1970

Investment Planning for the Electric Power Industry: An Integer Programming Approach

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RESEARCH REPORT 7035

INVESTMENT PLANNING FOR THE ELECTRIC POWER INDUSTRY: AN INTEGER PROGRAMMING APPROACH

by

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A paper to be presented at the Econometric Society Meetings in Detroit, December, 1970

Thanks are due to my advisers, Charles R. Frank, Jr. and Larry E. Westphal, for their help with theoretical and computational problems, and also to a score of officials within the Government of India for their help with the data.

November, 1970

This paper is concerned with the question of investment planning for the electric power industry, specifically the decisions about what types of plant to build, how large to build them and when to build them. Spatial decisions such as plant location and investment in long distance transmission capacity are not considered; the area under consideration is therefore assumed to be relatively small.[1] For larger areas, the model can be easily extended to include these decisions; this, however, will not be discussed here.

The problem is formulated here in terms of integer programming. This is an obvious extension of the work done at Electricité de France,[2] where the problem was formulated in terms of linear programming. Most of their continuous variables appear in our model also; our modifications lie in the use of zero-one variables to represent construction decisions for a variety of hydro sites and also to represent the existence of economies of scale in construction costs for conventional thermal and nuclear plants. The algorithm used in the solution is the branch-and-bound algorithm of Davis, Kendrick, and Weitzman.[3]

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[1] The area considered here is the State of Madras (Tamilnadu) in Southern India. Its size is roughly one-fourth that of France. There are, of course, possibilities for interstate transmission but this model is based on the assumption of self-sufficiency, which appears to be the current attitude of the Madras State Electricity Board. A multi-area model, not discussed here, considers the problem of coordinated investment planning for the entire Southern Electricity Region, the four States of Madras, Andhra Pradesh, Mysore and Kerala.

[2] See P. Massé and R. Gibrat, "An Application of Linear Programming to Investments in the Electric Power Industry," Management Science, January 1957, pp. 149-166. The most recent complete linear programming model used by Electricité de France is L'Etude a Long Terme des Plans Investissements a l'Aide de la Programmatione Linéaire: "Modele des trois Plans, Note des Etudes Economiques Générales de l'Electricite de France, Paris, 1962.

[3] See R. E. Davis, D. A. Kendrick, and M. Weitzman, A Branch and Bound Algorithm for Zero-One Mixed Integer Programming Problems, Memorandum No. 69, Project for Quantitative Research in Economic Development, Harvard University, September 1967.

Background

The questions of what types of plant to build and the timing and scale of construction can be distinguished conceptually, but in the actual process of decision-making they are all interrelated.

The choice of technology involves at least three alternatives: conventional thermal (coal-burning) power plants, hydro power plants, and nuclear power plants. Conventional thermal plants have relatively low costs of construction, but they have relatively high costs of operation (fuel costs). Hydro projects on the other hand have relatively high construction costs but they have negligible operating costs. The third type, nuclear plants, are like hydro projects in that they have very high construction costs and relatively low operating costs.

A related problem concerns the size of plant to be built. This problem is complicated by the existence of economies of scale in construction costs for all technologies,[4] i.e., the per-unit-capacity cost of building a large plant is less than that of building a small plant. There are, of course, limits to the advantages of large scale plants, which are most obvious when surplus capacity appears.

Closely related to the plant size problem is the timing problem. On the one hand, it would seem economical to build a few very large plants, widely separated in time, in order to take advantage of the economies of scale; but on the other hand, this strategy would involve periodic overcapacity while demand caught up with the periodic large increases in capacity. The problem is thus to balance (in some sense) the economies of large scale plants with the diseconomies of temporary overcapacity.

Other factors making the investment planning problem more complicated

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[4]See S. Ling, Economies of Scale in the Steam Electric Power Generating Industry: An Analytic Approach, North Holland Publishing Co., Amsterdam, 1964. See also M. Galatin, Economies of Scale and Technological Change in Thermal Power Generation, North Holland Publishing Co., Amsterdam, 1968.

include several characteristics of the electric power industry that are not immediately obvious but are of special importance. First, the demand for electric power varies greatly depending on the time of day and season of the year, but since it is not storable the output of power must be varied according to the demand. Second, the supply capacity of hydro plants, being largely dependent on water flow, varies from season to season and from year to year. Finally, of great importance is the fact that the fuel costs of a given plant depend upon how much the plant is operated which, in turn, depends upon how the given plant fits into the system.

Since demand varies greatly from hour to hour (and in some systems from season to season) and since power cannot be stored (except perhaps in the form of water in hydro plant reservoirs) electric power systems must have sufficient capacity to meet peak demand, even though much of this capacity will lie idle during non-peak periods. Some capacity, therefore, will typically be utilized only during periods of peak demand; this contributes to the higher long-run marginal cost of meeting peak demand, compared with the long-run marginal cost of meeting non-peak demand. The importance of this can be lessened by the existence of hydro plants with reservoirs which can store water (electric power, in a sense) during non-peak hours for use during peak hours.[5]

The typical electric power system therefore has significant variations in demand over the span of a day, and some systems have significant variations in demand from season to season.[6] In addition to variations in demand

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[5]The existence of reservoirs, especially seasonal reservoirs, introduces problems of reservoir management analogous to those of optimal inventory policy. The problem is further complicated in the case of multiple purpose hydro projects; for example, when the project provides for irrigation as well as power, the target of cheap electric power might conflict with that of sufficient water for irrigation. These problems will however be generally ignored in this paper.

[6]The model discussed below has no seasonal variation in demand, although there is significant seasonal variation in (hydro) supply capacity.

from season to season, some electric power systems (with significant amounts of hydro power capacity) have variations in supply capacity, because of variations in rainfall and water flow, from season to season and from year to year. This variation in rainfall and water flow will have less of an influence on supply capacity the larger the available reservoir capacity since the reservoirs could act as buffers, absorbing large inflows of water in wet seasons and wet years and storing them until a time when natural water flows are greatly reduced. But even in systems with very large reservoir capacity, the natural variations in water flow will have an important impact on the supply capacity of hydro plants.

The fuel costs over the life of a given plant depend, of course, upon how much the plant is to be operated; this, however, cannot be determined without knowledge of what other plants are on the system and at least a rudimentary idea of how the system will be operated. Since a system with enough power capacity to meet peak demands will have idle capacity in off-peak periods and since the different plant types have different fuel costs and different possibilities for storage, the question of what plant types to operate for the various levels of demand is fairly complicated. An answer to this question, however, is necessary when calculating the costs of building and operating a given plant. Thus, exactly how an individual plant on a given system should be operated depends on the configuration of the system; that is, it depends on how costly it is to operate the plant relative to other plants on the system, how much capacity the system has and needs at various times, how much storage capacity the hydro reservoirs have, and so forth. Comparisons of different investment possibilities therefore must include both the different costs of building plant type A or plant type B and also the different costs of operating the system with plant type A or the system with plant type B.[7]

Preliminaries to the Model

The planning horizon of the model is 15 years; the model is divided into 5 three-year periods (t = 1,2,3,4,5). Investment decisions are made concerning which hydro projects to build and what amount of thermal and nuclear capacity should be constructed in each of the first four three-year periods.[8] Operating decisions (what plants should produce what outputs for the different load conditions) provide a rudimentary idea of how the system will operate and therefore what fuel costs will be; these decisions are made for each of the last four three-year periods.[9]

The costs incurred in each three-year period are the construction costs of investment decisions, the fixed costs of operation and maintenance, and the variable costs of operation. Costs in the different time periods are discounted to present value,[10] using a discount rate of 10% per annum.[11] Costs occurring beyond the 15 year horizon are also included in the model by the assumption that the maintenance and operating costs of the last three-year

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[7]For a discussion of this, see M. Boiteux and F. Bessiere, "Sur l'emploi des methodes globale et marginale dans le choix des investissements, Revue Francaise de Recherche Operationnelle, 5 (1961), No. 20.

[8]Investment decisions are not made for the last three-year period because capacity built in the period would not be ready in time for its benefits to be considered explicitly by the model.

[9]Operating decisions are not made for the first three-year period because this is a question concerning the operation of the existing system, and not a question related to investment planning.

[10]For purposes of discounting to present value, costs in a three-year period will be assumed to occur in the middle of the relevant time period. For example, all the costs occurring in the second three-year period are assumed to occur in year 5, and are discounted accordingly.

[11]Discount rates of 12½% per annum and 15% per annum will also be examined.

period (t=5) are repeated for the post-terminal years.[12] (This latter assumption is employed to avoid biasing the model, especially in the later periods, against hydro or nuclear investments which have higher initial costs but lower costs of operation and maintenance.)

The demand for electricity is assumed to grow only in very discrete jumps: there is no annual variation within each three year period, but the demand increases by 33% from one three-year period to the next.[13] Within each three-year time period the only variation assumed is the hourly variation between peak and non-peak times (there is no seasonal variation in demand). It is further assumed that this variation can be represented by a load curve with three distinct levels of demand (index d = 1,2,3):

1) the non-peak level (d=1), which has a duration of 22,500 hours in each three-year period (approximately 85% of the total number of hours in the three-year period);
2) the normal-peak level (d=2), which has a duration of 3,180 hours in each three-year period (approximately 13% of the total number of hours);
3) the super-peak level (d=3), which has a duration of 600 hours in each three-year period (approximately 2% of the total number of hours).

Non-peak demand is 54% of normal-peak demand and super-peak demand is 15% greater than normal-peak demand. The super-peak demand level serves the

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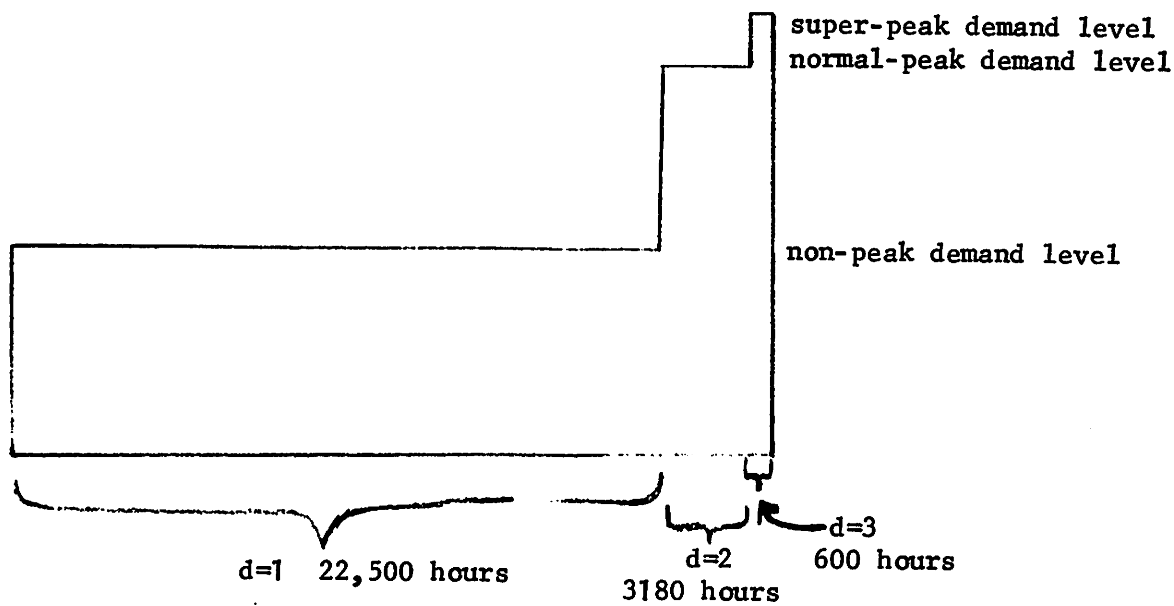
[12] These post-terminal costs will not be very large when discounted to present value at 10% per annum: one Rupee spent in each three-year period t = 6,7,8,..., to infinity. when discounted will <u>sum</u> to only .96 Rupee in present value.

[13] Current demand growth rates have been averaging 10% per year; this compounds to 33% for a three-year period. Other demand growth rates will also be examined, corresponding to annual growth rates of 7½% and 12½%.

function of ensuring that the system will have sufficient reserve power capacity.

Figure 1



duration of demand levels (d=1,2,3)
in hours per 3-year period

It is assumed that the demand for electricity at any time in a given three-year period will be at one of these levels; furthermore, each of these levels increases by 33% from one three-year period to the next.

Within a given three-year period, therefore, the demand levels do not change. This is not true of supply capacity within a given three-year period, in particular, the power (megawatts, MW) and energy (megawatt-hours, MW-hrs) capacities[14] of hydro plants. Due to seasonal differences in rainfall and water flow, the energy capacity of hydro plants is significantly greater in the wet season than in the dry season; the power capacity of hydro plants is also greater in the wet season than in the dry season. The model therefore distinguishes between two seasons (of equal duration), the wet season and the dry season; operating decisions are made separately for each season. In addition to seasonal variation there is significant annual variation, from a "wet" hydrological year to a "dry" one. The model distinguishes between three types of hydrological years, each with an assumed probability of occurrence (hydro-year index h = 1,2,3):

h = 1: a wet hydro-year, with probability .15 and hydro energy capacity 15% greater than that in a normal hydro-year

h = 2: a normal hydro-year, with probability .8

h = 3: a dry hydro-year, with probability .05 and hydro energy capacity 45% less than that in a normal hydro-year[15]

(Hydro power capacity is not assumed to vary from one hydro-year to the next.) Operating decisions are made separately for each season in each of the three possible hydro-years in each of the latter four three-year periods (t = 2,3,4,5).

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[14]Following conventional usage, "power" is to be interpreted throughout this paper in terms of megawatts (MW) while "energy" is to be interpreted as megawatt-hours (MW-hrs.).

[15]These figures were calculated using 60 years water flow data for the Krishna River (a major river in Southern India), using the assumption that the variable was normally distributed.

Investment Decision Variables

The focus of the model is the question of optimal investment decisions, that is, the types of plant to build and the timing and scale of their construction. In this regard, thermal and nuclear plant investments are treated differently from hydro plant investments. For thermal and nuclear investment the problem is to determine the timing and scale of new capacity. For hydro investments however, the decision is not simply the timing and scale of new hydro capacity in the abstract, but rather it's a question of which of a variety of specific hydro sites should be developed. Unlike thermal and nuclear investments, the power and energy capacities of possible hydro projects are largely determined by topography and water flow and they vary greatly from one hydro site to another, as does the per-megawatt and per-megawatt-hour cost of construction.

For hydro investments, the choice will be from a number of specific proposed hydro sites, each of which has given power capacities in the wet and dry season, given energy capacities in the wet and dry season of each possible hydro-year, and a given cost of construction. Corresponding to each of these proposed sites (indexed by project = p) and to each of the first three time periods (t = 1, 2, 3)[16] there is a zero-one variable (Ypt); its value equals one or zero depending on whether or not, respectively, construction of that hydro project (p) is to be undertaken in that time period (t)[17] .

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[16] Construction decisions for the last two three-year periods (t = 4, 5) are not made because hydro plant construction is assumed to take six years and hydro construction begun in the last two periods would not be finished in time for its benefits to be explicitly considered by the model.

[17] For operation beginning in time period t + 2 since hydro plant construction is assumed to take six years.

For thermal and nuclear investments,[18] decisions are made concerning the amount of capacity to be built in each of the first four time periods (t = 1,2,3,4). Each of these decisions is represented by a non-negative, continuous variable (Zjt) whose value equals the power capacity, in megawatts, of that plant type (j) to be built in that time period (t).[19] The power capacity of these investments do not vary between seasons (or hydro-years, for that matter) and their energy capacity is limited only by their power capacity (unlike hydro projects). Construction costs are given for thermal and for nuclear investments but the per-megawatt cost decreases as the size of plant increases, that is, there are economies of scale in construction costs. This is taken account of by assuming fixed-charge cost functions for thermal and nuclear investments, which in turn implies an additional "fixed-charge" variable (Fjt) corresponding to each construction decision variable (Zjt) for thermal or nuclear investment. The fixed-charge variable (Fjt) is restricted to the values one or zero depending on whether or not, respectively, the associated construction variable (Zjt) is positive or zero.

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[18] Actually, in the model discussed below, investment decisions are not made for conventional (coal-burning) thermal plants, but instead for plants using lignite, a low grade soft coal available in Madras.

[19] For operation beginning in time period t+1 since thermal and nuclear plant construction is assumed to take three years.

Operating Decision Variables

Although the focus of the model is on the investment decision variables, additional variables are included in the model in order to approximate the pattern of system operation and thereby estimate the fuel costs of alternative investments. These additional variables are the operating decision variables--briefly the output levels for the different plant types for the various demand conditions.

As mentioned above, operating decisions are made separately for each season and for each possible hydro-year in each of the last four time periods ($t = 2, 3, 4, 5$). In each of these instances, the demand is assumed to be represented by the load curve in Figure 1 ; this load curve, incidentally, is the same for both seasons and all possible hydro-years in a given time period. Output level decisions, in each of these instances, are defined not for each demand level ($d = 1, 2, 3$) but in a slightly different way, for each "mode" of operation ($i = 1, 2, 3$):[19a]

i=1 continuous operation, producing for non-peak, normal peak and super-peak demand levels

i=2 production for normal-peak and super-peak demand levels only

i=3 production for super-peak demand levels only.

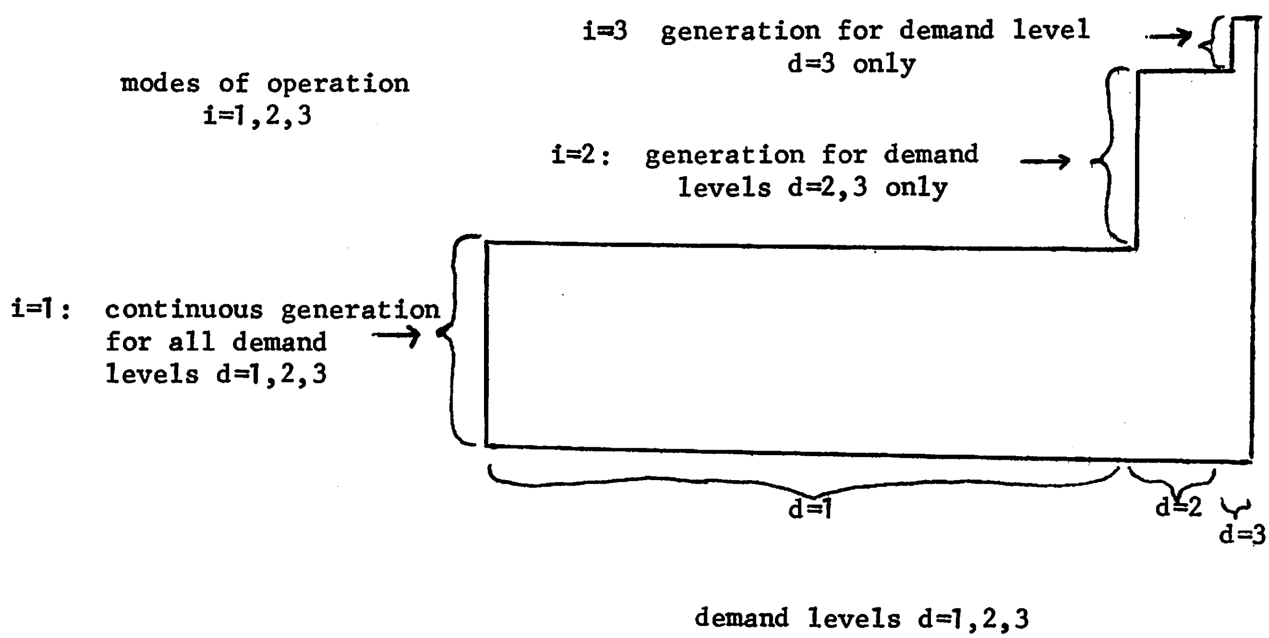
The durations of the three modes of operation are, of course, different from the durations of the three demand levels; the mode durations are as follows (per season per three-year period):

mode	i=1	13140	hours	(11250 + 1590 + 300)
	i=2	1890	hours	(1590 + 300)
	i=3	300	hours	(300)

To clarify the difference between demand levels and modes of operation, we depict the assumed demand curve again in Figure 2:

[19a]This is similar to the method used in the paper by G. F. de la Garza and A. S. Manne, "A Model for Planning Investments in Generating Facilities, Central-East-West Systems of CFE [Mexico], 1975-95," mimeo., Stanford University, Operations Research House, 1969.

Figure 2



Thus, for each plant type (j)[20] for each mode of operation (i), in each season (s), in each possible hydro year (h) in each time period (t=2,3,4,5), there is an operating decision variable (Xjisht) whose value equals the MW output to be sustained in that mode of operation (i) by that plant type (j) under those conditions (s, h, t).

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[20] In the model discussed below, operating decisions are made for hydro, nuclear, and lignite plants (for which investment decisions are also made), and also for conventional thermal plants which are already in existence, and which use coal imported from Andhra Pradesh.

List of Indices used in the Names of Variables

p refers to proposed hydro project p; in our example there are ten proposed hydro sites p=1, 2, ..., 10

j refers to plant type j
- j=1 conventional thermal
- j=2 lignite
- j=3 nuclear
- j=4 hydro

in our example, operating decisions are made for all j=1, 2, 3, 4; but (non-hydro) construction decisions are made for j=2, 3

d refers to demand level d
- d=1 non-peak demand level
- d=2 normal-peak demand level
- d=3 super-peak demand level

i refers to mode of operation i
- i=1 continuous operation, for all demand levels d=1, 2, 3
- i=2 generation only for demand levels d=2, 3
- i=3 generation only for demand level d=3

s refers to season s
- s=1 wet season
- s=2 dry season

h refers to possible hydrological-year h
- h=1 a wet hydro-year
- h=2 a normal hydro-year
- h=3 a dry hydro-year

t refers to the three-year period t
- t=1 years 1, 2, 3
- t=2 years 4, 5, 6
- t=3 years 7, 8, 9
- t=4 years 10, 11, 12
- t=5 years 13, 14, 15

Names of Variables

$Y_{pt} = 0$ or 1	investment decision concerning whether or not to start construction on hydro project p(p=1, ..., 10) in period t(t=1, 2, 3) for operation beginning in period t+2.
$F_{jt} = 0$ or 1	fixed-charge variable corresponding to the construction variable Z_{jt} (j=2, 3; t=1, 2, 3, 4); $F_{jt} = 0$ if $Z_{jt} = 0$ and $F_{jt} = 1$ if $Z_{jt} > 0$.
$Z_{jt} \geq 0$	MW capacity of plant type j (j=2, 3) to be built in time period t(t=1, 2, 3, 4), for operation beginning in period t+1.
$X_{jisht} \geq 0$	MW output level to be sustained by plants of type j (j=1, 2, 3, 4) under mode of operation i (i=1, 2, 3) during season s (s=1,2) of hydro-year h (h=1, 2, 3) in time period t (t = 2, 3, 4, 5).

Constraints

There are seven types of constraints in the model:

1) supply $\geq$ demand

2) power output $\leq$ power capacity

3) energy output $\leq$ energy capacity

4) constraints forcing the fixed charge to be incurred if a thermal or nuclear plant is built

5) constraints preventing hydro projects from being "built" more than once

6) zero-one constraints for the zero-one variables

7) non-negativity constraints for the continuous variables

The supply-demand constraints require that for each level of demand (d) in each season (s) in each hydro-year (h) of each time period (t = 2,3,4,5), the amount of power in MW supplied by the various plants is no less than the demand level in that time period:[21]

$$\sum_{i=1}^{d} \sum_{j=1}^{4} C_{1j} X_{jisht} \geqq \text{demand level d in period t}$$

There are 72 of these constraints: 3 demand levels, 2 seasons, 3 hydro-years, 4 time periods.

The power capacity constraints require that the power output (in MW) sustained by each plant type (j) in each season (s) of each possible hydro-year (h) in each time period (t = 2, 3, 4, 5) does not exceed the power capacity (in MW) available in that plant type (j) in that season (s) of that hydro-year (h) and time period (t = 2, 3, 4, 5)[22]:

for plant types j = 1, 2, 3

$$\sum_{i=1}^{3} X_{jisht} \leqq C_{2j} \left\{ \sum_{t'=1}^{t-1} Z_{jt'} + \begin{array}{l}\text{MW capacity initially avail-} \\ \text{able in plant type j}\end{array} \right\}$$

for hydro (j = 4)

$$\sum_{i-1}^{3} X_{jisht} \leqq C_{2j} \left\{ \sum_{p} \sum_{t'=1}^{t-2} Y_{pt'} \begin{pmatrix}\text{MW capacity,} \\ \text{season s,} \\ \text{project p}\end{pmatrix} + \begin{array}{l}\text{initially available} \\ \text{hydro MW capacity} \\ \text{in season s}\end{array} \right\}$$

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[21] The constants appearing in these constraints (C_{1j}) represent a correction for the difference between gross generation and net generation by plant type j: $C_{11} = C_{12} = C_{13} = .92$; $C_{14} = .97$.

[22] The constants appearing in these constraints (C_{2j}) represent a correction for that part of capacity of plant type j which is periodically inoperable due to scheduled maintenance: $C_{21} = C_{22} = C_{23} = .94$; $C_{24} = .97$.

There are 96 of these constraints: 4 plant types, 2 seasons, 3 hydro-years and 4 time periods.

The energy (MW-hr.) capacity constraints refer only to energy capacity of hydro plants; there are no explicit energy capacity constraints for thermal, lignite or nuclear plants. (Since thermal, lignite and nuclear plants could be operated virtually continuously if enough fuel were provided, the power capacity constraint provides the effective energy capacity constraint for thermal, lignite and nuclear plants.) However, since the energy capacity of most hydro plants is much too small to allow continuous operation, a separate constraint must be imposed to prevent hydro plants from producing more energy (MW-hrs.) than they have water available. Thus, for each season (s) in each hydro-year (h) of each time period (t = 2, 3, 4, 5) the energy output (MW-hrs.) of hydro plants must not exceed the hydro energy capacity available in that season of that hydro-year in that time period:

$$\sum_{i=1}^{3} \begin{pmatrix}\text{duration} \\ \text{of mode } i \\ \text{in season } s\end{pmatrix} X_{4isht} \leq \sum_{p} \sum_{t'=1}^{t-2} Y_{pt'} \begin{pmatrix}\text{MW-hr. capacity,} \\ \text{season } s\text{, hydro-year} \\ h\text{, project } p\end{pmatrix} + \begin{matrix}\text{initially available hydro energy} \\ \text{capacity in season } s \text{ of hydro-year } h\end{matrix}$$

There are 24 of these constraints: 2 seasons, 3 hydro-years, and 4 time periods.

The fourth type of constraint is concerned with the fixed-charge variables. For each fixed-charge zero-one variable (Fjt) there is a constraint which (in addition to the zero-one restriction) forces the variable to the value "one" if the corresponding construction variable (Zjt) is positive. Letting 5000 MW be an <u>a priori</u> upper bound on the value of Zjt, we have the constraint

$$5000\ Fjt - Zjt \geq 0.$$

There are 8 of these constraints, one for each fixed-charge variable Fjt, (j = 2, 3,; t = 1, 2, 3, 4).

For each proposed hydro proposed hydro project (p) there are three zero-one variables (Ypt) representing construction decisions in the first three time periods (t = 1, 2, 3); in order to prevent the model from "building" an especially atractive hydro project more than once, there is a constraint for each hydro project (p) which prevents the sum of the three relevant zero-one variables (Ypt) from exceeding the value "one" ($\sum_{t=1}^{3} Ypt \leq 1$). There are ten of these constraints in our example, one for each of the proposed hydro projects.

The zero-one constraints require that each hydro investment variable (Ypt) and each fixed-charge variable (Fjt) have either the value "zero" or the value "one":

$$Ypt = 0 \text{ or } 1$$

$$Fjt = 0 \text{ or } 1$$

The non-negativity constraints require that the remaining variables (Zjt and Xjisht) be non-negative:

$$Zjt \geq 0$$

$$Xjisht \geq 0$$

The Objective Function

The objective function, to be minimized, is the sum of construction costs for hydro, lignite, and nuclear plants plus the fixed costs of operation plus the variable costs of operation (fuel costs); of all four plant types. All costs are discounted to present value according to the time period in which they occur:

1) hydro construction costs: when a given hydro plant is to be started in period t the construction costs are divided equally between period t (discounted by the factor[23] $1/(1.10)^{3t-1}$) and period t+1 when the project is completed (discounted by the factor[24] $1/(1.10)^{3t+2}$):

$$\sum_{p} \sum_{t=1}^{3} Y_{pt} \cdot \begin{bmatrix}\text{construction} \\ \text{cost of} \\ \text{project p}\end{bmatrix} \cdot \left(\frac{1}{2}\right) \cdot \left[\frac{1}{1.10^{3t-1}} + \frac{1}{1.10^{3t+2}}\right]$$

2) construction costs for lignite plants: the construction cost, in million Rupees, for a lignite plant is assumed to be approximated by the function 45 + 1.6 times the MW capacity; the objective function entry is therefore

$$\sum_{t=1}^{4} \frac{1}{1.10^{3t-1}} \left[45\ F_{2t} + 1.6\ Z_{2t}\right]$$

3) construction costs for nuclear plants: the construction cost, in million Rupees for a nuclear plant is assumed to be approximated by the function 84 + 2.66 times the MW capacity; the objective function entry therefore is:

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[23] This discount factor assumes that these costs occur in the middle of the 3 year period t and that the discount rate is 10% per annum.

[24] This discount factor assumes that these costs occur in the middle of the 3 year period t+1 and that the discount rate is 10% per annum.

$$\sum_{t=1}^{4} \frac{1}{1.10^{3t-1}} \left[84\, F_{3t} + 2.66\, Z_{3t}\right]$$

4) fixed costs of operation and maintenance: in each period, t=2, ..., 5 and beyond,[25] fixed costs are incurred in maintaining the newly constructed capacity; these costs, in million Rupees, per MW of installed capacity are .18 for lignite, .09 for nuclear and .054 for hydro plants:

$$\sum_{t=2}^{\infty} \frac{1}{1.10^{3t-1}} \left\{ \left[\sum_{t'=1}^{t-1} .18\, Z_{2t'} + .09\, Z_{3t'}\right] + \left[\sum_{t'=1}^{t-2} \sum_{p} .054\, Y_{pt'} \begin{pmatrix}\text{MW capacity,}\\ \text{project } p\end{pmatrix}\right]\right\}$$

5) variable costs of operation (fuel costs): the costs, in Rupees, of operating the various plant types for one MW-hr. are 55 for thermal, 30 for lignite, 10 for nuclear, and .1 for hydro; the costs for a given time period equal the costs of operating the system in each possible hydro-year times the probability of that hydro-year; they are discounted according to the time period in which they occur:

for periods t = 2, 3, 4:

$$\sum_{t=2}^{4} \frac{1}{1.10^{3t-3}} \sum_{j=1}^{4} \sum_{h=1}^{3} \sum_{i=1}^{3} \sum_{s=1}^{2} \begin{bmatrix}\text{fuel cost}\\ \text{of 1 MW-hr.}\\ \text{plant type } j\end{bmatrix} \cdot Xjisht \cdot \begin{bmatrix}\text{probability of}\\ \text{hydro-year } h\end{bmatrix}$$

for t=5 and beyond[26]

$$\sum_{t=5}^{\infty} \frac{1}{1.10^{3t-3}} \sum_{j=1}^{4} \sum_{h=1}^{3} \sum_{i=1}^{3} \sum_{s=1}^{2} \begin{bmatrix}\text{fuel cost of}\\ \text{1 MW-hr.}\\ \text{plant type } j\end{bmatrix} \cdot Xjish5 \cdot \begin{bmatrix}\text{probability of}\\ \text{hydro-year } h\end{bmatrix}$$

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[25] Note that post-terminal fixed costs are included and appropriately discounted.

[26] Note that the pattern of system operation and fuel costs during period t=5 are assumed to be repeated during periods t=6, 7, ..., ∞ and are discounted accordingly.

Data

Hydro Capacity.

The initially available (t=1) hydro power capacity is assumed to be 1225 MW in the wet season and 855 MW in the dry season. The initially available (t=1) hydro energy capacity in a normal hydro-year is assumed to be 4.27 million MW-hrs. in the wet season and 2.91 million MW-hrs. in the dry season; the energy figures for a wet hydro-year are 15% greater and for a dry hydro-year they are 45% less.

There are ten proposed hydro sites (p=1, ..., 10). Given below are their respective power capacities in the wet and dry seasons, their energy capacities in the wet and dry seasons of a normal hydro-year,[27] and their construction cost.

Name of Project	Power Capacity in MW		Energy Capacity in a normal hydro-year in 10^3 MW-hrs.		Construction Cost in Million Rupees
	wet season	dry season	wet season	dry season	
p=1. Pandiar-Punnapuzha	100	75	380	250	150
p=2. Cholathipuzha	60	45	205	137	65
p=3. Kadamparai	35	27	109	72	62
p=4. Paralayar	35	27	96	64	41
p=5. Suruliyar	35	27	106	70	40
p=6. Coonoor-Kallar	50	38	91	61	84
p=7. Lower Moyar	70	52	143	97	118
p=8. Upper Manimuthar	90	45	246	120	123
p=9. Upper Amaravathy	70	35	263	129	141
p=10. Upper Thambarapani	200	100	296	144	250

[27] The energy capacities for a wet hydro-year are 15% greater than for a normal hydro-year; for a dry hydro-year they are 45% less than for a normal hydro-year.

Conventional thermal, lignite, and nuclear capacity.

The initially available (t=1) power capacity of conventional thermal plants is 470 MW; further investment in conventional thermal capacity is ruled out because lignite plants have roughly the same construction costs but significantly lower fuel costs. With regard to lignite plants, the initially available power capacity is 600 MW. As for nuclear capacity, there is assumed to be no capacity available initially; however, there is a 400 MW plant presently under construction which will begin operation in period t=3.

Demand levels.

The three demand levels for period t=2 are 845 MW for non-peak demand, 1564 MW for normal-peak demand, and 1835 MW for super-peak demand. Each of these levels is assumed to grow by 33% from one three-year period to the next. (This represents an estimated growth rate of 10% per annum; other demand growth rates are examined below: 7 1/2% per annum or 24% every three years, and 12 1/2% per annum or 42% every three years.)

Results[28]

1) Optimal Investment Program

In the first 3-year period (t=1):
start construction of seven hydro sites (p=1,2,3,4,5,7,8) with a combined installed capacity of 425 MW; these hydro sites will be ready for operation in period t=3. In addition, build a 215 MW lignite plant; this will be ready for operation in period t=2.

In the second 3-year period (t=2):
start construction on one hydro site (p=6) which has an installed capacity of 50 MW; this will be ready for operation beginning in period t=3. No additional construction will begin in this period but the seven hydro sites started in period t=1 will be completed.[28a]

In the third 3-year period (t=3):
construct an 890 MW lignite plant, for operation starting in period t=4. In addition, the hydro site started in period t=2 will be completed.

In the fourth 3-year period (t=4):
construct a 700 MW lignite plant and a 540 MW nuclear plant, both of which will start operation in period t=5.

The optimal investment pattern for this system includes three different types of plant coming into the system. At the start of the second three-year period a relatively small (215 MW) lignite plant comes into operation. At the start of the third three-year period seven hydro plants, with combined installed capacity of 425 MW, come into operation, as does a 400 MW nuclear plant whose construction was assumed exogenously. At the start of the fourth three-year period, a large (890 MW) lignite plant comes into operation. Finally, at the start of the fifth three-year period, a moderate sized lignite plant (700 MW) and a moderate sized nuclear plant (540 MW) come into operation.

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[28] As read into the computer, this problem had 237 rows, 629 columns, and 2774 matrix entries; the solution procedure required that 25 linear programming sub-problems be solved, taking approximately two minutes on an IBM 360/85 computer.

[28a] A 400 MW nuclear plant (presently under construction) is assumed to be completed in period t=2, for operation beginning in period t=3.

2) Pattern of System Operation

Conventional thermal plants. The 420 MW capacity, which was available at the start of the model, is to be used as little as possible because of its high variable costs of operation. It is to be used in each of the three possible hydro years but only during the dry season (never in the wet season) and only for the normal-peak and super-peak demand levels (never for non-peak demand). In the first three-year period of operation in the model (t=2) the conventional thermal capacity is used at partial strength (30% of capacity) during the normal-peak period in the dry season, and at full capacity during the super-peak period of the dry season; this is true in each of the three hydro-years in this period. In the later three-year periods (t=3,4,5), when lignite capacity has been increased, conventional thermal capacity is used only for super-peak periods in the dry season (at full capacity);[29] this usage is followed in each of the three hydro-years in these time periods.

Nuclear power plants. There are two nuclear plants in operation within this model: one (of approximately 400 MW) which is presently under construction and which is assumed to begin operation in time period t=3, and another which is to be constructed in period t=4 for operation beginning in period t=5 (the size of the latter is to be 540 MW). This capacity is to be operated continuously at full capacity in each season of each possible hydro-year of time periods t=3, 4, 5. (The only exception to this is in the wet season of a wet hydro-year when there is such abundant hydro energy that nuclear power is less than fully utilized during non-peak demand.[30]

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[29] Actually, in time period t=3 the model uses a negligible amount (27 MW) of conventional thermal capacity during the normal-peak period of the dry season for all hydro-years.

[30] In period t=3, nuclear capacity is not used at all during the non-peak period of the wet season in a wet hydro year; in periods t=4 and t=5 nuclear capacity is utilized at roughly 20% and 75%, respectively, under these conditions.

and is only used to full capacity during the normal peak and super-peak periods.) The pattern of use reflects the very low variable costs of operation for nuclear plants and also, in a sense, the relatively high construction costs for such plants. That is, construction of nuclear plants is only economical when the capacity can be utilized fully at all times (or at almost all times). Nuclear construction would not be economical if the capacity would lie idle much of the time; in such cases, lignite capacity would be more economical with its lower construction costs yet higher operating costs.

Hydro power plants. The same pattern of operating hydro plants is followed in each of the time periods, but with proportionately greater amounts of output in the later periods (t=3, 4, 5) as the newly constructed hydro plants come into operation. In normal hydro-years, hydro power capacity is fully utilized during normal-peak and super-peak periods in both seasons; during the non-peak period in both seasons, hydro power capacity is run at about 75% to 80% capacity.[31] In wet hydro-years the pattern of operation is similar to normal hydro-years, but during non-peak periods (in both seasons) hydro power capacity is 85% to 90% utilized; the extra amounts of hydro energy in wet hydro-years are thereby used up. In dry hydro-years the pattern of operation is again similar to normal hydro-years but in this case hydro power capacity is only 25% to 35% utilized during non-peak periods (of both seasons); this of course reflects the reduced energy capacity available in dry hydro-years. Note that this pattern of hydro operation utilizes all the available hydro energy in each season of each hydro-year in each time period and that

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[31] Although this pattern of capacity utilization holds for both seasons the actual amounts of power output are greater in the wet season than in the dry season because of the assumption that hydro power capacity is greater in the wet season than in the dry season. This is, of course, true also for hydro energy output (almost by definition of wet season and dry season).

it utilizes all the available hydro power for the normal peak and super-peak demand levels in each season, hydro-year, and time period; the differing amounts of hydro energy available in the various hydro-years are taken account of by varying the percentage of hydro power capacity utilization during the non-peak demand periods (from 30% in dry hydro-years, to 80% in normal hydro-years, to 90% in wet hydro-years).

Lignite plants. Given the great variability of hydro output between seasons and between hydro-years, the system has varying needs for non-hydro power. As mentioned above, the output of conventional thermal capacity is kept to a minimum because of the high variable costs: it is only used in peak periods of the dry season (in all hydro-years). As also mentioned above, for a nuclear plant's construction to be economical it must be utilized fully at all times (or at nearly all times). The burden of varying output to complement the variability of hydro output therefore falls upon the lignite capacity.[32] The output of lignite plants therefore varies inversely with that of hydro plants. In a normal hydro-year, lignite capacity is fully utilized (during peak demand periods) in the dry season but not in the wet season. In the dry season of a normal hydro-year it's used at anywhere from 20% to 35% capacity during non-peak periods and at full capacity during the normal-peak and super-peak periods.[33] In the wet season of a normal hydro-year, lignite capacity is not used at all to

[32] As mentioned previously, the initially available lignite capacity is 550 MW. To this is added a 215 MW plant in period t=1, an 890 MW plant in period t=3, and 700 MW plant in period t=4.

[33] Actually, in time periods t=4, 5 it's fully utilized only for the super-peak period and 90% utilized in the normal-peak period.

satisfy non-peak demand, it's used at about 50% capacity during normal-peak demand, and at about 80% capacity for super-peak demand. In a wet hydro-year, the pattern of operation is very similar to a normal hydro-year, the main difference being that lignite capacity is about 15% utilized during non-peak demand in the dry season (compared with 20% to 35% utilization under similar conditions in a normal hydro-year). In a dry hydro-year the pattern is similar in that lignite capacity is fully utilized (during peak demand periods) in the dry season but not in the wet season; but the pattern shifts toward much greater use of lignite capacity during non-peak periods. Thus in the dry season of a dry hydro-year capacity utilization increases to above 50% in non-peak periods (compared to roughly 30% in a normal hydro-year); in the wet season, capacity utilization during non-peak demand is roughly 30% (compared with no lignite outputs under such conditions in a normal hydro-year). Note again that this variability by season and hydro-year in lignite output is inversely related to the variability in hydro output.

3) A Graphic Explanation of the Pattern of System Operation

On the following page are a set of graphs depicting the levels of output from the different plant types under all possible conditions in one time period, t=4. This period was chosen as representative of the others (t=2, 3, 5); both seasons in each of the three possible hydro-years are depicted.

Within each graph the output levels for the different plant types are shown for each "mode of operation": continuous operation, production for normal-peak and super-peak demand only, and production for super-peak demand only.[34] Vertical distances are roughly proportional, although horizontal distances are distorted (for the sake of visual presentation) with the result that the durations of normal-peak demand and especially super-peak demand appear much greater than they are in fact.

To take one graph as an example, consider the dry season in a normal hydro year (of time period t=4). This graph says that the output of nuclear power is maintained continuously at 350 MW. Hydro power output is 821 MW during non-peak demand and is 1104 MW (821 + 283) during normal-peak and super-peak demand. Lignite power output is 406 MW during non-peak demand, 1487 MW (406 + 1081) MW during normal-peak demand, and 1588 MW (406 + 1081 + 101) during super-peak demand. Thermal power output is nil except during super-peak demand when it is 420 MW.

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[34] Note that the output levels shown are gross output, without correction for the differential between gross and net output which is made in the supply-demand constraints. This is the reason why the sums of the gross output levels from the different plant types exceed the respective demand levels.

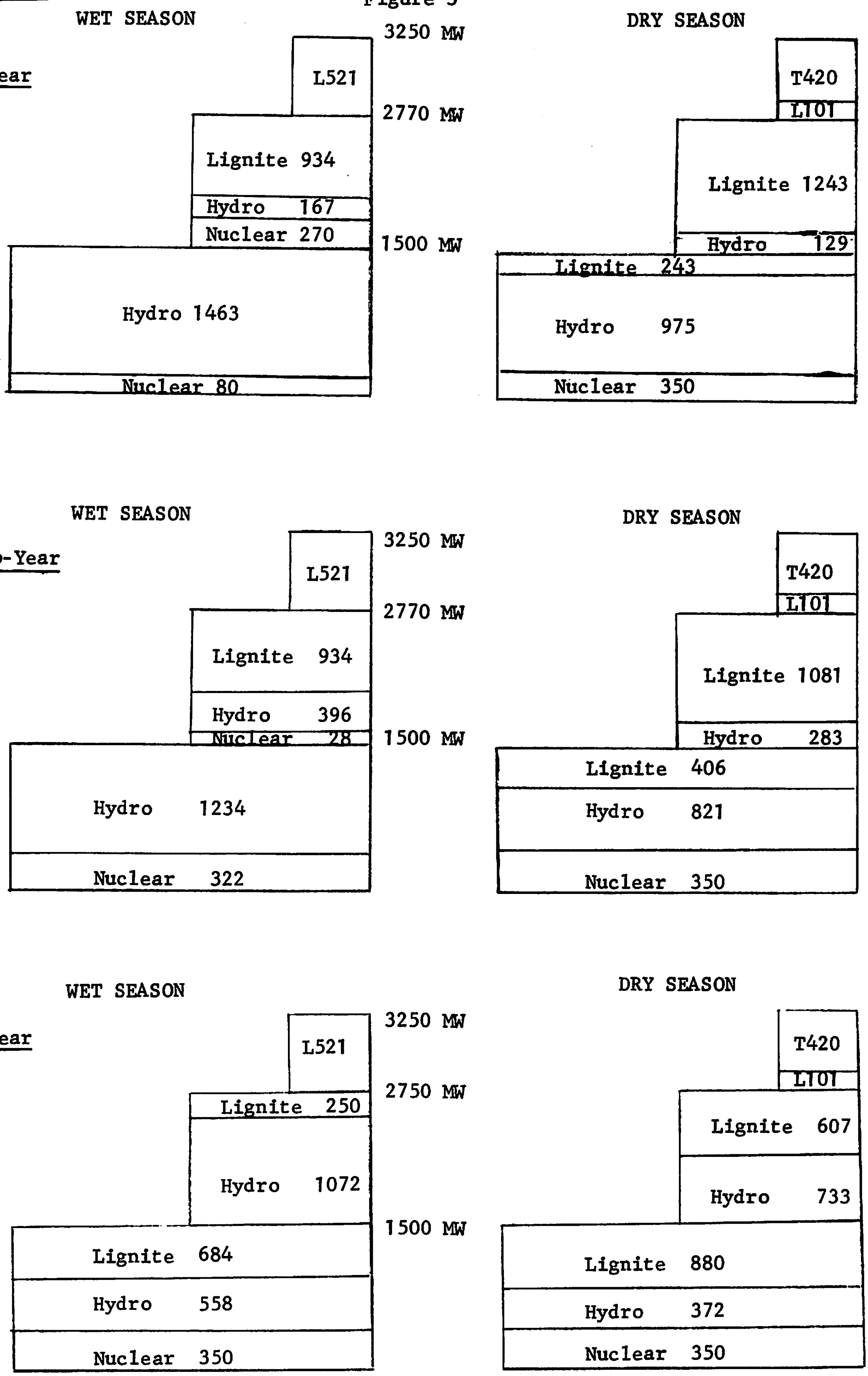
TIME PERIOD t=4:

Figure 3

Wet Hydro-Year
t = 4

Normal Hydro-Year
t = 4

Dry Hydro-Year
t = 4



A comparison of the graphs is also useful in seeing the pattern of complementarity between hydro and lignite output from season to season and from a wet hydro-year to a normal or a dry hydro-year. For example, observe the decreased hydro output for all demand levels in going from wet to dry seasons and the corresponding increase in lignite output. Also observe the shift in hydro output from mostly continuous generation in wet hydro-years to more and more generation for peak periods only in normal and dry hydro-years.

As mentioned above, hydro energy capacity is fully utilized in each season, as is hydro power capacity during normal-peak and super-peak demand. Nuclear power capacity (and energy capacity, for that matter) is fully utilized at all times. Thermal power capacity is left idle except for super-peak demand in the dry season. Lignite power capacity is partly idle for the entire wet season and is only fully utilized in the super-peak period of the dry season; lignite energy capacity is therefore underutilized in both seasons.

Parametric Variations

Inasmuch as the values of several different parameters had to be estimated in a very rough way it would be important to see how sensitive are the model's results when certain of these parameters are varied slightly. Two parameters which are important but whose values are not known with any certainty are the rate of growth of demand and the discount rate at which future costs should be evaluated.

The rate of growth of demand was assumed to be 10% per annum, or 33% per three-year period. Since this is only a rough estimate from the actual average growth rate over the past several years it would be well to examine the results of alternative demand growth rates. Two alternatives were examined: 24% per 3-year period and 42% per 3-year period.[35] (Note that each of these alternatives was examined while keeping the time discount rate at its original value, 10% per annum.)

The time discount rate (10% per annum) was likewise a rough estimate. Two alternative discount rates were examined: 12½% per annum and 15% per annum. (Note that each of these alternatives was examined while keeping the demand growth rate at its original value, 10% per annum.)

a) Demand Growth Rate of 24% per 3-year period (7½% per annum)

The results for the model using a 7½% per annum demand growth rate (and a 10% per annum discount rate)were basically a scaled-down version of the results for the model using a 10% annual growth rate for demand, with the main difference being that no nuclear plant is to be built in period t=5 in this case.

The same hydro sites are selected for construction but only three of them (p=2,4,5), with a combined installed capacity of 170 MW, are to be started in period t=1; the other five (p=1,3,6,7,8), with combined installed capacity of 345 MW, are to be started in period t=2. In the original model described

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[35] Corresponding to annual growth rates of 7½% and 12½%, respectively.

above the same eight hydro sites were chosen but seven of them (p=1, 2, 3, 4, 5, 7, 8) with combined installed capacity of 425 MW were to be started in period t=1 and the eighth (p=6) with installed capacity of 50 MW was to be started in period t=2.

As for non-hydro construction, the same pattern is followed on a smaller scale, except for the fact that there is to be no nuclear construction in period t=5. In the first time period t=1 a smaller lignite plant (71 MW) is to be built than in the original model (215 MW). In the second time period t=2 only hydro plants are to be built (as in the original model).[36] In the third time period t=3 a much smaller lignite plant (326 MW) is to be built than in the original model (890 MW). Finally, in the fourth time period the only plant to be built is a 730 MW lignite plant, which is larger than the lignite plant built in the original model (698 MW) but there is to be no nuclear construction in t=4, compared with a 540 MW nuclear plant in the original model.

The value of the objective function is likewise scaled down from the original model: 2656 in this case versus 4798 in the original run.

The pattern of system operation is basically the same as in the original model. Conventional thermal capacity is used as little as possible, only for peak periods in the dry season. Nuclear capacity (from the plant exogenously assumed to begin operation in period t=3) is typically utilized continuously at full capacity in both seasons of all hydro-years. Hydro output varies greatly between seasons and hydro-years and the same basic pattern of the original model is followed. Lignite capacity, as in the original model, is operated in complementary fashion to hydro capacity.

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[36] As in the original model, a medium sized (400 MW) nuclear plant, which is presently under construction is assumed to come into operation for time period t=3.

In summary, the main difference between these results and those of the original model is the original model's preference for a nuclear plant to be built in the fourth time period. This difference can perhaps be explained intuitively by saying again that nuclear plants will only be economical when there is sufficient need for non-hydro power on a continuous basis, i.e. all seasons and hydro-years.

b) Demand Growth Rate of 42% per 3-year period (12½% per annum)

The results for the model using a 12 1/2% per annum demand growth rate (and a 10% discount rate) are largely a scaled-up version of the results for the original model, although there are some qualitative changes as well.

Hydro construction is much the same as in the original model but now there is an additional site chosen for construction (p=10, a 200 MW project). Seven sites are to be started in period t=1 (p=1, 2, 3, 4, 5, 6, 7) with a combined installed capacity of 385 MW; this is roughly the same as the original model which had seven sites (p=1, 2, 3, 4, 5, 7, 8) with a combined installed capacity of 440 MW. In the second period t=2 there are two more sites to be started (p=8, 10), with a combined installed capacity of 290 MW; this is much greater than the 50 MW of hydro capacity started in t=2 in the original model.

The timing and scale of lignite facilities are also different from the original model. In this case, a moderate-sized lignite plant (360 MW) built in period t=1 (compared with a 215 MW plant originally), another plant (263 MW) in period t=2 (compared with no lignite capacity added in t=2 originally), and a very large lignite plant (1920 MW) in period t=4 (compared with two moderately large plants, 890 MW in t=3 and 698 MW in t=4, in the original model).

Nuclear construction also plays a slightly different role in this model.[37] Instead of a moderate sized (540 MW) nuclear plant to be built in t=4 as in the original model, this model selects a large nuclear plant (1194 MW) to be built in period t=3. Thus, not only is a much larger nuclear plant to be built but it's to be built earlier so that its benefits will be available sooner. This of course reflects the fact that the demand levels to be met are much greater, especially in the later periods, and therefore the amounts of non-hydro power required continuously are much greater, so that nuclear power would be more attractive than in the original model.

The value of the objective function is of course much greater in this case: 7377 versus 4798 in the original model.

The pattern of system operation is again fairly similar to the original model. The larger amounts of hydro, lignite, and nuclear capacity and the higher demand levels do not affect the basic pattern of operation.

c) Discount Rate of 12 1/2% per annum

The results for the model using a discount rate of 12% per annum (and a demand growth rate of 10% per annum) are almost identical to those of the original model with its 10% discount rate.

The hydro sites chosen are the same as the original model, the only difference being that one hydro project (p=6), with installed capacity of 50 MW, is started one period later (t=3) than in the original model (t=2). The same seven hydro sites (p=1, 2, 3, 4, 7, 8) as in the original model, with a combined installed capacity of 425 MW, are started in the first period t=1.

The timing and scale of lignite construction is virtually the same as the original model. A 215 MW lignite plant is built in the first period t=1

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[37] As in the original model, a medium sized (400 MW) nuclear plant which is presently under construction is assumed to come into operation for time period t=3.

(the same as before); a large lignite plant (931 MW) is built in the third period t=3 (compared with an 890 MW plant in the original model), and a moderately large (653 MW) plant is built in period t=4 (compared with a 698 MW plant in the original model).

Similarly for nuclear construction: a moderately large nuclear plant (544 MW) is to be built in period t=4, which is virtually identical to the 540 MW nuclear plant to be built in t=4 originally. In addition, of course, the 400 MW nuclear plant presently under construction is assumed to begin operation in period t=3.

The value of the objective function in this case is of course lower than the original model: 3957 compared with 4798.

The pattern of system operation is also very similar to that in the original model. The only differences are slight quantitative differences in the amounts of output under the various conditions; these are due to slight differences in the timing and scale of construction of the various plant types.

d) Discount Rate of 15% per annum

The results of the model using a discount rate of 15% (and a demand growth rate of 10%) are quite different from those of the original model. The shift is away from nuclear and hydro plants and towards lignite plants. This presumably reflects that greater reliance on lignite plants would be economical when their greater fuel costs in the future are discounted more heavily; conversely, the higher construction costs of hydro and nuclear plants will not be economical if their lower variable costs are discounted too heavily in the future.

Among the possible hydro sites only four of those chosen originally are now chosen. These four (p=1,2,4,5) have a relatively small combined installed capacity, 230 MW; they are all to be started in the first period t=1. Note that in the original model, twice as many sites with more than twice as much capacity were to be constructed.

There is to be no nuclear construction in this example[38], unlike the original model which had a 540 MW plant scheduled for period t=4.

Lignite capacity is to be greatly increased in this example. A small plant is to be built in period t=1 (215 MW, as before) and also in period t=2 (138 MW). A large plant is to be built in period t=3 (931 MW) and an even larger plant in period t=4 (1238 MW). Note that in the original model an 890 MW lignite plant was scheduled for period t=3 and both a 700 MW lignite plant and a 540 MW nuclear plant for period t=4.

The value of the objective function in this case is lower than in the original case, as expected: 2936 compared with 4798.

The output levels by plant types are likewise different inasmuch as the capacities of the various plant types are different. The basic pattern, however, still applies: using the nuclear plant continuously, the thermal capacity only for peak demand in the dry season, and the complementary use of hydro and lignite capacity.

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[38]The 400 MW nuclear plant presently under construction is still assumed to come into operation for period t=3.

e) Table IV-1: Comparison of Investment Results

Case 1: original model with 10% per annum demand growth rate and 10% discount rate

Case 2: demand growth rate of 7 1/2% per annum and 10% discount rate

Case 3: demand growth rate of 12 1/2% per annum and 10% discount rate

Case 4: demand growth rate of 10% per annum and 12 1/2% discount rate

Case 5: demand growth rate of 10% per annum and 15% discount rate

	Case 1	Case 2	Case 3	Case 4	Case 5
objective function (million Rupees, present value)	4798	2656	7377	3957	2936
investment in t=1:	hydro 425 MW lignite 215 MW	hydro 170 MW lignite 71 MW	hydro 385 MW lignite 359 MW	hydro 425 MW lignite 215 MW	hydro 230 MW lignite 215 MW
investment in t=2:	hydro 50 MW	hydro 345 MW	hydro 290 MW lignite 263 MW	nil	lignite 138 MW
investment in t=3:	lignite 890 MW	lignite 326 MW	nuclear 1194 MW	hydro 50 MW lignite 931 MW	lignite 931 MW
investment in t=4:	lignite 698 MW nuclear 540 MW	lignite 730 MW	lignite 1919 MW	lignite 653 MW nuclear 544 MW	lignite 1238 MW

Conclusions

This paper began with the linear programming model for planning investment in generating facilities developed at Electricité de France and it extended that analysis to a mixed-integer programming framework by using zero-one variables to represent the choice of hydro sites and also to reflect economies of scale in thermal and nuclear construction.

The data used were for the State of Madras (Tamilnadu) in Southern India. The results are meant to be more illustrative of the working of the model than to be taken at face value. In particular, the optimal values of the operating decision variables are not intended as an answer to the very complex problem of load dispatching but are only utilized to give a rough estimate of the fuel costs associated with different investment strategies. In addition, the optimal values for the investment decision variables do not provide the definitive word on investment planning but rather an input into the decision.