are debtors in the first period of their life must repay these debts in the second period.

As in the existing literature on asset-pricing, this study will be restricted to the examination of stationary equilibria. Since the variable (z_t) summarizes all information concerning the state relevant variables at time (t), such as population composition and capital returns, the analysis will seek to characterize the price of capital by a pricing function

$$P_t = P^*(z_t),$$

which is a time-autonomous function of the state variable (z_t). It can now be stated, in a precise manner, exactly what is meant by a competitive equilibrium for such an economy.

<u>Definition</u>: A competitive equilibrium is a continuous function $R^*: Z \times Z \to (0,\infty)$ and a continuous function $P^*: Z \to (0,1)$ such that:

(i) There exist functions $q^{\ell*}$, $q^{b*}: Z \to (0,1)$ such that equations (1) through (6) hold identically when $R_t(\cdot,\cdot) \equiv R^*(\cdot,\cdot)$ and $q^j(\cdot) \equiv q^{j*}(\ )$ $j = b,\ell$.

(ii) Equations (7) and (8) hold identically when $R_t(\cdot,\cdot) \equiv R^*(\cdot,\cdot)$, $q^j(\cdot) \equiv q^{j*}(\cdot)$, $j = b,\ell$, and $P_t \equiv P^*(z_t)$.

The characterization of a competitive equilibrium for the present economy is non-trivial because of the difficulty in characterizing the functions R^* and P^*. However, there exists an alternative technique which, while being somewhat illuminating itself, will yield a proof of the existence of a competitive equilibrium.

Before proceeding, it should be clear that members of a given generation can engage in trade or risk sharing in order to maximize expected utility. Naturally, it is assumed that debts incurred by any agent in the first period of his or her life will be repaid in the following period. Hence, the competitive equilibrium is one in which there are no ex ante redistributions among members of a given generation which would yield higher expected utility without lowering the expected utility level of some other agent. Therefore, the competitive equilibrium results in an optimum weighted average of expected utilities of agents who are members of the same generation.

With this in mind, consider the following artificial planning problem. A planner may, at each time (t), seek to maximize a weighted average of the expected utilities of agents born at that date, subject to certain resource constraints. In the context of the model described above, the resource constraints take the form of feasibility constraints on consumption allocations which are determined, in part, by the total endowments and yields on capital. The planner will be restricted to solving the following problem. The planner, at each date t, must maximize a weighted average of the expected utilities of agents born at time (t) subject to the constraint that members of generation (t) must purchase the K units of productive capital from the agents who are members of generation (t-1). This will then permit the determination of the price of capital as it is sold from one generation to another. Specifically, the planner's problem at time (t) is to

$$\text{maximize } E_t\{\omega^{\ell}(z_t)[U_{\ell}(c_{1t}^{\ell})+V_{\ell}(c_{2t}^{\ell})]+\omega^{b}(z_t)[U_b(c_{1t}^{b})+V_b(c_{2t}^{b})]\} \quad (9)$$

subject to the resource constraints

$$N\, c_{1t}^{\ell} + M^b(z_t)c_{1t}^{b} \leqq N - P_tK \quad (10)$$

$$N\, c_{2t}^{\ell} + M^b(z_t)c_{2t}^{b} \leqq M^b(z_t) + [P_{t+1} + d(z_{t+1})]K \quad (11)$$

where $\omega^{\ell}(\ )$ and $\omega^{b}(\ )$ are relative welfare weights.

These weights may be restricted such that $\omega^{\ell}(z_t) \epsilon (0,1)$ and $\omega^{\ell}(z_t) + \omega^{b}(z_t) = 1$. The weights are further written as continuous functions of the state variable z_t because there appears to be no criteria for choosing a single set of time invariant weights.

The formulation in equations (9) through (11) also treats equally, all members of a single generation who have identical endowments in the same period. In other words, all identical individuals of the same generation are treated equally. Then, keeping in mind that $N = K$, the resource constraints can be rewritten in the form

$$c_{1t}^{\ell} + \theta(z_t)c_{1t}^{b} \leq 1 - P_t x$$

$$c_{2t}^{\ell} + \theta(z_t)c_{2t}^{b} \leq \theta(z_t) + [P_{t+1} + d(z_{t+1})]x,$$

with the restriction that $x \epsilon [0,1]$, with $x = 1$ in equilibrium.[5] As will be shown this formulation will be beneficial because it will permit the straightforward characterization of the price of capital as a function of the state vector z_t. In fact, the planner will be restricted to choosing allocations in which this form for the pricing function results. As a result, should there occur at any two distinct dates, the same realization of the state variable z_t, the young agents alive at these dates will solve the same optimization problem, and capital will have the same value. As well, this technique is useful because, as will be shown, the equilibrium price of capital will then reflect the quantity of the consumption good that a member of generation (t) would be willing to pay for an extra unit of capital.

It will also be beneficial to write the consumption allocations of members of generation t as functions of the state variables which affect these agents. Hence the planner's problem may be rewritten again as

$$\text{maximize} \int_Z \{\omega^{\ell}(z_t)[U_{\ell}(c_1^{\ell}(z_t)) + V_{\ell}(c_2^{\ell}(z_t,z_{t+1}))] + \omega^b(z_t)[V_b(c_1^b(z_t)) + V_b(c_2^b(z_t,z_{t+1}))]f(z_{t+1}|z_t)dz_{t+1} \tag{12}$$

subject to the constraints

$$c_1^{\ell}(z_t) + \theta(z_t)c_1^b(z_t) \leqq 1 - P_t x \tag{13}$$

$$c_2^{\ell}(z_t,z_{t+1}) + \theta(z_t)c_2^b(z_t,z_{t+1}) \leqq \theta(z_t) + [P_{t+1} + d(z_{t+1})]x \tag{14}$$

$\forall\, z_{t+1} \in Z$

Again, x is restricted to be $\in [0,1]$ with $x=1$ in equilibrium. The interior solution to the planner's problem then takes the following form.

$$c_1^{\ell}(z_t):\quad \omega^{\ell}(z_t)U_{\ell}'(c_1^{\ell}(z_t)) - \lambda(z_t) = 0 \tag{15}$$

$$c_1^b(z_t):\quad \omega_t^b(z_t)U_b'(c_1^b(z_t)) - \lambda(z_t)\theta(z_t) = 0 \tag{16}$$

$$c_2^{\ell}(z_t,z_{t+1}):\quad \omega_t^{\ell}(z_t)V_{\ell}'(c_2^{\ell}(z_t,z_{t+1}))f(z_{t+1}|z_t) - \mu(z_t,z_{t+1}) = 0\,,\ \forall z_{t+1} \in Z \tag{17}$$

$$c_2^{\ell}(z_t,z_{t+1}):\quad \omega_t^b(z_t)V_b'(c_2^b(z_t,z_{t+1}))f(z_{t+1},z_t) - \mu(z_t,z_{t+1})\theta(z_t) = 0, \forall z_{t+1} \in Z \tag{18}$$

$$x:\quad -P_t\,\lambda(z_t) + [P_{t+1} + d(z_{t+1})]\mu(z_t,z_{t+1}) = 0, \forall z_{t+1} \in Z \tag{19}$$

$$\lambda(z_t):\quad c_1^{\ell}(z_t) + \theta(z_t)c_1^b(z_t) = 1 - P_t \tag{20}$$

$$\mu(z_t,z_{t+1}):\quad c_2^{\ell}(z_t,z_{t+1}) + \theta(z_t)c_2^b(z_t,z_{t+1}) = \theta(z_t) + [P_{t+1} + d(z_{t+1})], \forall z_{t+1} \in Z \tag{21}$$

Here $\lambda(\tilde{z})$ is the single multiplier associated with equation (13) when the state vector at time (t) is $z_t = \tilde{z}$. $\mu(\tilde{z},z')$ is the multiplier associated with equation (14) when the state at time (t) is $z_t = \tilde{z}$ and the realized state at time (t+1) is $z_{t+1} = z'$.

It will be useful to combine these equations in the following form. Combining equations (15), (17) and (19) and integrating with respect to (z_{t+1}) yields

$$P_t U_\ell'(c_1^\ell(z_t)) = \int_Z V'(c_2^\ell(z_t, z_{t+1}))(P_{t+1} + d(z_{t+1}))f(z_{t+1}|z_t)dz_{t+1}. \tag{22}$$

Similarly, equations (16), (18) and (19) yield

$$P_t U_b'(c_1^b(z_t)) = \int_Z V'(c_2^b(z_t, z_{t+1}))(P_{t+1} + d(z_{t+1}))f(z_{t+1}|z_t)dz_{t+1}. \tag{23}$$

Equations (15) and (16) together yield

$$\omega^\ell(z_t)\theta(z_t)U_\ell'(c_1^\ell(z_t)) = \omega^b(z_t)U_b'(c_1^b(z_t)) \tag{24}$$

Multiplication of equations (17) and (18) by $(P_{t+1} + d(z_{t+1}))$ and integrating yields

$$\omega^\ell(z_t)\theta(z_t)\int_Z V_\ell'(c_2^\ell(z_t, z_{t+1}))(P_{t+1} + d(z_{t+1}))f(z_{t+1}|z_t)dz_{t+1}$$

$$= \omega^b(z_t)\int_Z V_b'(c_2^b(z_t, z_{t+1}))(P_{t+1} + d(z_{t+1}))f(z_{t+1}|z_t)dz_{t+1}. \tag{25}$$

Equations (22) and (23) are the familiar intertemporal optimization conditions for equilibrium in the asset market. These continue to hold despite the presence of a heterogeneous population. Equations (24) and (25) are the result of the optimization problem solved by the social planner. That is, these are conditions which determine optimal intragenerational allocations. And again, equations (20) and (21) are the intertemporal feasibility constraints. Equations (20)-(25) completely characterize an optimal equilibrium set of allocations which solve an artificial planning problem. In Appendix A it is shown that there exists a stationary equilibrium for this economy, in which there exists a function $P^*: Z \to (0,1)$ such that equations (20)-(25) hold with $P_t \equiv P^*(z_t)$. It will also be straightforward to verify that

$$\omega^\ell(z_t)\theta(z_t)V_\ell'(c_2^\ell(z_t, z_{t+1})) = \omega^b(z_t)V_b'(c_2^b(z_t, z_{t+1})) \tag{26}$$

for all z_t, $z_{t+1} \in Z$, which is the exact equation gained from combining equations (17) and (18).

In Appendix B, the second theorem of welfare economics is proven for this economy. That is, it is shown that for each of the sets of allocations derived from the previous problem, there is a competitive equilibrium which yields the identical allocations. Therefore, for each of the continuous functions $\omega^{\ell}(\cdot)$ used to characterize the Pareto optimum, there is a different competitive equilibrium with an appropriate redistribution of the endowments. Hence, this analysis describes a wide range of competitive equilibria.

Assumption (iii), which is used to show the existence of the pricing function, is similar to a condition used in Huberman [9], in an overlapping generations model with many capital assets. This may lead one to conjecture that the model in this paper could also be generalized to include many capital assets as well.

The present model is of interest because it emphasizes the rather complicated way in which future dividends are discounted, and hence capital assets are priced, in a model of heterogeneous agents. It is of interest to compare equations (20)-(25) with the comparatively simple formulations in an infinitely-lived representative agent model (Lucas [18]) or in a model of a sequence of identical overlapping generations (Huberman [9]). In these latter models, all agents who are faced with a non-trivial decision, are faced with the <u>same</u> optimization problem. Hence the price of the asset is determined as the amount such an agent would pay for an asset which yields a dividend according to some probability distribution. The price of the capital asset is formulated according to intertemporal considerations only. However, in the present model, these considerations are present as well, as is shown by equations (24) and (25). But there are also other factors, which may be termed intratemporal constraints. Equations (20) and

(21) are not budget constraints for a representative consumer. They are feasibility constraints for a generation of individuals who are not identical in terms of income profiles, or, possibly, utility functions. Equations (24) and (25) are not familiar when considering conventional capital asset pricing. These equations guarantee the endowments and capital returns are distributed in such a way that there is no incentive for agents who are members of the same generation to engage in further asset exchanges.

It is of interest to note that the equilibrium derived from the artificial planning problem, in which there is a stationary solution for the price of capital, is one in which the steady-state rate of return in the economy exceeds unity. However, the steady-state <u>growth</u> rate of the economy is equal to unity because there is no growth. It is well known in the study of overlapping generations economies that the competitive equilibrium of such an economy is Pareto optimal only if the stationary steady-state rate of return is not less than the economy's growth rate. It is therefore conjectured (though no proof is offered) that the solution to the artificial planning problem yields a competitive equilibrium which is Pareto optimal.

Hence, there are at least three distinguishing characteristics of the asset pricing formulas developed in this paper, as compared with existing models. First, this model illustrates the interconnectedness of the competitive equilibria and the Pareto optimal allocations. Secondly, the present model guarantees that not only is the intertemporal capital market in equilibrium (as in Lucas and Huberman) but also the intra-generational loan market is in equilibrium as well. More will be said about this in the next section. Thirdly, it is not necessary that all members of a single generation have identical utility functions.

IV. INSIDE AND OUTSIDE ASSETS

A novel feature of this environment is the existence of agents who are members of the same generation whose endowment patterns are different. The setup described in Section II is illustrative of a more general framework in which agents who are members of the same generation have different income streams, and possibly no agent has an endowment stream which coincides with the agent's desired consumption path. Hence, there is a reason for private borrowing and lending in order to "smooth" consumption patterns and achieve a higher level of expected utility. These arrangements also have an effect upon the equilibrium price of capital. Hence this economy may be thought of as having both inside assets (private loans) and outside assets (real capital).

Consider an environment in which

$$y_1^{\ell}(z_t) = y_2^{b}(z_t, z_{t+1}) \equiv 1 \tag{27}$$

and

$$y_1^{b}(z_t) = y_2(z_t, z_{t+1}) \equiv 0 \tag{28}$$

for all $z_t, z_{t+1} \varepsilon Z$. Further, let $[c_1^{\ell}(z_t)^*, c_2^{\ell}(z_t, z_{t+1})^*, c_1^{b}(z_t) c_1^{b}(z_t), c_2^{b}(z_t, z_{t+1})^*]$ denote the equilibrium consumption allocations for members of generation (t) which result from the solution to the competitive equilibrium (equations 1 - 8). The competitive equilibrium for such an economy could be interpreted as one in which, at time (t), the lenders each purchase one unit of capital at a price $\tilde{P}(z_t)$. In addition, they each loan $[1 - \tilde{P}(z_t) - c_1^{\ell}(z_t)^*]$ units of the consumption good to the borrowers of the same generation. These borrowers each consume $[1 - \tilde{P}(z_t) - c_1^{\ell}(z_t)^*]/\theta(z_t) = c_1^{b}(z_t)$ units of the consumption good. In the second period of this generation's life, the borrowers repay the loan to the lenders in the amount $\theta(z_t)[1 - c_2^{b}(z_t, z_{t+1})^*]$ $= [c_2^{\ell}(z_t, z_{t+1})^* - \tilde{P}(z_{t+1}) - d(z_t)]$. The ex post rate of return on such loans is then

$$\frac{\theta(z_t)[1 - c_2^b(z_t, z_{t+1})^*]}{[1 - \tilde{P}(z_t) - c^\ell(z_t)^*]} .$$

(The ex ante rate of return on these loans is derived straight from equations (22) and (23).) The borrowers then consume what is left of their endowments and the lenders consume the proceeds from the private loans plus the dividend yielded by the security $d(z_{t+1})$, and the proceeds from the sale of the asset $P(z_{t+1})$. It should be noted that because there are complete markets, the repayment of the private loan is state-contingent. Seen in this light, it is easy to see that the quantity of private loans will affect the price of capital. This is illustrated with the following example.

Example 1: Let the utility functions be

$$U(c^j_{1,t}) + V(c^j_{2,t}) = \ln(c^j_{1,t}) + \ln(c^j_{2,t}) , \qquad j=\ell,b.$$

Further, suppose $d_t = 1\ \forall t$ and the only uncertainty is introduced through the variable θ, where

$$\theta = \begin{cases} 1 & \text{with probability } 1/2 \\ 2 & \text{with probability } 1/2 \end{cases}$$

There then exists an equilibrium in which the state variables take on values as shown in Table 1.

TABLE 1

State	$\theta = 1$	$\theta = 2$
$P(z_t)$	.363	.285
$q^\ell(z_t)$	0.5	0.5
$q^b(z_t)$	-.137	-.108

The lenders save the same amount in each state. The price of capital is highest when there are few borrowers. An increase in the amount of private borrowing depresses the price of capital and hence raises the economy's interest rate. Hence private borrowing and lending can affect the price of real assets.

The next example considers an identical economy in which there exists a (legal?) restriction on private borrowing.

Example 2: The economy is exactly the same as in Example 1 except that the restriction is imposed that $-.05 \leq q^b \leq 0$ in all states. In this case

$$P(z_t) = \begin{cases} 0.45 & \text{when } \theta = 1 \\ 0.40 & \text{when } \theta = 2 \end{cases}$$

This borrowing restriction raises the price of capital (and lowers the economy's interest rate) because lenders must seek other assets, other than private loans, in which to hold their wealth.

The formal model developed in Sections II and III did not require that borrowers and lenders have identical preferences. However, a restriction on absolute risk aversion was sufficient to guarantee the existence of an equilibrium pricing function. It is then of some interest to study economies in which cohorts do differ with respect to endowments and preference structures.

Consider the economy developed in Sections II and III in which the endowments are given by equations (27) and (28), and the preferences of borrowers are linear:

$$U_b(c_1^b(z_t)) + V_b(c_2^b(z_t, z_{t+1})) = c_1^b(z_t) + \rho\, c_2^b(z_t, z_{t+1})$$

where $\rho \in (0,1)$. Equations (22) to (24), with a version of equation (26) now become

$$P_t U'_\ell(c_1^\ell(z_t)) = \int_z V'(c_2^\ell(z_t, z_{t+1}))(P_{t+1} + d(z_{t+1}))f(z_{t+1}|z_t)dz_{t+1} \tag{29}$$

$$P_t = E_t\{\sum_{i=1}^{\infty} \rho^i d(z_{t+i})\} \tag{30}$$

$$\omega^\ell(z_t)\theta(z_t)U'_\ell(c_1^\ell(z_t)) = \omega^b(z_t) \tag{31}$$

$$\omega^\ell(z_t)\theta(z_t)V'_\ell(c_2^\ell(z_t, z_{t+1})) = \omega^b(z_t)\rho \tag{32}$$

Equation (18) is merely the conventional linear asset pricing formula with a constant discount rate. This is simply the obvious notion that when there exists even one agent with linear utility in the economy at every date, the price of capital will be governed by this formula. In addition, risk-neutral agents will then be willing to accept all risk. This is clear from equation (32) which says that second-period consumption of the risk-averse agent will be independent of the realization of z_{t+1}. That is, these agents bear no consumption risk. In this case, the loans paid from borrowers to lenders have no state-contingent component. Only second-period consumption of borrowers is state-contingent.

Presumably, in an environment in which agents exhibit differing degrees of risk aversion, but no person was risk neutral, there would be risk-sharing agreements among agents. Asset prices would then reflect these differing degrees of risk aversion which were present in the economy. If one generation's attitude toward risk was different from that of another generation, the prices of assets should behave differently over time. This could have been illustrated in the environment of Sections II and III by making some agent's risk aversion dependent upon the state vector (z_t). As long as risk aversion does not change too much, the equilibrium pricing function, derived in Appendix A, would then reflect this fact.

The following is another example in which agents do not have identical preferences.

Example 3: The economy is exactly the same as in Example 1 except that now

$$U(c^b_{1,t}) + V(c^b_{2,t}) = \ln(c^b_{1,t}) + c^b_{2,t}.$$

The resulting equilibrium is illustrated in Table 2.

TABLE 2

State	$\theta = 1$	$\theta = 2$
$P(z_t)$	.274	.156
$q^{\ell}(z_t)$	0.5	0.5
$q^b(z_t)$	-.226	-.172

In this case, when borrowers are risk-neutral with respect to second period consumption, they borrow more and hence the price of capital is lower than in Example 1.

In all the above examples, the price of capital exhibits variability even when its dividends are constant. The intragenerational loans exist because agents who are members of the same generation have different endowment profiles. Agents who are cohorts can arrange private loans, provided repayment can be guaranteed, with other agents who will be present at a future date to repay the loan. In the existing model, intergenerational private loans are not feasible because agents who are members of different generations do not meet in more than one time period. In a more general model, in which agents live for n periods, where $n > 2$, there could conceivably be both intergenerational and intragenerational loans. Such a model would have different implications for the pricing of a capital asset.

A common strategy in the study of asset pricing is to compare the behavior of a specific model when a structure for certain aggregate variables is imposed upon the model. For example, it may be convenient to utilize an infinitely-lived representative agent economy, in which a capital asset is priced. Further, one could impose upon the model the observed behavior of consumption for a particular actual economy. This is done in Grossman and Shiller [6] and Prescott and Mehra [20]. However, the analysis of this paper indicates that the distribution of consumption among different types of agents will be critical for determining factors influencing the price of capital, as well as interest rates.[6] In fact, it is relatively easy to construct a generational economy in which aggregate consumption is constant but the price of capital exhibits variability. Hence, it may be that deeper insights into the determinants of these prices, in an economy populated by heterogeneous agents, can only be gained by also considering the budget constraints faced by the different agents.

V. CONCLUSION

An economy has been constructed in which finite-lived agents of the same generation have possibly different endowment profiles and different utility functions. The agents can engage in intragenerational borrowing and lending which, in turn, affects the equilibrium price of capital in the economy.

To arrive at an equilibrium, a social planner's problem was solved and interpreted as a competitive equilibrium. It was also shown that for each such set of Pareto optimal allocations, within a certain class, there exists a competitive equilibria which gives rise to the allocations. The appendices make these ideas precise and may be of some technical interest.

It was shown, by way of example, that private borrowing, and any factors which influence the amount of private borrowing, can also exert some influence on the price of capital. This is in contrast to the work done by Scheinkman and Weiss who showed that restrictions on borrowing and lending can potentially influence the behavior of the price of capital. It remains to be seen as to whether a significant amount of the variability in asset prices can be attributed to the variability in private borrowing.

An obvious extension to this work would be to characterize the behavior of asset prices in an environment populated by agents with arbitrarily long lifetimes. However, recent work by Aiyagari [1] suggests that such a task would meet with some difficulty. The reason is that the state variable which describes all real variables in a given period may no longer be sufficient to characterize the price of capital when agents live for more than two periods. Hence, some discretion must be used in constructing the "appropriate" state vector for the economy.

Footnotes

[1]The notion that borrowing against future income would have implications for financial markets, was also mentioned by Brock.

[2]This assumption is innocuous. An alternative assumption would be that the aggregate endowments of all generations should be bounded with probability one. The existence proof in the appendix would then hold with a slight modification to assumptions (i) and (ii) below. The algebra in the appendix then becomes tedious. The restriction on endowments was imposed merely to simplify the calculations.

[3]For the purposes of assumption (ii), this definition is too restrictive. A slightly less restrictive condition could be obtained if $M^b(z_t)$ and $d(z_t)$ are bounded.

[4]There is no possibility of agents being asked to repay debts which are greater than their endowments. Since there are complete markets, agents in this economy are insured against this event.

[5]This is to guarantee that the maximization takes place over a compact set. The quantity of capital (x) is a choice variable because the planner must choose the "optimal" quantity of capital to purchase, in order to maximize the objective function.

[6]Prescott and Mehra mention this briefly.

REFERENCES

[1] Aiyagari, S. R., "Consumption and Interest Rates in a Stationary Overlapping Generations Model With 'Long' Lived Agents," University of Wisconsin-Madison, Manuscript (1984).

[2] Bewley, T., "Thoughts on Tests of the Intertemporal Asset Pricing Model," Northwestern University, unpublished (1982).

[3] Brock, William A., "Asset Prices in a Production Economy" in The Economics of Information and Uncertainty, ed. J. J. McCall. Chicago: University of Chicago Press (1982).

[4] Fama, Eugene F., "Efficient Capital Markets: A Review of Theory and Empirical Work," Journal of Finance 25 (May 1970): 387-417.

[5] Flavin, Marjorie A., "Excess Volatility in Financial Markets: A Reassessment of the Empirical Evidence," Journal of Political Economy 91 (December 1983): 929-956.

[6] Grossman, Sanford F., and Shiller, Robert J., "The Determinants of the Variability of Stock Market Prices," American Economic Review 71 (May 1981): 222-227.

[7] Hansen, L. P. and Singleton, K. J., "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models". Econometrics 50 (September 1982): 1269-1286.

[8] Hansen, L. P. and Singleton, K. J., "Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns," Journal of Political Economy 91 (April 1983): 249-265.

[9] Huberman, G., "Capital Asset Pricing in an Overlapping Generations Model," Journal of Economic Theory

[10] Huffman, G. W., "Adjustment Costs and Capital Asset Pricing," Journal of Finance (forthcoming).

[11] Huffman, G. W., "Asset Pricing with Capital Accumulation," University of Western Ontario Working Paper (March 1981).

[12] Huffman, G. W., "The Representative Agent, Overlapping Generations, and Asset Pricing," University of Western Ontario Working Paper.

[13] Kleidon, A. W., "Bias in Small Sample Tests of Stock Price Rationality," University of Chicago Graduate School of Business Working Paper, revised (May 1982).

[14] LeRoy, Stephen F., "Risk Aversion and the Martingale Property of Stock Prices," International Economic Review 14 (July 1973): 436-446.

[15] LeRoy, Stephen F., "Expectations Modelsof Asset Prices: A Survey of Theory," Journal of Finance 37 (March 1982): 185-217.

[16] LeRoy, Stephen F., and LaCivita, C. J., "Risk Aversion and the Dispersion of Asset Prices," Journal of Business 92 (October 1981): 535-547.

[17] LeRoy, Stephen F., and Porter, Richard O., "The Present-Value Relation: Tests on Implied Variance Bounds," Econometrica 49 (May 1981): 555-574.

[18] Lucas, Robert E., Jr., "Asset Prices in an Exchange Economy," Econometrica 46 (November 1978): 1429-1445.

[19] Merton, R. C., "An Intertemporal Capital Asset Pricing Model," Econometrica 41 (September 1973): 867-887.

[20] Prescott, Edward D., and Mehra, Rajnish, "A Test of the Intertemporal Asset Pricing Model," Federal Reserve Bank of Minneapolis, Staff Report 81 (1982).

[21] Scheinkman, J. A. and Weiss, L., "Borrowing Constraints and Aggregate Economic Activity," University of Chicago (1983).

[22] Shiller, R. J., "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividents?" American Economic Review 71 (June 1981): 421-436.

[23] Townsend, R., "Asset Return Anomalies: A Choice Theoretic, Monetary Explanation," Carnegie-Mellon University (revised: June 1982).

APPENDIX A

For convenience, equations (22)-(25) are rewritten in the following form, using equations (20) and (21)

$$P_t U'_\ell[\beta[z_t](1-P_t)] = \int_Z V'_\ell[\alpha[z_t,z_{t+1}](\theta(z_t)+P_{t+1}+d(z_{t+1}))](P_{t+1}+d(z_{t+1}))$$
$$f(z_{t+1}|z_t)dz_{t+1} \tag{A1}$$

$$P_t U'_b[(1-\beta[z_t])(1-P_t)] = \int_Z V'_b[(1-\alpha[z_t,z_{t+1}])(\theta(z_t)+P_{t+1}+d(z_{t+1}))]$$
$$(P_{t+1}+d(z_{t+1}))f(z_{t+1}|z_t)dz_t \tag{A2}$$

$$\omega^\ell(z_t)\theta(z_t)U'_\ell[\beta(z_t)(1-P_t)] = \omega^b(z_t)U'_b[(1-\beta(z_t))(1-P_t)] \tag{A3}$$

$$\omega^\ell(z_t)\theta(z_t) \int_Z V'_\ell[\alpha[z_t,z_{t+1}](\theta(z_t)+P_{t+1}+d(z_{t+1}))](P_{t+1}+d(z_{t+1})$$
$$f(z_{t+1}|z_t)dz_{t+1}$$
$$= \omega^b(z_t) \int_Z V'_b[(1-\alpha[z_t,z_{t+1}])(\theta(z_t)+P_{t+1}+d(z_{t+1})](P_{t+1}+d(z_{t+1}))$$
$$f(z_{t+1}|z_t)dz_t \tag{A4}$$

Equations (A1)-(A4) will hold for some $\beta[z_t]$, $\alpha_\ell[z_t,z_{t+1}] \in (0,1)$, as this is just a way of substituting equations (20) and (21) into equations (22) to (25). Notice that if equations (A1), (A2) and (A4) hold with $P_t \equiv \tilde{P}(z_t)$, for some function $\tilde{P}(\ )$, then equation (A3) automatically holds with equality, and hence is redundant.

Now define S to be the space of continuous functions such that $S = \{f|f:Z \to (-\infty,0]\}$ with norm

$$\|f\| = \sup_{z \in Z} |f(z)| .$$

It then becomes convenient to rewrite equations (A1), (A2), and (A3) in the following form, for $g \in S$,

$$e^{g(z_t)} U'_\ell[\beta[z_t](1-e^{g(z_t)})] = \int_Z V'_\ell[\alpha[z_t,z_{t+1}](\theta(z_t)\, e^{g(z_{t+1})} + d(z_{t+1}))]$$
$$(e^{g(z_{t+1})} + d(z_{t+1}))f(z_{t+1}|z_t)dz_{t+1} \quad \text{(A5)}$$

$$e^{g(z_t)} U'_\ell[(1-\beta[z_t])(1-e^{g(z_t)})] = \int_Z V'_\ell[(1-\alpha[z_t,z_{t+1}])(\theta(z_t) + e^{g(z_{t+1})} + d(z_{t+1}))]$$
$$(e^{g(z_{t+1})} + d(z_{t+1}))f(z_{t+1}|z_t)dz_{t+1} \quad \text{(A6)}$$

$$\omega^\ell(z_t)\theta(z_t)\int_Z V'_\ell[\alpha[z_t,z_{t+1}](\theta(z_t) + e^{g(z_{t+1})} + d(z_{t+1}))](e^{g(z_{t+1})} + d(z_{t+1}))$$
$$f(z_{t+1}|z_t)dz_{t+1}$$
$$= \omega^b(z_t)\int_Z V'_b[(1-\alpha[z_t,z_{t+1}])(\theta(z_t) + e^{g(z_{t+1})} + d(z_{t+1}))](e^{g(z_{t+1})} + d(z_{t+1}))$$
$$f(z_{t+1}|z_t)dz_{t+1} \quad \text{(A7)}$$

It should be clear that the function $\alpha[z_t,z_{t+1}]$ can always be chosen in such a way as to yield

$$\omega^\ell(z_t)\theta(z_t)V'_\ell[\alpha[z_t,z_{t+1}](\theta(z_t) + e^{g(z_{t+1})} + d(z_{t+1})] =$$
$$= \omega^b(z_t)V'_b[(1 - \alpha[z_t,z_{t+1}])(\theta(z_t) + e^{g(z_{t+1})} + d(z_{t+1})] \quad \text{(A8)}$$

$\forall\, z_t,z_{t+1} \in Z$. This will ensure that equations (17) and (18) will hold for all realizations.

Now, by the definition of S, and other elements in equations (A5)-(A8), the integrands in these equations are continuous. Since $f(\cdot|\cdot)$ is a density function, and by assumption (iii), there exists a $\bar{z} \in Z$ such that

$$\omega^\ell(z_t)\theta(z_t)V'_\ell[\alpha[z_t,\bar{z}](\theta(z_t) + e^{g(\bar{z})} + d(\bar{z})](e^{g(\bar{z})} + d(\bar{z}))$$
$$= \omega^b(z_t)V'_b[(1-\alpha[z_t,\bar{z}])(\theta(z_t) + e^{g(\bar{z})} + d(\bar{z})](e^{g(\bar{z})} + d(\bar{z}))$$
$$= \omega^b(z_t)\int_Z V'_b[(1-\alpha[z_t,z_{t+1}])(\theta(z_t)+e^{g(z_{t+1})} + d(z_{t+1}))](e^{g(z_{t+1})} + d(z_{t+1}))$$
$$f(z_{t+1}|z_t)dz_{t+1} \quad \text{(A9)}$$

Define the function $\eta(K_1,K_2)$ as the solution to the following two equations.

$$\eta\, U_\ell'[\beta(1-\eta)] = K_1 \tag{A10}$$

$$\eta\, U_b'[(1-\beta)(1-\eta)] = K_2 \tag{A11}$$

for some $\beta \in (0,1)$. The left side of (A10) is increasing in η and decreasing in β, while the left side of (A11) is increasing in both η and β. Hence, for given (K_1,K_2), there will exist unique values of η and $\beta \in (0,1)$ such that equations (A9) and (A11) hold with equality. Since $U_\ell(\cdot,\cdot)$ and $U_b(\cdot,\cdot)$ are twice continuously differentiable, $\eta: [0,\infty) \times [0,\infty) \to [0,1)$ is once differentiable. Differentiation of equations (A10) and (A11) reveal that the elasticities of $\eta(\cdot,\cdot)$ satisfy

$$0 \leq \left(\frac{\eta}{K_1}\right)\left(\frac{\partial\eta(K_1,K_2)}{\partial K_1}\right) \leq 1 \tag{A12}$$

$$0 \leq \left(\frac{\eta}{K_2}\right)\left(\frac{\partial\eta(K_1,K_2)}{\partial K_2}\right) \leq 1 \tag{A13}$$

Now equations (A7)-(A11) determine an operator T such that $T:S \to S$, which can be described in the following manner. For a given $g \in S$, equation (A7) determines a function $\alpha[\cdot,\cdot]$ such that this equality holds. There then exists a $\bar{z} \in Z$ such that (A9) holds. Let

$$K_1(z_t) = V_\ell'[\alpha(z_t,\bar{z})(\theta(z_t) + e^{g(\bar{z})} + d(\bar{z}))](e^{g(\bar{z})} + d(\bar{z})) \tag{A14}$$

and

$$K_2(z_t) = V_b'[(1-\alpha(z_t,\bar{z}))(\theta(z_t) + e^{g(\bar{z})} + d(\bar{z}))](e^{g(\bar{z})} + d(\bar{z})) \tag{A15}$$

as derived from equation (A9). Equations (A10) and (A11) then yield $\eta[K_1(z_t),K_2(z_t)]$.

Let

$$\tilde{g}(z_t) \equiv \ln\{\eta[K_1(z_t),K_2(z_t)]\} \tag{A16}$$

Clearly $\tilde{g}:Z \rightarrow (-\infty,0)$ and, as a continuous function of continuous functions, is continuous. Hence, equations (A7)-(A11) and (A14)-(A16) determine a mapping T from S into S. It will now be shown that this mapping has a fixed point. If there exists a $g^* \in S$ such that $Tg^* = g^*$, which satisfies (A7)-(A11), (A14)-(A16), then it also solves the system of equations (A1)-(A4) with

$$P_t = \tilde{P}(z_t) \equiv e^{g^*(z_t)}$$

It can now be established:

<u>Lemma</u>: $T:S \rightarrow S$ is a contraction mapping.

<u>Proof</u>: Noting that K_1 and K_2, from equations (A14) and (A15), are functions of $g(\cdot)$, we then get from the Mean Value Theorem then implies that, for $f, h \in S$

$$\|Tf(K_1(f),K_2(f) - Th(K_1(h),K_2(h))\|$$

$$\leq \|f-h\|\{\sum_{i=1}^{2} \| \frac{\partial Tf(K_1(f),K_2(f))}{\partial K_i} \cdot \frac{\partial K_i(f)}{\partial f} \|_{f=g}\}$$

for some function g in the convex hull of the two element set $\{f,h\}$.

Omitting, for convenience the arguments of $\alpha[\cdot,\cdot]$, $\beta[\cdot]$, $\theta(\cdot)$, and the utility functions, equation (A14) yields

$$\frac{dK_1(z_t)}{dg} = V'_\ell \cdot e^{g(\bar{z})} + V''_\ell \cdot (\alpha e^{g(\bar{z})})(e^{g(\bar{z})} + d(\bar{z}))$$
$$+ V''_\ell \cdot (e^{g(\bar{z})} + d(\bar{z}))(\theta + e^{g(\bar{z})} + d(\bar{z})) \frac{d\alpha}{dg} .$$

Differentiation of equation (A9) yields

$$\frac{d\alpha}{dg} = \frac{-[\theta\omega_\ell V''_\ell + \omega_b V''_b] \cdot (\alpha e^{g(\bar{z})})(e^{g(\bar{z})} + d(\bar{z})) + \omega_b V''_b \cdot (e^{g(\bar{z})} + d(\bar{z}))e^{g(\bar{z})}}{[\theta\omega_\ell V''_\ell + \omega_b V''_b](e^{g(\bar{z})} + d(\bar{z}))(\theta + e^{g(\bar{z})} + d(\bar{z}))} .$$

Hence, these latter two equations imply

$$\frac{dK_1(z_t)}{dg} = V'_\ell \cdot e^{g(\bar{z})} + \frac{V''_\ell \cdot \omega_b V''_b \cdot (e^{g(\bar{z})} + d(\bar{z}))e^{g(\bar{z})}}{[\theta\omega_\ell V''_\ell + \omega_b V''_b]} . \quad \text{(A17)}$$

A similar operation yields

$$\frac{dK_2(z_t)}{dg} = V_b' \; e^{g(\bar{z})} + \frac{V_b'' \cdot \theta\omega_\ell V_\ell'' \cdot (e^{g(\bar{z})} + d(\bar{z}))e^{g(\bar{z})}}{[\theta\omega_\ell V_\ell'' + \omega_b V_b'']} \; . \tag{A18}$$

Writing equations (A14) and (A15) as functions of (g), with the aid of equation (A16) yields

$$\tilde{g}(z_t) = \ell n\{\eta[K_1(g(z_t)), K_2(g(z_t))]\} \; ,$$

and hence

$$\frac{\partial\tilde{g}(z_t)}{\partial g} = [(\frac{K_1}{\eta})(\frac{\partial\eta}{\partial K_1})][(\frac{1}{K_1}) \frac{\partial K_1}{\partial g}] + [(\frac{K_2}{\eta})(\frac{\partial\eta}{\partial K_2})][(\frac{1}{K_2}) \frac{\partial K_2}{\partial g}]$$

$$\leq (\frac{1}{K_1}) \frac{\partial K_1}{\partial g} + (\frac{1}{K_2}) \frac{\partial K_2}{\partial g}$$

by equations (A12) and (A13). Equations (A17) and (A18) then imply

$$\frac{\partial\tilde{g}(z_t)}{\partial g} \leq \frac{2e^{g(\bar{z})}}{e^{g(\bar{z})} + d(\bar{z})} + (e^{g(\bar{z})})\gamma$$

$$\leq \frac{2}{1+\underline{d}} + \gamma < 1$$

by assumption. Hence we have $\|Tg-Tf\| \leq (\frac{2}{1+\underline{d}} + \gamma)\|f-g\|$, and so the desired fixed point exists.

APPENDIX B

In this appendix it is shown that, for each set of allocations, with a pricing function derived in Appendix A, there exists at least one corresponding competitive equilibrium which gives use to these allocations and price of capital.

Define $c_1^j(z_t)^*$ as the strictly positive first-period consumption of an agent of type j (=ℓ,b) in state z_t. Also, let $c_2^j(z_t,z_{t+1})^*$ be the strictly positive second-period consumption of an agent of type j when state (t) was (z_t) and state (t+1) is (z_{t+1}). These are the allocations derived as solutions to the social planners problem. Let $P(z_t)^*$ be the corresponding pricing function as well. From Appendix A, it is clear that these functions are continuous functions of the state variables. What is to be shown is that there exists functions:

(i) y_1^ℓ: $Z \rightarrow [0,1]$

(ii) y_2^ℓ: $Z \times Z \rightarrow [0,1]$

(iii) y_1^b: $Z \rightarrow [0, \theta(\ )^{-1}]$

(iv) y_2^b: $Z \times Z \rightarrow [0, \theta(\ )^{-1}]$

such that the allocations $(c_1^\ell(z_t)^*, c_2^\ell(z_t,z_{t+1})^*, c_1^b(z_t)^*, c_2^b(z_t,z_{t+1})^*)$ are in fact those which result from some competitive equilibrium described by equations (1) through (8), for $j=\ell$,b, i=1,2.

Define

$$\tilde{R}(z_t,z_{t+1}) \equiv \left(\frac{P(z_{t+1})^* + d(z_{t+1})}{P(z_t)^*}\right)$$

Clearly $\tilde{R}(\cdot,\cdot)$ is continuous.

For arbitrary $(z_t,z_{t+1}) \in Z \times Z$ define the correspondence

$$Q(z_t,z_{t+1}) \equiv \{(\tilde{m},\tilde{n}) \in [0,\theta(z_t)] \times [0,1] \mid \tilde{m} = c_2^\ell(z_t,z_{t+1})^* - R(z_t,z_{t+1})(n - c_1^\ell(z_t)^*)\}$$

That Q is non-empty is clear from equations (13) and (14). Because the functions $R^*(\cdot,\cdot)$, $c_2^{\ell}(\cdot,\cdot)^*$ and $c_1^{\ell}(\cdot)^*$ are continuous, $Q(\cdot,\cdot)$ is then a continuous correspondence from $Z \times Z$ to $[0,\theta(\cdot)] \times [0,1]$. For arbitrary $z_t \in Z$, define

$$n'(z_t) \equiv \sup_{z' \in Z} \left\{\frac{c^{\ell}(z_t,z')}{R(z_t,z')}\right\} + c_1^{\ell}(z_t),$$

and

$$n''(z_t) \equiv \inf_{z'' \in Z} \left\{\frac{c^{\ell}(z_t,z'') - \theta(z_t)}{R(z_t,z'')}\right\} + c_1^{\ell}(z_t)$$

Clearly $n'(z_t) > 0$, $n''(z_t) \leq n'(z_t)$, and it can be shown that equations (13) and (14) imply $n''(z_t) < 1$. If $n^* \in [\max[0,n''(z_t)], \min[1,n'(z_t)]]$ then, for any $z_{t+1} \in Z$ there exists $m^* \in [0,\theta(z_t)]$ such that $(m^*,n^*) \in Q(z_t,z_{t+1})$. In other words, n^* does not depend upon z_{t+1}. Therefore, there exist continuous functions $m^*: Z \times Z \to [0,\theta(z_t)]$ and $n^*: Z \to [0,1]$ such that $(m^*(z_t,z_{t+1}), n^*(z_t)) \in Q(z_t,z_{t+1}) \ \forall\ z_t,z_{t+1} \in Z$.

Hence an agent of type "ℓ" who maximizes utility subject to constraints 1 and 2, with $y_1^b(z_t) \equiv n^*(z_t)$ and $y_2^b(z_t,z_{t+1}) \equiv m^*(z_t,z_{t+1}) \ \forall\ z_t \in Z$, and who faces rate of return $\tilde{R}(z_t,z_{t+1})$, will consume $c_1^{\ell}(z_t)^*$ in the first period and $c_2^{\ell}(z_t,z_{t+1})^*$ in the second period. This, together with equations (7), (8), (10) and (11) imply that equations (3) and (4) will also give the desired allocation for agents of type "b", when $y_1^b(z_t) \equiv \theta(z_t)^{-1} - n^*(z_t)$ and $y_2^b(z_t,z_{t+1}) \equiv \theta(z_t)^{-1} - m(z_t,z_{t+1}) \ \forall\ z_t,z_{t+1} \in Z$. That equations (5) and (6) are satisfied is clear from equations (A1) and (A2). Lastly, equations (7) and (8) clearly hold as they are just equations (10), (11) together with equations (1) through (4).

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