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# International Comparison of Consumption: An Index-Number Approach

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In the international comparison of personal consumption levels, most of the empirical estimations take the form of index numbers of the Laspeyres and Paache type. This paper is an attempt to find an economic meaning for the index numbers of this type. Further, as we shall see later, a comparison of estimated index numbers may shed some light on the relationship between prices and quantities consumed of individual commodities.

Apart from Fisher's attempt to construct an ideal index number satisfying some statistical test criteria, the old debates on the cost-of-living index numbers have been centered around "the theory of limit" (Frisch) to true index numbers. The early literature came to conclude that in general the two cost-of-living indexes using the Laspeyres and Paache formulae were insufficient to provide limits to the true index numbers. To make the measurement complete, what indexes should be calculated in addition to the Laspeyres and Paache index numbers? Further, if one does not calculate these indexes, can one find the conditions under which the conventional Laspeyres and Paache index numbers do provide the limits to the true index numbers?

The theoretical framework in which we analyze the index number relies heavily on the theory of revealed preference of a single consumer. We thus assume that a consistent map of social indifference curves can be constructed from individual indifference curves through lump-sum redistribution of personal incomes. When the theory is applied to the international comparison, the assumption on the common taste between nations has also to be made. Further, all the argument is based on the static assumption that consumers spend all their income on perishable goods and services.

This article is divided into four parts. The first part reviews the past discussions of the index numbers and illustrates our problem in two-good case. The second part deals with a sufficiency condition under which the conventional measurements with the Laspeyres and Paache quantity indexes provide upper and lower boundaries to the true quantity indexes. The third part is an empirical attempt to measure the points of under- and over-compensated change in income and to find whether this sufficiency condition is met or not in international cross-section data. The last section points out some of the limitations in our empirical results and critically discusses Denison's view on "the economies of scale associated with income elasticities."

# I

## The Laspeyres and Paache Index Numbers and "The Theory of Limit"

Most of the empirical results of international comparison of consumption are shown in the form of index numbers

(1.1) $$Q_L \equiv \Sigma\ p_1x_2/\Sigma\ p_1x_1$$

and

(1.2) $$Q_P \equiv \Sigma\ p_2x_2/\Sigma\ p_2x_1\ ,$$

where p and x refer to the retail price and the per capita consumption of a commodity. Subscripts 1 and 2 indicate Country 1 and Country 2 under comparison. In an analogy to the time-series index numbers of a growing economy, let us call $Q_L$ and $Q_P$ the Laspeyres and Paache index numbers, respectively. Assuming that consumers are in equilibrium at two observations in Figure 1, $q_1$ represents the per capita consumption of X-good and Y-good of Country 1, and S′S the budget line. Similarly, $q_2$ and T′T indicate the consumption and the budget constraint of Country 2. Hence, $Q_L$ = OC/OS and $Q_P$ = OT/OE in Figure 1.

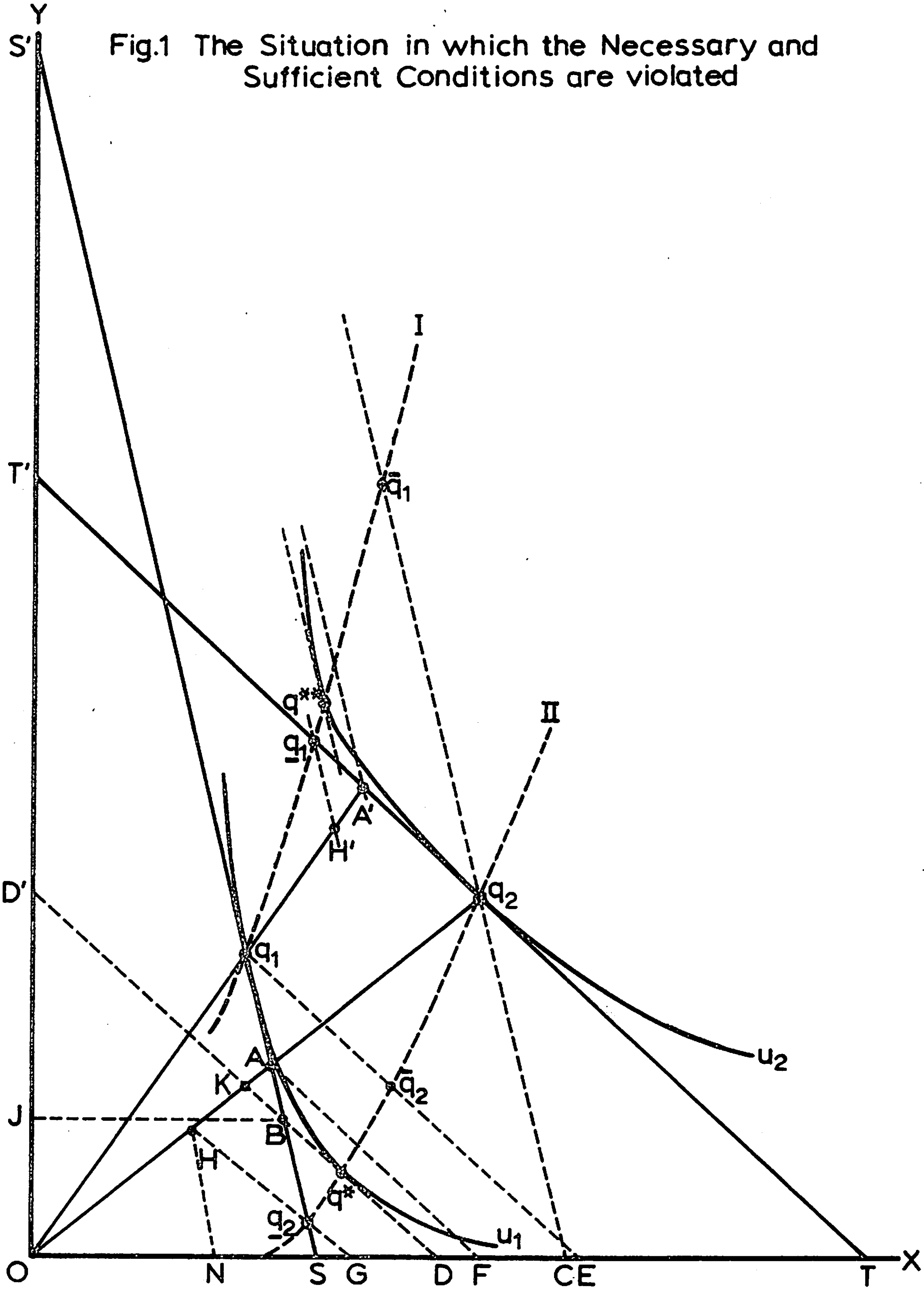
The pair of index numbers may be compared with a pair of "true index numbers." The true index numbers, $Q_1$ and $Q_2$, are the ratios between the expenditure at an observational point such as $q_1$ and $q_2$ and the expenditure at the point of exact-compensated change in income such as q* and q**. That is,

(1.3) $$Q_1 \equiv \Sigma\ p_1x^{**}/\Sigma\ p_1x_1$$

(1.4) $$Q_2 \equiv \Sigma\ p_2x_2/\Sigma p_2x^{*}$$

The true index numbers are theoretically more meaningful than the index numbers of the Laspeyres and Paache type. As well as meeting the "time" reversal test of Fisher, the true index numbers have a clear welfare connotation not found in the Laspeyres and Paache index numbers. They value the bundle of goods with equilibrium prices, and measure the differences in

Fig.1 The Situation in which the Necessary and Sufficient Conditions are violated



equilibrium expenditure between the same two preference scales ($u_1$ and $u_2$ in our figure).[1]

The early literature on index numbers was concerned with the question whether or not the actually estimated $Q_L$ and $Q_P$ can form the boundaries of the theoretical $Q_1$ and $Q_2$. Since $\Sigma\ p_2x_1$ in (1.2) equals the equilibrium expenditure $\Sigma\ p_2\bar{x}_2$ at $\bar{q}_2$, the point of over-compensated change in income from $q_1$, this expenditure is greater than $\Sigma\ p_2x^*$ in (1.4). Thus

(1.5) $$Q_P \leq Q_2 \ .$$

Similarly, since $\Sigma\ p_1x_2$ equals $\Sigma p_1\bar{x}_1$ at $\bar{q}_1$ and the latter is greater than $\Sigma\ p_1x^{**}$ at $q^{**}$, one obtains from (1.1) and (1.3) the relationship

(1.6) $$Q_L \geq Q_1 \ .$$

The "theory of limit" thus states that $Q_P$ is the lower limit to one true index number, and $Q_L$ the upper limit to the other true index number. But this of course does not mean that $Q_P$ and $Q_L$ form the lower limit and the upper limit, respectively, to both true index numbers. In international cross-sectional studies (as well as in time series of individual countries), the index number $Q_L$ is usually found to be greater than $Q_P$. But this does not exclude the possibility of having $Q_1 \leq Q_P \leq Q_L \leq Q_2$.

As Liviatan and Patinkin clearly demonstrated,[2] one needs to calculate the index numbers related to the points of under-compensated change in income,[3]

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[1]Of course, $Q_1$ and $Q_2$ represent different magnitudes, since a marginal utility of income is not independent of an income level and prices. But we shall not discuss the difference between $Q_1$ and $Q_2$, for we make no more restrictive assumptions on indifference curves than the ordinary ones such as convexity and non-intersection.

[2]Nissan Liviatan and Don Patinkin, "On the Economic Theory of Price Indexes," in Essays in the Quantitative Study of Economic Growth. Economic Development and Cultural Change, April 1961, pp. 502-536.

[3]P. A. Samuelson, "Consumption Theorems in Terms of Overcompensation rather than Indifference Comparisons," Economica, New Series, Vol. XX, No. 77 (February 1953), pp. 1-9.

in order for those limits to be complete. We define the index number measuring $\underline{q}_2$, the point of under-compensated change in income and $q_2$, the observational point along the income-expenditure path of Country 2 as

$$(1.7) \qquad \underline{Q}_2 \equiv \Sigma\, p_2 x_2 / \Sigma\, p_2 \underline{x}_2 \ .$$

In (1.7) $\underline{x}_2$'s refer to the quantities consumed per capita at $\underline{q}_2$. Since $\Sigma\, p_2 \underline{x}_2 \leq \Sigma\, p_2 x^*$, one obtains

$$(1.8) \qquad \underline{Q}_2 \geq Q_2$$

from (1.4) and (1.7). We further define the similar index number along the income-expenditure path of Country 1 as

$$(1.9) \qquad \underline{Q}_1 \equiv \Sigma\, p_1 \underline{x}_1 / \Sigma\, p_1 x_1 \ .$$

Comparing (1.9) with (1.3), one finds

$$(1.10) \qquad \underline{Q}_1 \leq Q_1 \ .$$ [4]

Thus the inequalities of (1.8) and (1.10) combined with (1.5) and (1.6) set the lower limit and the upper limit to both true index numbers.[5]

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[4] This incompleteness of the Laspeyres and Paache index numbers and the need for calculating the index numbers related to the point of under-compensated change in income have long been known since the early literatures on the cost-of-living index numbers. In fact, the points in diagrams appeared in earlier articles, for example, P" of Allen (p. 200), $Q_0$ of Staehle (Diagrams 4 and 5) and $Q_1^{\alpha}$ of Schultz (Figure 1), all indicate the point of under-compensated change in income. See R. G. D. Allen, "On the Marginal Utility of Money and Its Application," Economica, No. 40, May 1933, pp. 186-209; Hans Staehle, "A Development of the Economic Theory of Price Index Numbers," The Review of Economic Studies, June 1935, pp. 163-188; Henry Schultz, "A Misunderstanding in Index Number Theory: The True Konues Condition on Cost of Living Index Numbers and Its Limitations," Econometrica, January 1939, pp. 1-9.

[5] If one assumes a homothetic utility function so that an income-expenditure path is a straight line through the origin, then $\underline{Q}_1 = Q_P$ and $\underline{Q}_2 = Q_L$. Therefore, $Q_L$ does form the upper limit to both $Q_1$ and $Q_2$; and $Q_P$ the lower limit to both $Q_1$ and $Q_2$. But Engel curves in cross-sectional data are known to be non-linear. And we are not making this restrictive assumption.

Although the limits were completed by estimating two additional index numbers, the estimation of the additional index numbers requires one to trace the income-expenditure paths and find the points of under-compensated change in income. The estimation of $Q_L$ and $Q_P$, on the other hand, requires less information. The market data on the bundle of goods per capita of each country measured in prices of both countries are enough to construct $Q_L$ and $Q_P$. As no estimation on $Q_1$ and $Q_2$ have been tried in international cross-section studies, we have to rely on the available data, that is, $Q_L$ and $Q_P$. Then the question arises as to under what conditions $Q_P$ is lower than both true index numbers, and $Q_L$ higher than both true index numbers. If these conditions are actually prevalent, one may assume that two true index numbers are somewhere in the range between $Q_L$ and $Q_P$.

As we already know $Q_L \geq Q_1$ in (1.6), what remains to be found is the condition under which the inequality

(1.11) $\quad Q_L \geq Q_2$

holds. Similarly, one needs to find the condition under which

(1.12) $\quad Q_P \leq Q_1$

is met.

The condition for (1.11) in two-good case can be found in Figure 1. There, $Q_2 = OT/OD$. And $Q_L = OC/OS = Oq_2/OA = OT/OF$. Since $OF > OD$, the inequality (1.11) does not hold in Figure 1. In Figure 2, on the other hand, this inequality is satisfied. In Figure 1, the straight line $OKAq_2$ passes "north-west" of the point B, and in Figure 2 this line passes "south-east" of B. In other words, the ratio $D'K/D'D = T'q_2/T'T$ is smaller than $D'B/D'D$ in Figure 1. And the case is opposite in Figure 2. Let us denote the quantities of two goods with x and y, and their prices with p and 1. The income in equilibrium is represented by M. The ratio $T'q_2/T'T$ is the share of expenditure on X-good at $q_2$, that is, $p_2x_2/M_2$. On the other hand,

$D'B/D'D = JB/OD = ((M_1-M^*)/(p_1-p_2)/(M^*/p_2)$, where $M^* = p_2x^* + y^*$ is the income at $q^*$. Since $p_1 > p_2$ in both Figures 1 and 2, one finds the following condition.

$$(1.13) \quad Q_L \equiv (p_1x_2+y_2)/(p_1x_1+y_1) \geq (p_2x_2+y_2)/(p_2x^*+y^*) \equiv Q_2$$

is equivalent to

$$(1.14) \quad p_2x_2/M_2 \underset{(\leq)}{\geq} \frac{M_1-M^*}{M^*} \bigg/ \frac{p_1-p_2}{p_2} \text{ for } p_1 \underset{(<)}{>} p_2 .$$

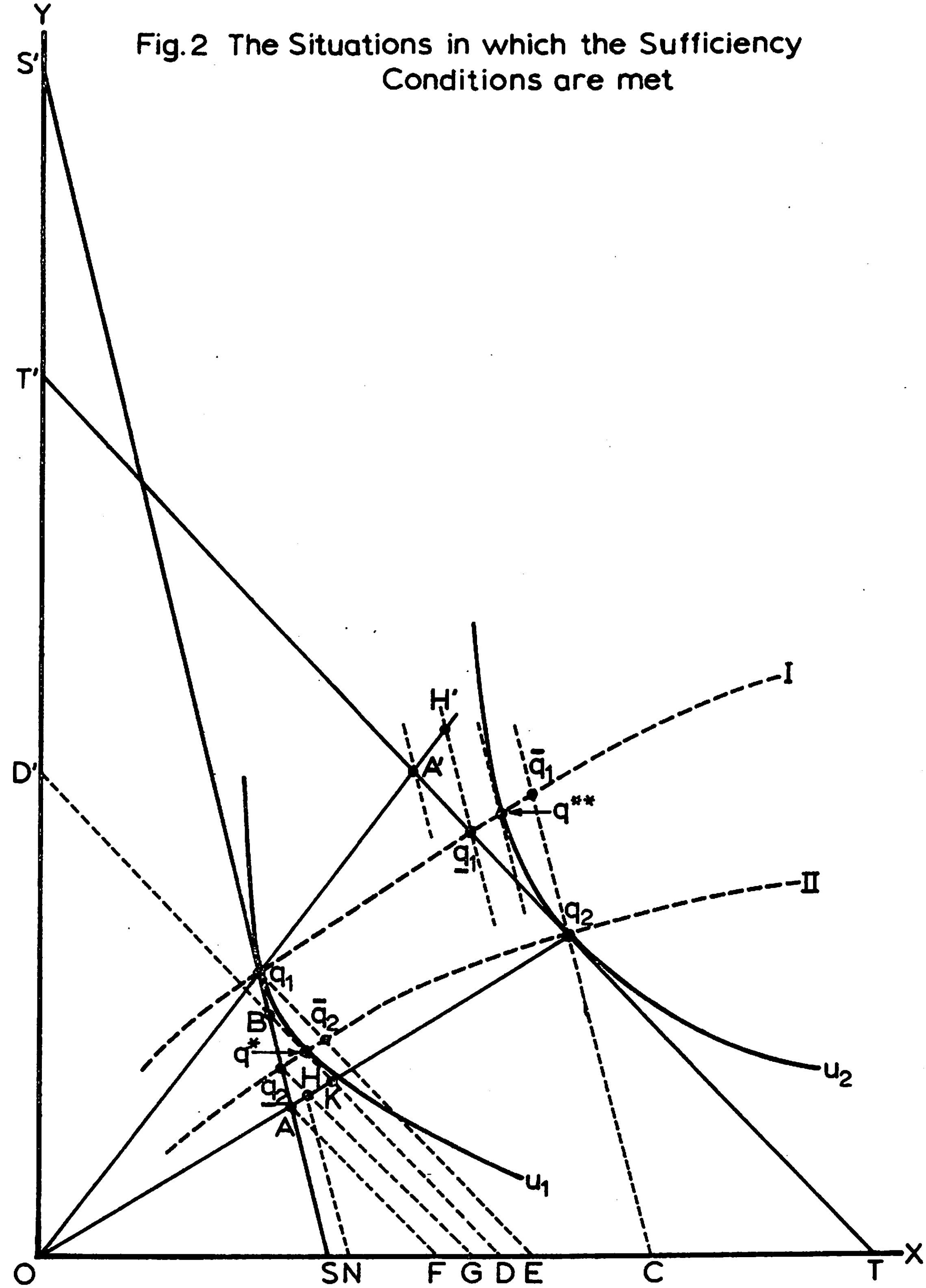
The necessary and sufficient condition (1.14) states that if the price of X-good is lower in Country 2 than in Country 1, Country 2's expenditure share of X-good is greater than the arc-price-elasticity of income compensation at the welfare level of Country 1. The more elastic the indifference curve $u_1$, the smaller is the arc-price-elasticity.

One may restate (1.14) by subtracting the expenditure share of X-good at $q^*$ from both sides of (1.14). Then (1.14) is equivalent to

$$(1.15) \quad p_2x_2/M_2 - p_2x^*/M^* \underset{(\leq)}{\geq} \frac{p_1(x_1-x^*) + y_1-y^*}{M^*} \bigg/ \frac{p_1-p_2}{p_2}$$

for $p_1 \underset{(<)}{>} p_2$. The term in the right-hand side of (1.14) is non-positive (non-negative) for $p_1 \underset{(<)}{>} p_2$. This is because $p_1(x_1-x^*) + y_1-y^*$ is non-positive due to the convexity of the indifference curve $u_1$. Assume that the income level is lower at the first observation than at the second observation so that $M^* < M_2$. And assume $p_1 > p_2$. In Figure 2 where X-good is a luxury, the condition (1.15) is met. If X-good is a necessity as shown in Figure 1, however, one can no longer be sure whether (1.15) is satisfied or not. The condition may be violated, if the substitution is not very elastic along the welfare level of the first observation and the income elasticity of X-good is low. This consideration leads one to the following sufficient condition.

Fig. 2 The Situations in which the Sufficiency Conditions are met



Theorem 1 (Sufficiency condition in two-good case)

(1.11) $Q_L \geq Q_2$

if

(1.16) $p_2 x_2/M_2 - p_2 x^*/M^* \underset{(\leq)}{\geq} 0$ for $p_1 \underset{(<)}{>} p_2$.[6]

Exactly parallel condition can be found for the Paache quantity index to be smaller than $Q_1$. Taking similar steps to (1.14) and (1.15), one obtains

Theorem 2 (Sufficiency condition in two-good case)

(1.12) $Q_P \leq Q_1$

if

(1.17) $p_1 x^{**}/M^{**} - p_1 x_1/M_1 \underset{(\leq)}{\geq} 0$ for $p_1 \underset{(<)}{>} p_2$,

where $x^{**}$ and $M^{**}$ represent the quantity consumed of X-good and the equilibrium income $(p_1 x^{**} + y^{**})$, respectively, at $q^{**}$, the point of exact-compensated change in income.

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[6] Needless to say, we are dealing with the case of discrete change. If the price change is infinitesimal, then the conditions (1.16) and (1.14) are equivalent. This is because, for the infinitesimal change in price from $p_2$, $p_2 x^*/M^* = \frac{\Delta M}{M^*} \Big/ \frac{\Delta p}{p_2}$. As Roy first showed, $-f_p/f_M = x$ where $f_p$ and $f_M$ are partial derivatives of the indirect utility function $u = f(p,M)$. (See H. S. Houthakker, "Compensated Changes in Quantities and Qualities Consumed," The Review of Economic Studies, Vol. XIX(3), No. 50, 1952-53, pp. 155-164.)

II

## Positive Income Effect as a Sufficiency Condition

In the case of more than two goods, the clear correspondence between the difference in price and the difference in expenditure share stated in the sufficiency conditions in the last section is blurred. But one can still have a meaningful condition if the correspondence is predominant for most goods.

At this stage it is convenient to introduce Hicks' definition of the income effect and the substitution effect in index numbers.[7] According to this definition, the difference between the Laspeyres quantity index and the Paache quantity index is decomposed into those two effects by putting $q^{**}$, the point of compensated change in income from the second observation, as an intermediate point. The income effect is obtained by constructing two quantity index numbers in different sets of prices to compare between an observational point and a point of compensated change in income along the same income-expenditure path and then by taking the difference between two index numbers. Namely,

$$(2.1) \qquad I_1 \equiv \Sigma\, p_1 x^{**} / \Sigma\, p_1 x_1 - \Sigma\, p_2 x^{**} / \Sigma\, p_2 x_1 .$$

Let us call this the income effect on the income-expenditure path of Country 1. On the other hand, the substitution effect at the welfare level of Country 2 is defined as

$$(2.2) \qquad S_2 \equiv \Sigma\, p_1 (x_2 - x^{**}) / \Sigma\, p_1 x_1 - \Sigma\, p_2 (x_2 - x^{**}) / \Sigma\, p_2 x_1 .$$

It is obvious that

$$(2.3) \qquad I_1 + S_2 = \Sigma\, p_1 x_2 / \Sigma\, p_1 x_1 - \Sigma\, p_2 x_2 / \Sigma\, p_2 x_1 \equiv Q_L - Q_P .$$

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[7] J. R. Hicks, A Revision of Demand Theory (Oxford, 1956), Chapter 19.

From our convexity assumption on an indifference curve $u_2$, this substitution effect is non-negative. On the other hand, the income effect can be either positive or negative. If $I_1$ is positive, this implies that $Q_L$ is greater than $Q_P$. However, an empirical finding that $Q_L$ is greater than $Q_P$ is not sufficient for $I_1$ to be positive, because a positive $S_2$ can be so large that it is more than enough to offset a negative $I_1$.

In parallel with the above, the difference between the reciprocal of the Paache index number and the reciprocal of the Laspeyres index number is decomposed into two effects by putting $q^*$, the point of compensated change in income from the first observation, as an intermediate point. They are $I_2$, the income effect on the income-expenditure path of Country 2, and $S_1$, the substitution effect at the welfare level of Country 1. That is,

$$(2.4) \qquad I_2 \equiv \Sigma p_2 x^* / \Sigma p_2 x_2 - \Sigma p_1 x^* / \Sigma p_1 x_2$$

$$(2.5) \qquad S_1 \equiv \Sigma p_2 (x_1 - x^*) / \Sigma p_2 x_2 - \Sigma p_1 (x_1 - x^*) / \Sigma p_1 x_2$$

$$(2.6) \qquad I_2 + S_1 = \Sigma p_2 x_1 / \Sigma p_2 x_2 - \Sigma p_1 x_1 / \Sigma p_1 x_2$$

$$\equiv 1/Q_P - 1/Q_L \ .$$

From our assumption $S_1$ is always non-negative, whereas $I_2$ can be either positive or negative.

From this definition of the income effect and the substitution effect, Marris derived the definition in relative terms.[8] According to the latter definition, the discrepancy which is to be divided into the income effect and the substitution effect is not the absolute difference $Q_L - Q_P$, but the

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[8] R. L. Marris, "Professor Hicks' Index Number Theorem," Review of Economic Studies, Vol. 25, No. 66 (October, 1957), pp. 25-40; Economic Arithmetic, (London and New York, 1958), Chapters 8 and 9.

relative difference $(Q_L - Q_P)/Q_L$. Further, this relative difference and its two components are shown to be expressed in covariance terms. That is to say,

$$(Q_L - Q_P)/Q_L \equiv (\Sigma\, p_1x_2/\Sigma\, p_1x_1 - \Sigma\, p_2x_2/\Sigma\, p_2x_1)/(\Sigma\, p_1x_2/\Sigma\, p_1x_1)$$

$$= \frac{1}{Q_L P_L}\left\{(\Sigma \frac{p_1x_1}{\Sigma p_1x_1}\ \frac{x_2}{x_1})(\Sigma \frac{p_1x_1}{\Sigma p_1x_1}\ \frac{p_2}{p_1}) - \Sigma \frac{p_1x_1}{\Sigma p_1x_1}\ \frac{x_2}{x_1}\ \frac{p_2}{p_1}\right\}$$

where $P_L$ represents the cost-of-living index number in the Laspeyres form, $\Sigma\, p_2x_1/\Sigma\, p_1x_1$. Although the prices and the quantities consumed are not randomly distributed variables, at least in its form the term in the brackets is the covariance (with a negative sign) between the rate of quantity change and the rate of price change with the expenditure share at the first observation as weights. Let us denote this with

$$(2.7)\qquad (Q_L - Q_P)/Q_L = -\,cov_1((x_2/x_1)/Q_L,\ (p_2/p_1)/P_L)\ .$$

Further, this relative difference can be shown to be the sum of two covariance terms, one covariance term, $i_1$, representing the relative income effect and the other, $s_2$, representing the relative substitution effect. That is,

$$(2.8)\qquad (Q_L - Q_P)/Q_L \equiv i_1(Q_1/Q_L) + s_2$$

where

$$(2.9)\qquad i_1 \equiv I_1/Q_1$$

$$= \frac{1}{Q_1 P_L}\left\{(\Sigma \frac{p_1x_1}{\Sigma p_1x_1}\ \frac{x^{**}}{x_1})(\Sigma \frac{p_1x_1}{\Sigma p_1x_1}\ \frac{p_2}{p_1}) - \Sigma \frac{p_1x_1}{\Sigma p_1x_1}\ \frac{x^{**}}{x_1}\ \frac{p_2}{p_1}\right\}$$

$$\equiv -\,cov_1((x^{**}/x_1)/Q_1,\ (p_2/p_1)/P_L)$$ [9]

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[9] This definition of the income effect in relative terms is slightly different from Marris' definition. The latter definition is $I_1/Q_L$.

and 

$$(2.10) \quad s_2 \equiv S_2/Q_L$$

$$= \frac{1}{Q_L P_L}\left\{(\Sigma \frac{p_1x_1}{\Sigma p_1x_1} \frac{x_2-x^{**}}{x_1})(\Sigma \frac{p_1x_1}{\Sigma p_1x_1} \frac{p_2}{p_1}) - \frac{p_1x_1}{\Sigma p_1x_1} \frac{x_2-x^{**}}{x_1} \frac{p_2}{p_1}\right\}$$

$$\equiv -\ \mathrm{cov}_1 \left(\frac{x_2-x^{**}}{x_1}/Q_L \ ,\ \frac{p_2}{p_1}/P_L\right)$$

Just by inverting $Q_L$ and $Q_P$, this relative difference can also be expressed in terms of another covariance,

$$(2.11) \quad (Q_L - Q_P)/Q_L = (1/Q_P - 1/Q_L)/(1/Q_P)$$

$$= -\ \mathrm{cov}_2\left(\frac{x_1}{x_2}/\frac{1}{Q_P}\ ,\ \frac{p_1}{p_2}/\frac{1}{P_P}\right)$$

where $P_P$ represents the cost-of-living index in the Paache form, $\Sigma p_2x_2/\Sigma p_1x_2$. Further, this covariance term can be decomposed into the relative income term along the income-expenditure path of Country 2 and the relative substitution term at the welfare level of Country 1. That is,

$$(2.12) \quad i_2 \equiv I_2Q_2 = -\ \mathrm{cov}_2\left(\frac{x^*}{x_2}/\frac{1}{Q_2}\ ,\ \frac{p_1}{p_2}/\frac{1}{P_P}\right)$$

$$(2.13) \quad s_1 \equiv S_1Q_P = -\ \mathrm{cov}_2\left(\frac{x_1-x^*}{x_2}/\frac{1}{Q_p}\ ,\ \frac{p_1}{p_2}/\frac{1}{P_P}\right)$$

and

$$(2.14) \quad (Q_L - Q_P)/Q_L = i_2(Q_P/Q_2) + s_1 .$$

Let us return to our original problem of finding the condition under which the Laspeyres index number, $Q_L \equiv \Sigma\, p_1x_2/\Sigma\, p_1x_1$, is greater than a true index number, $Q_2 \equiv \Sigma\, p_2x_2/\Sigma p_2x^*$. It is obvious that the relationship $Q_L \geq Q_2$ is equivalent to having

(2.15) $\Sigma\, p_2x^*/\Sigma\, p_2x_2 - \Sigma\, p_1x^*/\Sigma\, p_1x_2 \geq \Sigma\, p_1(x_1-x^*)/\Sigma\, p_1x_2$ .[10]

In (2.15) the terms in the left hand side of the inequality represent the income effect $I_2$ of (2.4). The term in the right hand side is a portion of the substitution effect of (2.5) and this term is non-positive from the convexity of an indifference curve. This immediately leads us to the following sufficiency condition.

Theorem 3 (Sufficiency condition in n-good case)

The inequality $Q_L \geq Q_2$ holds if the income effect on the income-expenditure path of Country 2 is non-negative, that is,

$$(2.16) \qquad I_2 \equiv \Sigma\, p_2x^*/\Sigma\, p_2x_2 - \Sigma\, p_1x^*/\Sigma\, p_1x_2 \geq 0 .$$

Alternatively, one may express this sufficiency condition in terms of covariance in (2.12).

$$(2.17) \qquad -i_2 \equiv \mathrm{cov}_2\left(\frac{x^*}{x_2}/\frac{1}{Q_2}\ ,\ \frac{p_1}{p_2}/\frac{1}{P_P}\right) \leq 0 .$$

This implies an inverse relation between the price ratio and the arc-income-elasticity of individual commodities. For an illustration, suppose the welfare level of Country 2 is higher than that of Country 1, so that $Q_2 > 1$. Then, the condition (2.17) means that there is a tendency that the relative price is lower in Country 2 than in Country 1 ($(p_2/p_1)/P_P < 1$) for a luxury ($(x_2/x^*)/Q_2 > 1$) and the relative price of Country 2 is higher for a necessity. It is clear that this condition corresponds with the condition (1.16) in two-good case.

Exactly parallel relationship is found as to the condition under which the Paache quantity index number is kept below the true index number under the prices $p_1$.

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[10] The condition (2.15), when it is applied to the two-good case, is equivalent to the necessary and sufficient condition expressed in (1.15).

Theorem 4 (Sufficiency condition in n-good case)

The inequality $Q_P \leq Q_1$ holds if

$$(2.18) \quad I_1 \equiv \Sigma p_1 x^{**}/\Sigma p_1 x_1 - \Sigma p_2 x^{**}/\Sigma p_2 x_1 \geq 0 .$$

This condition is restated in covariance term (2.9),

$$(2.19) \quad -i_1 \equiv \text{cov}_1((x^{**}/x_1)/Q_1, (p_2/p_1)/P_L) \leq 0 .$$

And this condition in n-good case corresponds with the condition (1.17) in two-good case.

It should be recalled that the positiveness of substitution effect was derived from the theoretical consideration on the shape of indifference map, but the positive income effect was not. It is entirely an empirical question whether the income effect turns out to be positive or negative in comparing two countries' consumption patterns. Further, an empirical finding that the Laspeyres index number is greater than the Paache index number cannot help to support the positiveness of income effect.

Testing of the two sufficiency conditions, however, requires one to measure the expenditures at the points of exact-compensated change in income ($q^*$ and $q^{**}$ in Figures 1 and 2). Of course, one cannot estimate them unless one assumes demand functions in some specific form, for example, the indirect addilog function of Konius-Leser-Houthakker, or the linear expenditure system of Klein-Rubin-Stone. In fact, the estimation of these functions enables one to construct the true index numbers. But we are not assuming this task in this paper. Our empirical research in the following section will be limited to a more modest goal, namely, to approximate the expenditure at the point of exact-compensated change in income with the expenditures at the points of under- and over-compensated change in income ($\underline{q}$'s and $\overline{q}$'s in Figures 1 and 2).

Thus we replace the income effect along the income-expenditure path of Country 2, $I_2$, with the approximate income effect, $\underline{I}_2$, by substituting the bundle of goods at the point of under-compensated change in income ($\underline{q}_2$) for the bundle of goods at the point of exact-compensated change ($q^*$). That is,

$$(2.20)\qquad \underline{I}_2 \equiv \Sigma\, p_2\underline{x}_2/\Sigma\, p_2x_2 - \Sigma\, p_1\underline{x}_2/\Sigma\, p_1x_2$$

$$= \Sigma\, p_2\underline{x}_2/\Sigma\, p_2x_2 - \Sigma\, p_1x_1/\Sigma\, p_1x_2 \;.$$

Further, dividing (2.20) with $\Sigma\, p_2\underline{x}_2/\Sigma\, p_2x_2 \equiv 1/\underline{Q}_2$ , this approximate income effect is expressed in covariance terms.

$$(2.21)\qquad \underline{i}_2 \equiv \underline{I}_2\cdot\underline{Q}_2 \equiv -\operatorname{cov}_2\left(\frac{\underline{x}_2}{x_2}/\frac{1}{\underline{Q}_2}\;,\;\frac{p_1}{p_2}/\frac{1}{P_P}\right)\;.$$

The only difference of (2.21) from the relative income effect of (2.12) is the definition of arc-income-elasticities.

Similarly, we replace the income effect $I_2$ with the approximate income effect $\overline{I}_2$ by substituting the bundle of goods at the point of over-compensated change in income for the bundle of goods at the point of exact-compensated change.

$$(2.22)\qquad \overline{I}_2 \equiv \Sigma\, p_2\overline{x}_2/\Sigma\, p_2x_2 - \Sigma\, p_1\overline{x}_2/\Sigma\, p_1x_2$$

$$= \Sigma\, p_2x_1/\Sigma\, p_2x_2 - \Sigma\, p_1\overline{x}_2/\Sigma\, p_1x_2$$

The expression of this income effect in covariance term is

$$(2.23)\qquad \overline{i}_2 \equiv \overline{I}_2\cdot Q_P \equiv -\operatorname{cov}_2\left(\frac{\overline{x}_2}{x_2}/\frac{1}{Q_P}\;,\;\frac{p_1}{p_2}/\frac{1}{P_P}\right)\;.$$

Similar to the positiveness of $I_2$, the positiveness of $\underline{I}_2$ can be considered as a sufficiency condition for $Q_L \geq Q_2$. This is because this inequality is equivalent to

$$(2.24)\qquad \Sigma\, p_2\underline{x}_2/\Sigma\, p_2x_2 - \Sigma\, p_1\underline{x}_2/\Sigma\, p_1x_2 \geq \Sigma\, p_2(\underline{x}_2 - x^*)/\Sigma p_2x_2 \;.$$

The terms in the left hand side of the inequality (2.24) represent $\underline{I}_2$. The term in the right hand side equals $1/\underline{Q}_2 - 1/Q_2$, and is non-positive. Thus the relationship $Q_L \geq Q_2$ holds if $\underline{I}_2$ is non-negative.[11]

But $\underline{I}_2$ in (2.20) equals $1/\underline{Q}_2 - 1/Q_L$. Therefore, the non-negative $\underline{I}_2$ is equivalent to $\underline{Q}_2 \leq Q_L$. And, as (1.8) showed, $\underline{Q}_2$ is known to be the upper limit to the true index number $Q_2$. Hence, the condition that $\underline{I}_2$ be non-negative actually overshoots the limit to a true index number.

In exactly the same way, one can find a sufficiency condition for $Q_P \leq Q_1$, that is,

$$(2.25)\qquad \underline{I}_1 \equiv \Sigma\, p_1\underline{x}_1/\Sigma\, p_1x_1 - \Sigma\, p_2\underline{x}_1/\Sigma\, p_2x_1 \geq 0$$

or

$$(2.26)\qquad -\underline{i}_1 \equiv \mathrm{cov}_1\left(\frac{\underline{x}_1}{x_1}/\underline{Q}_1 \,,\, \frac{p_2}{p_1}/P_L\right) \leq 0 \;.$$

This condition $\underline{I}_1 \geq 0$ is equivalent to saying $Q_P \leq \underline{Q}_1$. And, as (1.10) showed, $\underline{Q}_1$ is the lower limit to the true index number $Q_1$. Hence, this sufficiency condition also overshoots the limit to a true index number.

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[11] Contrary to $\underline{I}_2$, a non-negative $\bar{I}_2$ is not a sufficient condition for having the inequality $Q_L \geq Q_2$. This is because $Q_L \geq Q_2$ is equivalent to the inequality

$$\frac{\Sigma\, p_2\bar{x}_2}{\Sigma\, p_2x_2} - \frac{\Sigma\, p_1\bar{x}_2}{\Sigma\, p_1x_2} \geq \frac{\Sigma\, p_1(x_1 - x^*)}{\Sigma\, p_1x_2} + \frac{\Sigma\, p_2(\bar{x}_2 - x^*)}{\Sigma\, p_2x_2} - \frac{\Sigma\, p_1(\bar{x}_2 - x^*)}{\Sigma\, p_1x_2} \;.$$

The terms in the left hand side of the above inequality was defined as $\bar{I}_2$ in (2.22). The first term in the right hand side is non-positive due to the convexity of an indifference curve. But the sign of the second and third terms

$$\Sigma\, p_2(\bar{x}_2 - x^*)/\Sigma\, p_2x_2 - \Sigma\, p_1(\bar{x}_2 - x^*)/\Sigma\, p_1x_2 = -\,\mathrm{cov}_2\left(\frac{\bar{x}_2 - x^*}{x_2} \,,\, \frac{p_1}{p_2}/\frac{1}{P_P}\right)$$

is uncertain.

But to question whether or not the sign of $\underline{I}$ and $\overline{I}$ are the same as the sign of I seems to dwell on too minute details. From practical point of view, it appears extremely unlikely that the estimated results show different signs for three income effects, I, $\overline{I}$ and $\underline{I}$. This is because the covariance between relative prices and expenditure elasticities is most probably of the same sign, regardless of whether $(x^{**}/x_1)/Q_1$, $(\overline{x}_1/x_1)/Q_L$ or $(\underline{x}_1/x_1)/\underline{Q}_1$ is chosen as an arc-elasticity along the income-expenditure path of Country 1 (similarly, regardless of whether $(x^*/x_2)/Q_2$, $(\overline{x}_2/x_2)Q_P$, $(\underline{x}_2/x_2)\underline{Q}_2$ is chosen along the income-expenditure path of Country 2). Thus the positive income effects are far from being a necessary condition for $Q_L \geq Q_2$ and $Q_P \leq Q_1$.

## III

### Estimation in International Cross-section Data

Whether the income effect is positive or not in an international comparison is an open question. It may well be that the relative price of a consumer durable is high in low income countries and low in high income countries, due to the economies of scale in manufacturing industries. But take instead the medical care or the personal service (laundry, hair cuts and the like). Because of the labor intensiveness of those services its relative price is likely to be higher in a high income country than in a low income country. In abstract terms it remains inconclusive, therefore, whether or not the income effect is positive, or equivalently, whether or not the co-variance between the international price ratio of a commodity and its arc-income-elasticity of demand is negative.[12]

With the aim of finding the points of over- and under-compensation, we estimate the expenditure functions for individual groups of goods and services. We rely on Gilbert and Kravis' data and Gilbert's data which compare the consumption level between eight West European countries and

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[12]This ambiguity was already pointed out by Kuznets in his reference to the difference in the expenditure share of a commodity group between the United States and each of eight European countries, when he examined Gilbert's data. According to his calculations, the inter-country discrepancy in expenditure share is wider in the estimation in terms of the United States prices than in the estimation based on domestic prices of each country "for two of the categories characterized by high expenditure elasticity of demand--durable household goods and private passenger cars." The reverse is found "for other demand elastic categories, such as personal care and health, recreation, education and miscellaneous services." See Simon Kuznets, "Quantitative Aspects of Economic Growth of Nations: VII. The Share and Structure of Consumption," Economic Development and Cultural Change (January 1962, Part II).

Table 1

Correlation Coefficients between

Logarithm of Total Consumption Expenditure and

That of the Relative Price of Each Commodity Group

| | | |
|---|---|---|
| 10.0 | Communication Services | 0.4156 |
| 9.2 | Operation of Transport Equipment | -0.5178 |
| 12.0 | Health | 0.8232 |
| 7.0 | Household Goods | -0.7593 |
| 14.0 | Miscellaneous | 0.8793 |
| 1.7 | Non-alcoholic Beverages | -0.5435 |
| 9.1 | Purchases of Transport Equipment | -0.5667 |
| 6.0 | Fuel, light and Water | -0.7078 |
| 9.0 | Transportation | -0.6630 |
| 3.0 | Tobacco | -0.2330 |
| 4.1 | Footwear | -0.2764 |
| 4.0 | Clothing and Household Textiles, includ. Footwear | -0.4200 |
| 4.2 | Clothing and Household Textiles | -0.4224 |
| 5.0 | Housing | 0.7924 |
| 1.2 | Meats, Poultry and Fish | 0.0155 |
| 11.0 | Recreation and Entertainment | 0.5782 |
| 2.0 | Alcoholic Beverages | 0.6582 |
| 9.4 | Public Transport Services | 0.7323 |
| 13.0 | Education | 0.8156 |
| 8.0 | Household and Personal Services | 0.4169 |
| 1.3 | Dairy Products | 0.4534 |
| 1.4 | Fats and Oil | -0.1597 |
| 1.0 | Food | 0.2950 |
| 1.6 | Vegetables, Potatoes and Fruits | 0.5844 |
| 1.8 | Sugar, and Sugar Products | -0.4973 |
| 1.1 | Cereal and Cereal Products | 0.5323 |

the United States in 1950.[13] The figures of consumption expenditure are those of private households, except for the expenditure on education and medical care. And they are measured in market prices.

We assume that the per capita consumption expenditure on each commodity group of each country measured with constant prices is explained by two factors. One is the total per capita consumption expenditure of each country in terms of constant prices, and the other is the price of a commodity group in each country relative to its price in a specific country chosen as the base. Let us denote each country with superscript k (k=1,...,9), the country taken as the base for the comparison, i.e., the United States, with superscript A, the $i^{th}$ commodity group with subscript i, and the $j^{th}$ commodity within a commodity group with subscript j. With nine observations in data, we made an ordinary least square estimation on each commodity group in the log-log functional form,

$$(3.1) \quad \Sigma_j p_{ij}^A x_{ij}^k = C_i (\Sigma\Sigma p_{ij}^A x_{ij}^k)^{\alpha_i} \left( \frac{\Sigma_j p_{ij}^k x_{ij}^A}{\Sigma_j p_{ij}^A x_{ij}^A} \Big/ \frac{\Sigma\Sigma p^k x^A}{\Sigma\Sigma p^A x^A} \right)^{\beta_i}$$

Results of the estimation are presented in the appendix to this paper.

The sign of the income effect in index numbers may be reflected in the relationship between two explanatory variables in (3.1). If the income effect is positive, it implies that the higher the per capita expenditure of a country, the lower is the relative price of a luxury and the higher is the relative price of a necessity. Table 1 shows the correlation coefficients

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[13] Milton Gilbert and Irving B. Kravis, An International Comparison of National Products and the Purchasing Power of Currencies - A Study of the United States, the United Kingdom, France, Germany and Italy. (OEEC, 1954). Milton Gilbert and associates, Comparative National Products and Price Levels - A Study of Western Europe and the United States, (OEEC, 1958). See Tables 27-30 of Gilbert and Kravis and Tables 38-41 of Gilbert and associates. In the data of Gilbert and Kravis the United States per capita consumption expenditure was estimated as $1,259. But I took the figure $1,264 according to the estimation of Gilbert and associates.

between two explanatory variables in log-log form. Commodity groups are arranged in such an order that the group with the highest expenditure elasticity (Communication) appears at the top and the group with the lowest α (Cereal and Cereal Products) appears at the bottom. There are more commodity groups with a negative correlation among upper groups than groups near the bottom. On the other hand, more commodity groups with a positve correlation are found at the bottom than at the top. However, among the top groups there are some commodity groups with a positive correlation. They are rather service oriented, labor-intensive, groups such as Health and Miscellaneous. Therefore, the ambiguity remains as to the inverse relation between price ratios and income elasticities. Not until one estimates the weighted covariance, or the income effect, will the overall relationship be clear.

Our task is to find four points of over- and under-compensated change in income for each pair of eight binary comparisons between the United States and Western Europe.[14] Let $\bar{x}^E$ represent the quantity consumed per capita of a good at the point of over-compensated change in income from the United States income level due to the shift in prices to the price structure of a European country; and $\underline{x}^E$, the quantity at the corresponding point of under-compensation. Similarly, $\bar{x}^A$ denotes the point of over-compensation from a European country's income due to the price shift toward the United States price structure, and $\underline{x}^A$ the corresponding point of under-compensation. We shall explain the way in which $\bar{x}^A$ and $\underline{x}^A$ were estimated.

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[14] For the purpose of constructing index numbers it is sufficient to estimate only the points of under-compensated change in income, because Gilbert's data already presented the Laspeyres and Paache index numbers related to the points of over-compensation. But, in order to have approximate estimations of the income effect and the substitution effect, it is desirable to estimate points of both under- and over-compensation.

The two points of compensation, $\bar{x}^A$ and $\underline{x}^A$, lie on the income-expenditure path of the United States. The price variables in expenditure functions (3.1) are equal to one along this path. Thus the path is shown by

$$(3.2) \qquad \Sigma_j p^A x = \hat{C}_i (\Sigma\Sigma p^A x)^{\hat{\alpha}_i} \quad \text{for each } i.$$

The over-compensation point is defined as

$$(3.3) \qquad \Sigma\Sigma p^A \bar{x}^A \equiv \Sigma\Sigma p^A x^E$$

where $x^E$'s are the quantities at the observational point of a European country. That is, (3.3) represents the total consumption expenditure per capita of a European country measured in the U.S. prices, which is available in Gilbert's data. By substituting this total expenditure into the right-hand side of (3.2), one obtains $\Sigma_j p^A \bar{x}^A$ for each commodity group. This represents the point of over-compensation.

To estimate the point of under-compensation, one recalls its definition

$$(3.4) \qquad \Sigma\Sigma p^E \underline{x}^A \equiv \Sigma\Sigma p^E x^E .$$

We start with a tentative figure, say $\Sigma\Sigma p^A \hat{x}$, and calculate the expenditure of each commodity group, $\Sigma_j p^A \hat{x}$, from (3.2). Convert this expenditure into the expenditure in terms of the prices of a European country by using the inter-country price ratio in Gilbert's data.

$$(3.5) \qquad \Sigma_j p^E \hat{x} = (\Sigma_j p^E x^A / \Sigma_j p^A x^A)(\Sigma_j p^A \hat{x}) \quad \text{for each } i.$$

Aggregate the term in the left-hand side over commodity groups $\Sigma_i (\Sigma_j p^E \hat{x})$ and check how close this sum is to $\Sigma\Sigma p^E x^E$. Then restart these calculations with other tentative figures. After several iterations we reached the estimation close to $\Sigma\Sigma p^E x^E$ with .1 - .2% difference. In this method, however, the converting process of (3.5) is a little cheating. Ideally one should use the inter-country price ratio with quantities at the under-compensated point, $\underline{x}^A$'s, as weights. But it is impossible to obtain them because they are the goal of our estimation. So the underlying assumption of this estimation is

$$(3.6) \qquad \Sigma_j p^E \underline{x}^A / \Sigma_j p^A \underline{x}^A = \Sigma_j p^E x^A / \Sigma_j p^A x^A \ .$$

That is, the index number bias within a commodity group is not so serious that a price index number of each group will not be affected by the choice of quantities as weights, if the quantities are on the same income-expenditure path.[15]

In estimating the points of under- and over-compensation along the income-expenditure path of each country of Europe, the effect of the relative price of a commodity group of each country should be added to (3.2) to trace this path. The way in which the under-compensation point $\underline{x}^E$ was obtained is the same as the estimation method for the over-compensation point $\bar{x}^A$. The over-compensation point $\bar{x}^E$, on the other hand, was approximated by

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[15]Graphically, the shortcoming of our method can be seen in the following way. Suppose we have found the under-compensation point in terms of the U.S. prices $(\Sigma_j p^A \underline{x}^A)$ and converted it by the price ratio $(\Sigma_j p^E x^A / \Sigma_j p^A x^A)$ in (3.5). Then $(\Sigma_j p^E x^A / \Sigma_j p^A x^A)(\Sigma_j p^A \underline{x}^A) = (\Sigma_j p^E x^A)(\Sigma_j p^A \underline{x}^A / \Sigma_j p^A x^A)$. In Figures 1 and 2, one may assume that $x^A$'s and $x^E$'s are represented by the points $q_2$ and $q_1$ respectively. The above quantity ratio $\Sigma_j p^E \underline{x}^A / \Sigma_j p^A x^A$ is then shown by OG/OT = OH/O$q_2$ = ON/OC. The expenditure $\Sigma_j p^E x^A$ equals OC. The product of the two terms is thus equal to ON. We missed $\Sigma p^E \underline{x}^A$ which was equal to OS and, instead, ended up with ON. It appears that we were again trapped in the very bias we had wanted to eliminate. But the price ratio here refers not to the index number over all groups of goods and services, but to the price ratio of each commodity group. The bias is caused only by the within-group difference in quantities along an income-expenditure path.

the same iterative method with which the under-compensation point $\underline{x}^A$ was estimated. In the process of the conversion of the expenditure in terms of the U.S. prices into the expenditure in domestic prices of a European country, the quantities of this European country and not those of the U.S. should be used as weights in the price ratio. The underlying assumption in this converting process is again that the price ratio of each commodity group is not affected by the difference in quantity weights, whether the quantities are $x^E$'s, $\bar{x}^E$'s or $\underline{x}^E$'s. That is to say,

$$\Sigma_j p^E x^E / \Sigma_j p^A x^E = \Sigma p^E \bar{x}^E / \Sigma p^A \bar{x}^E = \Sigma p^E \underline{x}^E / \Sigma p^A \underline{x}^E \ .$$

Table 2 presents the index numbers of four different types. In Columns 1 and 2 are shown the index numbers of the Laspeyres and Paache formulae taken from the Gilbert data. Those in Column 1 are the index numbers measured in the United States prices, that is, $\Sigma p^A x^E / \Sigma p^A x^A = \Sigma p^A \bar{x}^A / \Sigma p^A x^A$. In analogy to the time-series of a growing economy, they may be regarded as the reciprocals of the Paache indexes. Those in Column 2 are the index numbers measured in terms of the prices of each European country, that is, $\Sigma p^E x^E / \Sigma p^E x^A = \Sigma p^E x^E / \Sigma p^E \bar{x}^E$, or the reciprocals of the Laspeyres indexes. The figures in Columns 3 and 4 are the index numbers related to the under-compensation points on the U.S. income-expenditure path, namely $\Sigma p^A \underline{x}^A / \Sigma p^A x^A$. They are all smaller than the corresponding index numbers in the first column, as they should be. It is because the numbers in the first column and the third-fourth columns measure the upper limit and the lower limit, respectively, to a true index number $\Sigma p^A x^{*A} / \Sigma p^A x^A$. Moreover, the figures in Columns 3 and 4 are all greater than the corresponding index numbers in Column 2. Therefore, this latter index number $\Sigma p^E x^E / \Sigma p^E \bar{x}^E$ not only forms the lower limit to a true index number $\Sigma p^E x^E / \Sigma p^E x^{*E}$, but is also lower than

Table 2

Per Capita Consumption Expenditures of European Countries relative to the United States in 1950 (in per cent)

| Country | Index Number in the U.S. Prices (related to the Over-Compensation Point) | Index Number in the prices of each European Country (related to the Over-Compensation Point) | Index Number in the U.S. Prices related to the Under-Compensation Point | | Index Number in the Prices of Each European Country related to the Under-Compensation Point | |
|---|---|---|---|---|---|---|
| E | $\Sigma p^A x^E / \Sigma p^A x^A = \Sigma p^A \bar{x}^A / \Sigma p^A x^A$ | $\Sigma p^E x^E / \Sigma p^E x^A = \Sigma p^E x^E / \Sigma p^E \bar{x}^E$ | $\Sigma p^A \underline{x}^A / \Sigma p^A x^A$ | | $\Sigma p^E x^E / \Sigma p^E \underline{x}^E$ | |
| | | | Based on Classification into 14 Commodity Groups | Based on Classification into 23 Commodity Groups | Based on Classification into 14 Commodity Groups | Based on Classification into 23 Commodity Groups |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| U.K. | 65.6 | 49.6 | 53.1 | 53.9 | 64.9 | 64.5 |
| Denmark | 64.5 | 51.0 | 52.0 | 52.6 | 66.3 | 66.1 |
| Belgium | 59.9 | 50.7 | 51.7 | 52.5 | 59.6 | 59.2 |
| Norway | 57.2 | 42.4 | 46.5 | 47.8 | 62.1 | 61.6 |
| France | 52.5 | 36.6 | 39.9 | 40.3 | 49.4 | 48.8 |
| Netherlands | 48.3 | 35.8 | 37.5 | 38.2 | 47.2 | 47.0 |
| W. Germany | 41.4 | 28.5 | 30.0 | 31.2 | 40.0 | 39.0 |
| Italy | 30.7 | 17.7 | 20.6 | 21.9 | 27.3 | 26.0 |

even the lower limit to another true index number $\Sigma p^A x^{*A}/\Sigma p^A x^A$. The figures in Columns 5 and 6 are the index numbers related to the under-compensation points on European income-expenditure paths, namely $\Sigma p^E x^E/\Sigma p^E \underline{x}^E$. They are all greater than the index numbers related to the over-compensation points, $\Sigma p^E x^E/\Sigma p^E \overline{x}^E$, in the second column. This is because the former forms the upper limit and the latter the lower limit to a true index number $\Sigma p^E x^E/\Sigma p^E x^{*E}$. Further, except for two countries, Denmark and Norway, the figures in the fifth-sixth columns are all smaller than the index numbers in the first column. Therefore, except for two cases this latter index number $\Sigma p^A \overline{x}^A/\Sigma p^A x^A$ not only forms the upper limit to a true index number $\Sigma p^A x^{*A}/\Sigma p^A x^A$ but is also higher than even the upper limit to another true index number.

Table 3 presents the income effect and the substitution effect according to Hicks' definition. The summation of those two effects constitutes the difference between the index numbers of the Laspeyres and Paache type. There are two ways of measuring them. One is to estimatethe income effect along the income-expenditure path of a European country ($I^E$) and the substitution effect at the welfare level of the U.S. ($S^A$). The other is to measure the income effect along the U.S. path ($I^A$) and the substitution effect at the welfare level of a European country ($S^E$). Since we did not attempt to estimate the point of compensated change in income ($x^*$'s), we substitute for it the point of under-compensation ($\underline{x}$'s) and the point of over-compensation ($\overline{x}$'s). Therefore, the effects approximated by the under-compensation point ($\underline{I}$ and $\underline{S}$) and the effects approximated by the over-compensation point ($\overline{I}$ and $\overline{S}$) were both calculated.

First, Table 3 shows that the substitution effects are all positive. This is a logical consequence of our method. We estimated the expenditure

Table 3

Estimation of the Income Effect and the Substitution Effect in Absolute Terms in the Comparison between the United States and Western Europe in 1950

(in per cent)

| Country | Overall Effect | Income Effect along European Path | | Substitution Effect at the Income Level of the U.S. | | Overall Effect | Income Effect along U.S. Path | | Substitution Effect at the Income Level of a European Country | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Approximation by Under-Compensation Point ($\underline{x}^E$) | Approximation by Over-Compensation Point ($\bar{x}^E$) | Approximation by Under-Compensation Point ($\underline{x}^E$) | Approximation by Over-Compensation Point ($\bar{x}^E$) | | Approximation by Under-Compensation Point ($\underline{x}^A$) | Approximation by Over-Compensation Point ($\bar{x}^A$) | Approximation by Under-Compensation Point ($\underline{x}^A$) | Approximation by Over-Compensation Point ($\bar{x}^A$) |
| | (1) = (2) + (4) = (3) + (5) | (2) | (3) | (4) | (5) | (6) = (7) + (9) = (8) + (10) | (7) | (8) | (9) | (10) |
| E | $\frac{\Sigma p^E x^A}{\Sigma p^E x^E} - \frac{\Sigma p^A x^A}{\Sigma p^A x^E}$ | $\underline{I}^E$ | $\bar{I}^E$ | $\underline{S}^A$ | $\bar{S}^A$ | $\frac{\Sigma p^A x^E}{\Sigma p^A x^A} - \frac{\Sigma p^E x^E}{\Sigma p^E x^A}$ | $\underline{I}^A$ | $\bar{I}^A$ | $\underline{S}^E$ | $\bar{S}^E$ |
| <u>Based on the Classification into 14 Commodity Groups</u> | | | | | | | | | | |
| U.K. | 49.3 | 5.1 | 9.7 | 44.2 | 39.6 | 16.0 | 3.5 | 1.7 | 12.5 | 14.3 |
| Denmark | 40.9 | -5.0 | -8.1 | 45.9 | 49.0 | 13.5 | 1.0 | .5 | 12.5 | 13.0 |
| Belgium | 30.4 | 2.9 | 3.3 | 27.5 | 27.0 | 9.2 | 1.0 | .0 | 8.2 | 9.2 |
| Norway | 61.0 | -4.3 | -8.9 | 65.2 | 69.9 | 14.8 | 4.1 | 1.2 | 10.7 | 13.6 |
| France | 82.3 | 21.0 | 29.2 | 61.3 | 53.1 | 15.8 | 3.3 | .6 | 12.6 | 15.2 |
| Netherlands | 72.3 | 6.1 | 9.7 | 66.3 | 62.6 | 12.5 | 1.6 | 1.5 | 10.9 | 11.0 |
| W. Germany | 109.9 | 7.5 | 8.7 | 102.3 | 101.2 | 12.9 | 1.6 | 1.2 | 11.4 | 11.8 |
| Italy | 239.0 | 50.8 | 83.0 | 188.3 | 156.0 | 13.0 | 2.9 | 2.5 | 10.1 | 10.5 |
| <u>Based on the Classification into 23 Commodity Groups</u> | | | | | | | | | | |
| U.K. | 49.3 | 6.0 | 10.0 | 43.3 | 39.3 | 16.0 | 4.3 | 2.7 | 11.7 | 13.3 |
| Denmark | 40.9 | -4.5 | -6.1 | 45.4 | 47.1 | 13.5 | 1.6 | 1.3 | 11.9 | 12.2 |
| Belgium | 30.4 | 3.9 | 5.2 | 26.5 | 25.1 | 9.2 | 1.8 | .8 | 7.4 | 8.4 |
| Norway | 61.0 | -2.7 | -5.9 | 63.6 | 66.8 | 14.8 | 5.4 | 2.8 | 9.4 | 12.0 |
| France | 82.3 | 23.7 | 33.7 | 58.6 | 48.6 | 15.8 | 3.6 | 1.1 | 12.2 | 14.7 |
| Netherlands | 72.3 | 7.0 | 11.5 | 65.3 | 60.8 | 12.5 | 2.4 | 1.9 | 10.1 | 10.7 |
| W. Germany | 109.9 | 13.6 | 20.3 | 96.2 | 89.6 | 12.9 | 2.8 | 2.5 | 10.2 | 10.5 |
| Italy | 239.0 | 70.4 | 110.4 | 168.6 | 128.7 | 13.0 | 4.2 | 3.9 | 8.8 | 9.1 |

functions on international cross-section data and, therefore, assumed the common taste among nations. Second, the income effects along the U.S. income-expenditure path ($I^A$) are all non-negative. Further, this income effect is greater when it is approximated by the under-compensation point than by the over-compensation point ($\underline{I}^A > \bar{I}^A$).[16] Fourth, the income effects along European income-expenditure paths ($I^E$) are positive except for two cases, Denmark and Norway. The negative income effects for Denmark and Norway were anticipated when the index numbers in Column 1 and Columns 5-6 of Table 2 showed the relationship $\Sigma p^A x^E / \Sigma p^A x^A < \Sigma p^E x^E / \Sigma p^E \underline{x}^E$. It is because this relationship is equivalent to $\underline{I}^E < 0$. Further, to six countries with positive $I^E$, their income effect is greater when it is approximated by an over-compensation point than by an under-compensation point ($\bar{I}^E > \underline{I}^E$). For Denmark and Norway where the negative $I^E$ is found, on the other hand, the income effect is smaller (algebraically) when it is approximated by an over-compensation point than by an under-compensation point ($\bar{I}^E < \underline{I}^E$).[17] Fifth, although the income effects are generally positive, they are far smaller than the positive substitution effects.

When the income effect and the substitution effect are calculated in absolute terms as in Table 3, it is difficult to compare each effect among

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[16]Since $\underline{I}^A$ and $\bar{I}^A$ are defined as $\underline{I}^A \equiv \Sigma p^A \underline{x}^A / \Sigma p^A x^A - \Sigma p^E \underline{x}^A / \Sigma p^E x^A$ and $\bar{I}^A \equiv \Sigma p^A \bar{x}^A / \Sigma p^A x^A - \Sigma p^E \bar{x}^A / \Sigma p^E x^A$, it follows that $\underline{I}^A - \bar{I}^A = \Sigma p^A (\underline{x}^A - \bar{x}^A) / \Sigma p^A x^A - \Sigma p^E (\underline{x}^A - \bar{x}^A) / \Sigma p^E x^A = -\mathrm{cov}_A \left( \frac{\underline{x}^A - \bar{x}^A}{x^A}, \ \frac{p^E}{p^A} \Big/ \frac{\Sigma p^E x^A}{\Sigma p^A x^A} \right)$

Therefore $\underline{I}^A \gtrless \bar{I}^A$ for $\mathrm{cov} \lesseqgtr 0$.

As $\underline{I}^A$ and $\bar{I}^A$ are both found to be non-negative, it is almost certain that this covariance is non-positive.

[17]The same as in Footnote 16 can be said as to the relationship between the inequality $\bar{I}^E \gtrless \underline{I}^E$ and the sign of a covariance.

countries. This is because the greater the difference in income level between two countries the wider may well be the disparity between the Laspeyres and Paache index numbers. Then the income effect and the substitution effect will not be independent of the scale. Further, Table 3 does not allow one to compare between the income effect along a European path and the income effect along the U.S. path. With this consideration Table 4 presents the income effect in relative terms. (Marris' definition.) This income effect is equivalent to the covariance between the arc-expenditure-elasticities and the relative prices of commodities, with the difference only in sign. In Table 4, as in the preceding tables, eight countries are ordered according to their per capita consumption expenditure (in terms of U.S. prices) in 1950. Column 1 presents the overall effect in relative terms, $(Q_L - Q_P)/Q_L$, which is equivalent to $-\text{cov}_A\left(\frac{x^E}{x^A}\Big/\frac{\Sigma p^A x^E}{\Sigma p^A x^A}, \frac{p^E}{p^A}\Big/\frac{\Sigma p^E x^A}{\Sigma p^A x^A}\right) = -\text{cov}_E\left(\frac{x^A}{x^E}\Big/\frac{\Sigma p^E x^A}{\Sigma p^E x^E}, \frac{p^A}{p^E}\Big/\frac{\Sigma p^A x^E}{\Sigma p^E x^E}\right)$.
Column 2 shows $\underline{i}^E$, the income effect in relative terms approximated by the under-compensation point along a European path. This effect is equivalent to $-\text{cov}_E\left(\frac{\underline{x}^E}{x^E}\Big/\frac{\Sigma p^E \underline{x}^E}{\Sigma p^E x^E}, \frac{p^A}{p^E}\Big/\frac{\Sigma p^A x^E}{\Sigma p^E x^E}\right)$. The figures in Column 3 are the same effect approximated by the over-compensation point, $\overline{x}^E$. Column 4 presents $\underline{i}^A$, the income effect in relative terms along the U.S. path approximated by the under-compensation point. This effect is equivalent to
$-\text{cov}_A\left(\frac{\underline{x}^A}{x^A}\Big/\frac{\Sigma p^A \underline{x}^A}{\Sigma p^A x^A}, \frac{p^E}{p^A}\Big/\frac{\Sigma p^E x^A}{\Sigma p^A x^A}\right)$. Column 5 shows the same effect approximated by the over-compensation point $\overline{x}^A$. Comparing eight figures in each column, one may find the general pattern that the lower the per capita consumption of a country the greater is the income effect in relative terms. In other words, the inverse correspondence between arc-income-elasticities and

Table 4

Estimation of the Income Effects in Relative Terms

(in per cent)

| Country | Overall Effect | Income Effect along European Path | | Income Effect along the U.S. Path | |
|---|---|---|---|---|---|
| | | Approximation by Under-Compensation Point ($\underline{x}^E$) | Approximation by Over-Compensation Point ($\bar{x}^E$) | Approximation by Under-Compensation Point ($\underline{x}^A$) | Approximation by Over-Compensation Point ($\bar{x}^A$) |
| | (1) | (2) | (3) | (4) | (5) |
| E | $\left(\frac{\Sigma p^A x^E}{\Sigma p^A x^A} - \frac{\Sigma p^E x^E}{\Sigma p^E x^A}\right) \Big/ \frac{\Sigma p^A x^E}{\Sigma p^A x^A}$ | $\underline{i}^E$ | $\bar{i}^E$ | $\underline{i}^A$ | $\bar{i}^A$ |
| **Based on Classification into 14 Commodity Groups** | | | | | |
| U.K. | 24.4 | 3.2 | 4.8 | 6.6 | 2.6 |
| Denmark | 20.9 | -3.3 | -4.1 | 2.0 | .8 |
| Belgium | 15.4 | 1.7 | 1.7 | 1.9 | .0 |
| Norway | 25.9 | -2.5 | -3.8 | 8.7 | 2.1 |
| France | 30.2 | 9.9 | 10.7 | 8.2 | 1.1 |
| Netherlands | 25.9 | 2.8 | 3.5 | 4.4 | 3.2 |
| W. Germany | 31.3 | 3.0 | 2.5 | 5.2 | 2.8 |
| Italy | 42.3 | 13.5 | 14.7 | 14.0 | 8.0 |
| **Based on Classification into 23 Commodity Groups** | | | | | |
| U.K. | 24.4 | 3.8 | 5.0 | 8.0 | 4.1 |
| Denmark | 20.9 | -3.0 | -3.1 | 3.0 | 2.0 |
| Belgium | 15.4 | 2.3 | 2.7 | 3.4 | 1.4 |
| Norway | 25.9 | -1.5 | -2.5 | 11.2 | 4.8 |
| France | 30.2 | 11.1 | 12.4 | 9.0 | 2.2 |
| Netherlands | 25.9 | 3.3 | 4.1 | 6.3 | 3.8 |
| W. Germany | 31.3 | 5.3 | 5.8 | 8.8 | 5.9 |
| Italy | 42.3 | 17.8 | 19.5 | 19.1 | 12.6 |

relative prices is more pronounced in Italy-U.S. comparison than in U.K.-U.S. comparison. This pattern is not detected very clearly, however. There are some exceptions, notably France.[18]

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[18] The division into the income effect and the substitution effect varies with the degree of aggregation of commodity groups. We classified total consumption expenditure into 14 groups in one estimation. Further disaggregation into 23 groups was used in the other estimation. As Tables 3 and 4 show, the estimation of income effect is always greater (algebraically) when the total consumption is divided into 23 groups than into 14 groups. This result seems quite plausible. As we explained, the income effect can be expressed by the covariance between elasticities and prices of individual commodities. When individual commodities are grouped together and the total expenditure is composed of several commodity groups, this covariance of individual commodities can also be expressed as the between-group covariance plus the weighted mean of within-group covariances (with expenditure share of each group as weight). Since it is likely that both between-group covariance and within-group covariance are negative, the absolute value of covariance of individual goods is greater than the absolute value of between-group covariance. Therefore, the more disaggregated the commodity groups the greater is the income effect. But this difference in income effect tapers off with further disaggregation. Indeed, if there is a proportionate change either in all prices (as in Hicks' definition of the commodity group), or in all quantities within a group, then the within-group covariance vanishes.

IV

## Concluding Remarks

Our findings in the preceding section must be taken with reservation due to some difficulties in estimating the points of compensated change in income. Even within the classical, static framework, the expenditure functions we estimated do not contain the prices of related commodity groups. And, as the table in appendix shows, some $\bar{R}^2$'s are low. Further, the functional form of (3.1) assumes constant elasticities. Thus it is only at the geometric average of observations that the weighted mean of expenditure elasticities equals unity. But the expenditure shares of luxuries increase as the income level rises. Around the points $\underline{x}^E$ and $\bar{x}^E$ near the income level of the United States, the weighted mean of expenditure elasticities exceeds unity and, therefore, the sum of dependent variables exceeds the value of independent variable. For the same reason the sum of dependent variables falls short of the value of independent variable around some of the low points $\underline{x}^A$ and $\bar{x}^A$. Although we assumed the sum of dependent variables as the total expenditure in our estimation of the compensation points, much work remains to improve the estimation method.[19]

Even in the present functional form the estimation will be improved if the household budget data of individual countries are pooled. Then one may add dummy variables representing different countries to the expenditure

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[19] In estimating $\bar{x}^A$ we assume the value in the left-hand side of $p_i^A \bar{x}_i^A = C_i(\Sigma p_i^A x_i)^{\alpha_i}$ as the over-compensation point. There $\Sigma p^A \bar{x}^A$ equals $\Sigma p^A x^E$ by definition. Similarly, in estimating $\underline{x}^A$ we assume the value in the left-hand side of $p_i^A \underline{x}_i^A = C_i(\Sigma p_i^A x_i)^{\alpha_i}$ as the under-compensation point. There $\Sigma(p_i^E/p_i^A)(p_i^A \underline{x}_i^A)$ equals $\Sigma p^E x^E$ by definition.

elasticity in (3.1) (possibly, dummies also to the intercept), and seek for the compensation point along each country's income-expenditure path with different expenditure elasticities. If this is done, the sign of the substitution effect will gain the meaningfulness as a test for the similarity of tastes among nations. With no available data this approach was not taken.[20]

With these limitations in mind we summarize the findings in the preceding section.

(1) Index numbers in terms of European prices in Gilbert's data $(\Sigma p^E x^E / \Sigma p^E x^A)$ are lower than index numbers in the U.S. prices related to the under-compensation points $(\Sigma p^A \underline{x}^A / \Sigma p^A x^A)$. Since the latter index numbers form the lower limits to true index numbers in the U.S. prices, one can conclude that those index numbers in Gilbert's data not only form the lower limits to true index numbers in terms of European prices but are also lower than even the lower limits to another true index number.

(2) Except for two cases (Denmark and Norway), index numbers in the United States prices in Gilbert's data $(\Sigma p^A x^E / \Sigma p^A x^A)$ are higher than index numbers in European prices related to the under-compensation points $(\Sigma p^E x^E / \Sigma p^E \underline{x}^E)$. Since the latter index numbers form the upper limits to true index numbers in European prices, one can conclude that those six index numbers (out of

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[20] A tentative result of our comparison of urban consumption between Russia in 1913 and the United States in 1901 shows that the substitution effects are positive and thus suggests that one cannot reject the hypothesis of the same tastes between two nations under comparison. The family budget data of each country were used to trace each income-expenditure path. This way of testing the hypothesis on similarity in tastes is in the same spirit as the ordinary test by weak axiom. But the test based on the sign of the substitution effect is useful when the income level of one country turns out to be higher than the level of the other, no matter which country's prices are chosen as weights.

eight) in Gilbert's data not only form the upper limits to true index numbers in terms of the U.S. prices but are also higher than even the upper limits to another true index number. From (1) and (2), therefore, one may say that by and large both true index numbers are located within the range between the Laspeyres and Paache index numbers.

(3) Approximate income effects along the U.S. income-expenditure path are all non-negative. Approximate income effects along European income-expenditure paths are also non-negative for six countries. However, the income effects are so small that they are outweighed by the substitution effects.

(4) When the income effect is put in relative term, the positive income effect is equivalent to the negative covariance between the price ratios of two countries and the arc-expenditure-elasticities from an observation point to a compensation point (Marris' finding). There is some evidence that this covariance is the higher (in absolute value), the lower the income level of a country under comparison with the United States.

In international comparisons as well as time series data, it is widely found that the Laspeyres index number is greater than the Paache index number. From (2.7) and (2.11), this means that the covariance between relative prices and quantity ratios comparing two observational points is predominantly negative. In contrast, the positiveness of the income effect, or the negative covariance between relative prices and income elasticities, is a more restrictive hypothesis. The limitation to the universal acceptability of this hypothesis in regard to time is evident, as the hypothesis relies on the effect of economies of scale and technological progress to the prices of manufactured consumer goods. Not until the Industrial Revolution did the production process of consumer goods change dramatically and exerted the different degrees of economies-of-scale effect to consumer durables and food. The hypothesis is also restrictive in terms of space. When one compares the consumption pattern between a communist country and a capitalist country, the result might be contrary to what is found in the comparison between capitalist countries. One reason for this conjecture is that the structure of indirect taxes may be more progressive in a communist country than in a capitalist country from the consideration of social equity and work incentives.[21] The other reason is the lag of technological change in light industries behind the progress in heavy industries in communist countries. As a consequence of these two factors, the income effect may turn out to be negative if one compares the consumption between a communist country and a capitalist country whose income level is lower than the level of the former.

In his monumental work on the sources for the differences in growth rates between the United States and Western Europe, Denison ascribed one major source for the differences in growth rates to what he called "economies of scale associated with income elasticities."[22] These "economies of scale" were

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[21] The question whether the Soviet turn-over tax structure has been regressive or progressive is still an unsettled question, since one has no access to the family budget data in the Soviet period which disclose the detailed consumption pattern of the families in differenct income levels. Among the economists interested in this matter, Dobb thinks that the turn-over tax structure was progressive. And he dismisses the charge of high tax rates on agricultural foodstuff on the ground that they did not lead to high retail prices, but were caused by low procurement prices. Holzman maintains, however, that the issue is not that simple. His findings on the turn-over tax structure in 1935-37 conclude that the intracommodity rates (rates on different grades and qualities in a commodity) were progressive, whereas the intercommodity rates for industrial products had no clearly defined pattern. The intercommodity rates for agricultural products, however, were marked by a high tax rate on bread, the most important product in the Russian diet. But in the years since 1950 the prices and taxes on food products relative to industrial consumer goods have been declining rapidly. And this has contributed to the increasing progressiveness of the tax structure. See Maurice Dobb, Soviet Economic Development since 1917, (New York, 1948), pp. 371-372, footnote, and F. D. Holzman, Soviet Taxation, The Fiscal and Monetary Problems of a Planned Economy (Cambridge, U.S.A., 1955), pp. 146-158.

[22] Edward F. Denison, assisted by Jean-Pierre Poullier, Why Growth Rates Differ (Washington, 1967), Chapter 17.

found as the gap between the European growth rates of consumption valued in each country's own prices and their growth rates measured in the United States prices. Since Denison's arguments were drawn from Gilbert's data of purchasing power parities, a comparison with our findings based on the same data would be of some interest.

His view may be summarized as follows. (1) The growth rates of European countries are higher when they are measured in European prices than in the U.S. prices. (2) This discrepancy between two measurements of the growth rate can chiefly be attributed to "economies of scale associated with income elasticities." This is due to the fact that "As European per capita consumption has risen, the increases have been concentrated in products that have high income elasticity and high relative prices in Europe as compared with the United States."[23] (3) The attribution of this discrepancy to a scale effect can be evidenced by the empirical finding that the discrepancy is the larger, the lower the per capita income level and the higher the rate of growth of a European country.

This first point has long been known as "the Gerschenkron hypothesis" in index numbers by economists working on the Soviet data. The output index of the Soviet heavy industry for the period of first two Five-Year Plans was constructed by Gerschenkron with the use of prices prevailing in the United States in 1939. This estimation of the Soviet industrial growth turned out to be much lower than the official index number in terms of 1926-27 Soviet prices. This discrepancy, Gerschenkron argued, may not have been wholly due to "bias" of the official index number, but to economic factors as well, since

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[23]Edward F. Denison, "Sources of Postwar Growth in Nine Western Countries," American Economic Review, Vol. LVII, No. 2 (May 1967), pp. 330-331.

the price structure of the United States industry resembled more the post-industrialization prices than the pre-Five-Year-Plan prices of the Soviet industry.[24]

A difficulty arises in Denison's second point, where he attempted to interpret the disparity in growth rates and attributed it to "economies of scale associated with income elasticities." It is clear that Denison's "economies of scale associated with income elasticities" is nothing but a positive income effect, or a negative covariance between income elasticities and price ratios. But when the consumption of expenditure on automobiles is higher in the United States than in Italy not only because the income level is higher but also the relative price is lower in the former than in the latter, one should have subtracted from the difference in the consumption the portion attributed to the difference in price and then should have mentioned "the income elasticities". Denison is ambiguous in his distinction between the overall effect--the covariance between price ratios and quantity ratios actually consumed in two countries--and the income effect - the covariance between price ratios and income elasticities. This ambiguity seems to have led a reviewer to sense "an element of cheating," although he considered Denison's reasoning as "the most controversial and theoretically novel section in the book".[25]

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[24]Alexander Gerschenkron, assisted by Alexander Erlich, A Dollar Index of Soviet Machinery Output (Santa Monica, 1951). See also Alexander Gerschenkron "Soviet Heavy Industry: A Dollar Index of Output, 1927/28-1937," Review of Economics and Statistics (May 1955), reprinted in Economic Backwardness in Historical Perspective (Cambridge, U.S.A., 1962); Ira O. Scott, Jr., "Gerschenkron Hypothesis of Index Number Bias," Review of Economics and Statistics, Vol. XXXIV, (November 1952); Abram Bergson, The Real National Income of Soviet Russia Since 1928 (Cambridge, U.S.A., 1961), p. 33.

[25]R. C. O. Matthews, "Why Growth Rates Differ," Review Article, Economic Journal, Vol. LXXIX No. 314 (June 1969), pp. 261-268.

However, Denison was apparently aware of the fact that the disparity consisted of two components: the substitution effect and the income effect.[26] But he seems to have been convinced that the substitution effect in inter-country comparison would be negligibly small if the per capita consumption level of the United States and European countries are close together. He reached this convergence hypothesis when he extrapolated the relationship between indexes of Laspeyres and Paache formulae of eight binary comparisons to a hypothetical situation where a country stands at the same level of per capita consumption as the United States. His estimation of the log-log regression on Gilbert's data shows that if a hypothetical country is at the same per capita consumption level as the United States in terms of the prices of that country, its consumption level measured in the United States prices will be 101.5% of the United States. Conversely, if the consumption levels of two countries are the same in terms of the United States prices, the consumption level of that hypothetical country will be 98% of the United States in terms of the prices of that country. The disparity between the two comes to a very small magnitude. Since virtually no income effect is existing in this hypothetical situation, this very small gap means that the substitution effect is also negligibly small.

No one can test Denison's convergence hypothesis directly, since there is no country whose income level matches the level of the United States. What we did in the preceding section was to estimate a European consumption pattern at the point where their income level parallels the level of the United States under the assumption that European price structure remained the same. Naturally our estimation of the substitution effect was large. This is all one can do from cross-sectional data with the United States as the

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[26] Denison, Why Growth Rates Differ, p. 236.

basis for comparison. Quite apart from testing the convergence hypothesis, if one is interested in whether the "economies of scale associated with income elasticities" have been higher, say, in Italy than in the United States, one may estimate the demand functions on time-series data of each country, calculate the income effect between the initial period and the terminal period for each series and compare the effect between two countries. The difficulty in extrapolating the cross-sectional findings and attaching them the implication of the change over time is too well known. As the proposer of "the Gerschenkron hypothesis" cautioned, "the assumed affinity between the temporal and spatial aspects of the index number problem implies that backward countries follow in their development charted by the more advanced countries. This is only a half truth at best."[27]

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[27] Gerschenkron, Economic Backwardness in Historical Perspective, p. 253.

# APPENDIX

## Estimation of Expenditure Functions in the International Cross-sectional Data of Gilbert

Results of the estimation in the double-log expenditure functions (3.1) are presented in the following table. The expenditure elasticities are all positive, ranging from the low figure for the expenditure on food, particularly on cereal and cereal products, to the high figure for the expenditure on household goods, medical services and the purchase and operation of transportation equipment. Although there are no inferior commodity groups in the Table, there are five groups where the hypothesis of zero expenditure elasticity cannot be rejected with 90% significance level. The elasticities with respect to the price of its own group are negative for all groups except for one (Miscellaneous). But there are many commodity groups whose demand is inelastic to the price change. The hypothesis of zero price elasticity cannot be rejected at 90% significance level for as many as 16 commodity groups out of 26 in total. The positive own-price elasticity of the Miscellaneous groups is awkward. But this group is rather a heterogeneous group of goods and services, which is comprised of the net tourist expenditure abroad, legal fees, burial expenses, etc. Further, the estimated price elasticity is so small that, when the price variable is dropped off, the estimated expenditure elasticity increases only slightly, from 2.2620 in the following Table to 2.2811 (with standard error 0.8027). Therefore, whether one retains the price variable or eliminates it has very little effect on the estimation of compensation points.

Table

Estimates of Expenditure Functions

under the Assumption of Constant Elasticities

| | | Expenditure elasticity |
|---|---|---|
| 1.0 | Food | 0.5491<br>(0.0383) |
| 1.1 | Cereal and cereal products | 0.0066<br>(0.1398) |
| 1.2 | Meats, poultry and fish | 1.1545<br>(0.1395) |
| 1.3 | Dairy products | 0.7196<br>(0.1924) |
| 1.4 | Fats and oil | 0.6943<br>(0.2653) |
| 1.5 | Vegetables, potatoes and fruits | 0.4862<br>(0.2860) |
| 1.6 | Non-alcoholic beverages | 1.7057<br>(0.3035) |
| 1.7 | Sugar and sugar products | 0.1220<br>(0.2688) |
| 2.0 | Alcoholic beverages | 1.0706<br>(0.8453) |
| 3.0 | Tobacco | 1.4122<br>(0.4149) |
| 4.0 | Clothing and household textiles | 1.1998<br>(0.2180) |
| 4.1 | Footwear | 1.4118<br>(0.1363) |
| 4.2 | Clothing and household textiles | 1.1851<br>(0.2500) |
| 5.0 | Housing | 1.1816<br>(0.2518) |

Table (continued)

| | Own-price elasticity | $R^2$ corrected with degree of freedom |
|---|---|---|
| 1.0 | -0.2663<br>(0.1566) | 0.9588 |
| 1.1 | -0.1648<br>(0.2425) | -0.3636 |
| 1.2 | -0.5935<br>(0.2106) | 0.8898 |
| 1.3 | -0.1265<br>(0.3610) | 0.5960 |
| 1.4 | -0.4103<br>(0.3547) | 0.4154 |
| 1.5 | -1.5120<br>(0.4784) | 0.4383 |
| 1.6 | -0.4380<br>(0.2667) | 0.8694 |
| 1.7 | -1.5750<br>(0.4090) | 0.6818 |
| 2.0 | -1.1992<br>(0.5329) | 0.1925 |
| 3.0 | -0.3019<br>(0.3054) | 0.5703 |
| 4.0 | -0.6022<br>(0.6110) | 0.8181 |
| 4.1 | -0.3488<br>(0.2207) | 0.9332 |
| 4.2 | -0.5023<br>(0.7085) | 0.7595 |
| 5.0 | -0.0402<br>(0.1598) | 0.8508 |

Table (continued)

| | | Expenditure elasticity |
|---|---|---|
| 6.0 | Fuel, light and water | 1.5268<br>(0.3478) |
| 7.0 | Household goods | 2.2720<br>(0.7343) |
| 8.0 | Household and personal services | 0.8455<br>(0.1746) |
| 9.0 | Transport equipment and services | 1.4813<br>(0.2906) |
| 9.1 | Purchase of transport equipment | 1.6066<br>(0.7368) |
| 9.2 | Operation of transport equipment | 2.8218<br>(0.4307) |
| 9.3 | Public transport services | 0.9703<br>(0.4611) |
| 10.0 | Communication services | 2.9713<br>(0.2798) |
| 11.0 | Recreation and entertainment | 1.1114<br>(0.3542) |
| 12.0 | Health | 2.8092<br>(0.6265) |
| 13.0 | Education | 0.8715<br>(0.3401) |
| 14.0 | Miscellaneous | 2.2620<br>(1.8205) |

Table (continued)

| | Own-price elasticity | $R^2$ corrected with degree of freedom |
|---|---|---|
| 6.0 | -0.9884<br>(0.3053) | 0.9151 |
| 7.0 | -1.2614<br>(1.1654) | 0.7925 |
| 8.0 | -0.6475<br>(0.2998) | 0.6945 |
| 9.0 | -0.8072<br>(0.5248) | 0.8802 |
| 9.1 | -3.7629<br>(1.1500) | 0.7788 |
| 9.2 | -1.5966<br>(0.6635) | 0.9051 |
| 9.3 | -0.9701<br>(0.7346) | 0.1456 |
| 10.0 | -0.8697<br>(0.3015) | 0.9260 |
| 11.0 | -0.2989<br>(0.3719) | 0.4822 |
| 12.0 | -2.2171<br>(1.0492) | 0.7340 |
| 13.0 | -0.4478<br>(0.3972) | 0.4131 |
| 14.0 | 0.0474<br>(3.9850) | 0.3036 |