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Economic Growth and International Trade with Transport Costs

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I. Introduction

The influence of economic growth on international trade and *vice versa* has been examined fairly often using either a comparative-static or a dynamic type of analysis. The dynamic approach was taken, e.g., by D. M. Bensusan-Butt, J. Bhagwati, J. Black, H. Brems, H. G. Johnson, J. Schumann, and T. N. Srinivasan.[1] But only in 1965/66 three articles were published by H. Oniki and H. Uzawa and by P. K. Bardhan[2] dealing with "fully-fledged" neo-classical two-country two-commodity two-factor models.

Oniki, Uzawa and Bardhan consider two economies having the usual neo-classical properties which may exchange newly produced goods without trade impediments. International factor migrations and investment abroad, however, are excluded.[3] They first analyze the relationship between the national total factor intensities and the characteristics of the corresponding two-country equilibria and then the variation in time of these intensities as brought about by net investment and the increase of the labour forces. Furthermore, assuming proportionate saving functions, they show that the system's set of steady states is non-empty and globally stable provided that the exogenously given growth rates of the labour supply are the same in both countries, and that either the consumption good is always more capital intensive than the capital good or that otherwise all elasticities of substitution are not smaller than unity. If, in addition, the countries have identical production techniques and the same saving functions they will possess a steady autarky state and, therefore, the per-capita volume of international trade will decrease to zero at least from some range of initial positions. Obviously, international differences in technology and saving behaviour normally will ensure the possibility of trade between the countries in the long run, the degree and form of specialization depending on the amount and nature of these differences.

In the present paper we study a model of the Bardhan-Oniki-Uzawa type with international trade impediments in the form of transport costs and with a third industry, the transport industry.[4] The case of import or export tariffs can be dealt with in a similar but much more simpler manner if we suppose that no factors of production are needed to collect these tariffs.[5] The main results are the same as those given below.

II. The Main Assumptions

We make the following assumptions:

i) There are two countries $i = 1,2$ which may trade with each other.

ii) In each country three homogeneous, internationally identical commodities $j = 1,2,3$ may be produced. These are the capital good, the consumption good and the transportation service, respectively. The two goods are intranationally completely but internationally incompletely mobile.

iii) To ship a unit of good j into country i σ_{ij} units of transportation services are needed ($\sigma_{ij} > 0$) which have to be supplied by the importing country i.[6]

iv) The factors of production, capital and labour, are malleable, intranationally completely mobile and internationally completely immobile. Their supply has a zero price elasticity.

v) The production functions, being defined for all positive, finite input values, are linear homogeneous and have continuous partial derivatives of the first and second order. Their iso-product curves are strictly concave from above.

vi) Perfect competition prevails within each country as well as between them.

vii) In both countries the same abstract unit of account is used to measure "monetary" quantities.[7] A positive demand for a commodity in a country implies that its accounting price is there greater than zero.

viii) All labour income is consumed and all capital income invested in the country in which they originate.

ix) Commodity inventories do not exist.

x) The balance of payments of each country is always balanced.

xi) The rates of growth of the labour forces are positive and constant.

xii) The capital stocks depreciate at positive, constant rates.

xiii) For every factor price ratio is the capital intensity in the transportation industry lower than in the consumption good industry and higher than in the capital good industry.[8,9]

It should be noted that we do not assume that the production functions of an industry, the growth rates of the labour force and the depreciation rates of the capital stock to be the same in both countries.

The following notations will be used:

K_i, A_i	total supply of capital and labour, respectively, in country i
K_{ij}, A_{ij}	input of capital and labour, respectively, in industry j of country i (i.e., in industry ij)
$\rho_i = K_i/A_i$	total capital intensity in country i
$\rho_{ij} = K_{ij}/A_{ij}$	capital intensity of industry ij
F_{ij}	production function of industry ij
Y_{ij}	rate of output of industry ij
N_{ij}	rate of total demand for commodity j in country i
N^a_{ij}	rate of import demand ($N^a_{ij} > 0$) or export supply ($N^a_{ij} < 0$) of good j in country i

R_i, L_i monetary capital rent and monetary wage rate, respectively, in country i

r_{ij}, ℓ_{ij} marginal physical product of capital and labour, respectively, in industry ij

P_{ij} accounting price of commodity j in country i

$p_{ij} = P_{ij}/P_{i1}$ relative price of commodity j in country i in terms of the capital good

λ_i rate of growth of the labour force in country i

δ_i rate of capital depreciation in country i

$a_{ij} = A_{ij}/A_i$, $n^a_{ij} = N^a_{ij}/A_i$, $f_{ij}(\rho_{ij}) = F_{ij}(\rho_{ij},1)$

These assumptions imply that the following basic relations hold: (If not otherwise stated the index i takes the values 1, 2 and the index j the values 1, 2, 3).

Assumption (v).[10]

$$r_{ij} = \frac{\partial F_{ij}}{\partial K_{ij}} = f'_{ij}, \quad \ell_{ij} = \frac{\partial F_{ij}}{\partial A_{ij}} = f_{ij} - \rho_{ij} f'_{ij} \tag{1}$$

$$\left.\begin{array}{l} 0 < f_{ij} < \infty, \quad 0 < f'_{ij} < \infty , \\ 0 < f_{ij} - \rho_{ij} f'_{ij} < \infty, \ - \infty < f''_{ij} < 0 \end{array}\right\} \ 0 < \rho_{ij} < \infty \tag{2}$$

For certain types of production functions, e.g., for CES functions with an elasticity of substitution greater than unity, industry ij might produce with one factor only. At $\rho_{ij} = 0$ and $\rho_{ij} = \infty$, however, the values of the functions in (2) may become zero or infinity. Since this possibility would cause some complications in our analysis we suppose that the sectoral factor intensities take only positive, finite values.

Assumptions (iii, vi, vii):

$$P_{kj} = P_{ij} - \sigma_{ij}P_{i3} > 0, \quad i \neq k, \; j = 1,2, \text{ if } N^a_{ij} > 0 \tag{3}$$

i.e., the accounting prices of a good which is internationally traded differ in the import and the export country by the transport costs.

$$N_{ij} = Y_{ij} + N^a_{ij}, \quad j = 1,2 \tag{4}$$

$$N_{i3} = Y_{i3} = \sigma_{i1}\max(N^a_{i1},0) + \sigma_{i2}\max(N^a_{i2},0) \tag{5}$$

$$N^a_{1j} + N^a_{2j} = 0, \quad j = 1,2 \tag{6}$$

$$P_{ij}r_{ij} = R_i, \quad P_{ij}\ell_{ij} = L_i, \quad \text{if } A_{ij} > 0 \tag{7}$$

$$K_{i1} + K_{i2} + K_{i3} = K_i, \quad A_{i1} + A_{i2} + A_{i3} = A_i \tag{8}$$

Assumption (viii):

$$P_{i1}N_{i1} = R_iK_i, \quad P_{i2}N_{i2} = L_iA_i \tag{9}$$

Assumption (x):

Either

$$N^a_{i1} = N^a_{i2} = 0 \tag{10}$$

or

$$N^a_{i1} > 0, \; N^a_{i2} < 0, \; (P_{i1} - \sigma_{i1}P_{i3})N^a_{i1} + P_{i2}N^a_{i2} = 0 \tag{11}$$

or

$$N^a_{i1} < 0, \; N^a_{i2} > 0, \; P_{i1}N^a_{i1} + (P_{i2} - \sigma_{i2}P_{i3})N^a_{i2} = 0 \tag{12}$$

i.e., a country is either self-sufficient or imports one good and exports the other. In the second case its expenditures for imports net of transport costs equal its receipts from exports.

Assumptions (xi, xii):

$$\dot{A}_i = \lambda_iA_i \quad (13) \qquad \dot{K}_i = N_{i1} - \delta_iK_i = \left(\frac{R_i}{P_{i1}} - \delta_i\right)K_i \tag{14}$$

Assumption (xiii):

$\rho_{i2} > \rho_{i3} > \rho_{i1}$ if country i is incompletely specialized[11] (15)

$\rho_{i2} > \rho_{i3}$ if country i produces no capital goods (16)

$\rho_{i3} > \rho_{i1}$ if country i produces no consumption goods (17)

III A Single Open Economy - Incomplete Specialization

Consider first one country only and suppose that it is incompletely specialized. Relations (1) and (7) imply:

$$f'_{i1} = p_{i2}f'_{i2} = p_{i3}f'_{i3} \tag{18}$$

$$f_{i1} - \rho_{i1}f'_{i1} = p_{i2}(f_{i2} - \rho_{i2}f'_{i2}) = p_{i3}(f_{i3} - \rho_{i3}f'_{i3}) \tag{19}$$

$$\frac{\ell_{i1}}{r_{i1}} = \frac{\ell_{ij}}{r_{ij}}, \quad j = 2,3 \tag{20}$$

The equations in (20) have a solution if, and only if, the three marginal product ratios take the same values for at least some values of the sectoral capital intensities. We can simplify the analysis without altering the main results by making

Assumption (xiv): For $0 < \rho_{i1}, \rho_{i2}, \rho_{i3} < \infty$ the ratios ℓ_{i1}/r_{i1}, ℓ_{i2}/r_{i2}, and ℓ_{i3}/r_{i3} have the same range of values.[12]

From (1), (2), (15), (18)-(20) follows:

$$0 < \frac{d\rho_{ij}}{d\rho_{i1}} = \frac{f_{i1}f''_{i1}}{p^2_{ij}f_{ij}f''_{ij}} < \infty, \quad j = 2,3 \tag{21}$$

$$-\infty < \frac{dp_{ij}}{d\rho_{i1}} = \frac{d}{d\rho_{i1}}\frac{f'_{i1}}{f'_{ij}} = \frac{(\rho_{ij} - \rho_{i1})f''_{i1}}{f_{ij}} < 0, \quad j = 2,3 \tag{22}$$

Hence, ρ_{i1}, ρ_{i2} and ρ_{i3} are monotonically decreasing and p_{i3} is a monotonically increasing function over a certain open interval $I(p_{i2})$ of admissible prices p_{i2} defined by (18)-(20).[13]

(18) and (19) moreover imply:

$$(\rho_{i3}-\rho_{i2})f_{i1} + p_{i2}(\rho_{i1}-\rho_{i3})f_{i2} + p_{i3}(\rho_{i2}-\rho_{i1})f_{i3} = 0 \tag{23}$$

We shall make use of this equation later.

Taking $Y_{ij}/A_i = a_{ij}f_{ij}$ and $K_{ij}/A_i = \rho_{ij}/a_{ij}$ into account we can transform (4), (5), (8) and (9) into the following system to determine the labour-input ratios a_{i1}, a_{i2}, a_{i3} and the per-capita trade volumes n^a_{i1}, n^a_{i2}:

$$a_{i1} + a_{i2} + a_{i3} = 1 \tag{24}$$

$$\rho_{i1}a_{i1}+\rho_{i2}a_{i2}+\rho_{i3}a_{i3} = \rho_i \tag{25}$$

$$f_{i1}a_{i1} + n^a_{i1} = \rho_i f'_{i1} \tag{26}$$

$$f_{i2}a_{i2} + n^a_{i2} = f_{i2}-\rho_{i2}f'_{i2} \tag{27}$$

$$f_{i3}a_{i3}-\sigma_{i1}\max(n^a_{i1},0) - \sigma_{i2}\max(n^a_{i2},0) = 0 \tag{28}$$

$$a_{i1} > 0,\ a_{i2} > 0,\ a_{i3} \geq 0 \tag{29}$$

The last inequality derives from the assumption made in the present context that country i is incompletely specialized. Obviously, the five quantities mentioned above and p_{i2} together with the given values of A_i and K_i of the total factor supply are sufficient to determine all other demand, supply and input variables.

(24), (25) and (28) imply:

$$a_{i1} = \frac{1}{\rho_{i2}-\rho_{i1}}[\rho_{i2}-\rho_i + a_{i3}(\rho_{i3}-\rho_{i2})] \tag{30}$$

$$a_{i2} = \frac{1}{\rho_{i2}-\rho_{i1}}[\rho_i-\rho_{i1} + a_{i3}(\rho_{i1}-\rho_{i3})] \tag{31}$$

$$a_{i3} = \frac{1}{f_{i3}}[\sigma_{i1}\max(n^a_{i1},0) + \sigma_{i2}\max(n^a_{i2},0)] \tag{32}$$

By inserting (30)-(32) into (26) and (27) and using (23) to simplify the first results we get:

$$(\rho_{i3}-\rho_{i2})f_{i1}[\sigma_{i1}\max(n^a_{i1},0) + \sigma_{i2}\max(n^a_{i2},0)] + (\rho_{i2}-\rho_{i1})f_{i3}n^a_{i1} = g_i f_{i3} \tag{33}$$

$$(\rho_{i1}-\rho_{i3})f_{i2}[\sigma_{i1}\max(n^a_{i1},0) + \sigma_{i2}\max(n^a_{i2},0)] + (\rho_{i2}-\rho_{i1})f_{i3}n^a_{i2} = -\frac{1}{p_{i2}} g_i f_{i3} \tag{34}$$

where

$$g_i = p_{i2}\rho_i f_{i2} - \rho_{i2} f_{i1} \tag{35}$$

With reference to the three possible cases stated in (10)-(12) relations (33) and (34) can be solved. Using

$$h_{i1} = (\rho_{i2}-\rho_{i1})f_{i3} + \sigma_{i1}(\rho_{i3}-\rho_{i2})f_{i1} \tag{36}$$

$$h_{i2} = p_{i2}[(\rho_{i2}-\rho_{i1})f_{i3} + \sigma_{i2}(\rho_{i1}-\rho_{i3})f_{i2}] \tag{37}$$

as abbreviations we find:

(a) Suppose $n^a_{i1} = n^a_{i2} = 0$. Then (33) and (34) are satisfied provided that $g_i = 0$.

(b) Suppose $n^a_{i1} > 0$, $n^a_{i2} < 0$. (11), (33) and (34) imply:

$$n^a_{i1} = \frac{g_i f_{i3}}{h_{i1}}, \quad n^a_{i2} = -\frac{1-\sigma_{i1}p_{i3}}{p_{i2}} n^a_{i1} \tag{38}$$

Conditions: $g_i h_{i1} > 0$ (39), $1 - \sigma_{i1}p_{i3} > 0$ (40)

(c) Suppose $n^a_{i1} < 0$, $n^a_{i2} > 0$. (12), (33) and (34) imply:

$$n^a_{i2} = -\frac{g_i f_{i3}}{h_{i2}}, \quad n^a_{i1} = -(p_{i2} - \sigma_{i2}p_{i3})n^a_{i2} \tag{41}$$

Conditions: $g_i h_{i2} < 0$ (42), $p_{i2} - \sigma_{i2}p_{i3} > 0$ (43)

Next we have to find out for which values of the variables n^a_{i1} and n^a_{i2} described by (38) and (41) there exists an equilibrium for country i with incomplete specialization, i.e., over which domain I_i they are unique functions of p_{i2} satisfying condition (29) and, at every point, one of the conditions (10)-(12). The size and position of I_i depend, as will be seen below, on the total factor intensity ρ_i and the transport coefficients σ_{i1} and σ_{i2}.

First we show that to every positive, finite value of ρ_i there corresponds a unique autarky price p_{i2}^{aut} at which g_i and therefore n_{i1}^a and n_{i2}^a vanish, and that this price belongs to I_i.

From (18), (19) and (35) we get:

$$g_i = (\rho_{i2}-\rho_{i1})\rho_i f'_{i1} - (\rho_{i2}-\rho_i) f_{i1} \tag{44}$$

$$g_i = p_{i2}[(\rho_i-\rho_{i1}) f_{i2} - (\rho_{i2}-\rho_{i1})(f_{i2}-\rho_{i2}f'_{i2})] \tag{45}$$

According to these equations g_i is positive if $\rho_i \geq \rho_{i2}$ and negative if $\rho_i \leq \rho_{i1}$. Since there exist p_{i2}-values with $\rho_i = \rho_{i1}$ and $\rho_i = \rho_{i2}$, respectively, $g_i(\cdot, \rho_i)$ vanishes at least once. This and the fact that

$$\left.\frac{\partial g_i}{\partial p_{i2}}\right|_{g_i=0} = \rho_i f_{i2} - \left[\frac{f_{i1}}{f_{i2}}(f_{i2}-\rho_{i2}f'_{i2}) + \rho_{i2}f'_{i1}\frac{d\rho_{i1}}{d\rho_{i2}}\right]\frac{d\rho_{i2}}{dp_{i2}} > 0 \tag{46}$$

establishes the uniqueness of $p_{i2}^{aut}(\rho_i)$.[14] Since for this value $a_{i3} = 0$ and $\rho_{i1} < \rho_i < \rho_{i2}$ we see from (30) and (31) that (10) and (29) are satisfied and that consequently p_{i2}^{aut} is lying in I_i. Moreover, the following relations hold:

$$g_i \gtreqless 0 \text{ if } p_{i2} \gtreqless p_{i2}^{aut} \quad (47) \qquad \frac{dp_{i2}^{aut}}{d\rho_i} < 0 \quad (48)$$

Now we have to ask where the conditions (40) and (43) are fulfilled.

(22) implies

$$\frac{d(1-\sigma_{i1}p_{i3})}{dp_{i2}} < 0 \tag{49}$$

and for $p_{i2} - \sigma_{i2}p_{i3} > 0$, i.e., $\sigma_{i2} < p_{i2}/p_{i3}$

$$\frac{d(p_{i2}-\sigma_{i2}p_{i3})}{dp_{i2}} > 1 - \frac{p_{i2}}{p_{i3}}\frac{dp_{i3}}{dp_{i2}} = \frac{(\rho_{i3}-\rho_{i2})f_{i1}}{p_{i3}(\rho_{i1}-\rho_{i2})f_{i3}} > 0 \tag{50}$$

Therefore, two uniquely determined values $p_{i2}(\sigma_{i1})$ and $p_{i2}(\sigma_{i2})$ exist such that $1-\sigma_{i1}p_{i3} > 0$ if, and only if, $p_{i2} < p_{i2}(\sigma_{i1})$ and $p_{i2}-\sigma_{i2}p_{i3} > 0$ if, and only if, $p_{i2} > p_{i2}(\sigma_{i2})$. They may be boundary values of the interval $I(p_{i2})$ of all admissible p_{i2}-values. It is easy to infer from (23), (36) and (37) that

$$h_{i1} > \frac{p_{i2}}{p_{i3}}(\rho_{i3}-\rho_{i1})f_{i2} > 0 \quad \text{if} \quad p_{i2} < p_{i2}(\sigma_{i1}) \tag{51}$$

$$h_{i2} > \frac{p_{i2}}{p_{i3}}(\rho_{i2}-\rho_{i3})f_{i1} > 0 \quad \text{if} \quad p_{i2} > p_{i2}(\sigma_{i2}) \tag{52}$$

Thus we have shown that the conditions (39/40) or (42/43) are satisfied if $p_{i2}^{aut} < p_{i2} < p_{i2}(\sigma_{i1})$ or $p_{i2}(\sigma_{i2}) < p_{i2} < p_{i2}^{aut}$, respectively. This means that the functions n_{i1}^{a} and n_{i2}^{a} are single-valued over the closed interval I_i^z with $\min\left[p_{i2}(\sigma_{i2}), p_{i2}^{aut}\right]$ as lower and $\max\left[p_{i2}(\sigma_{i1}), p_{i2}^{aut}\right]$ as upper boundary point and that at every point of I_i^z one of the conditions (10)-(12) holds. However, I_i^z is not the set of incomplete specialization we are interested in since (29) need not be satisfied everywhere in I_i^z.

It should be noted that the subset of I_i^z to the right of the autarky price will be empty if the value of the transportation coefficient σ_{i1} exceeds a certain positive level which depends on ρ_i. This has the consequence that given the value of the total capital intensity there exist prohibitively high real expenditures for transporting capital goods into country i and that therefore this country cannot be incompletely specialized and import capital goods. The same holds true with respect to the transport coefficient σ_{i2} and consumption goods. At worst country i may have to remain self-sufficient.

(46) implies that n^a_{i1} and $-n^a_{i2}$ possess positive left-hand and right-hand derivatives with respect to p_{i2} at p^{aut}_{i2} so that at least near the autarky price these two functions are monotonically increasing. But it is not possible to assert whether or not they have positive derivatives over the whole interval I^z_i. To simplify the analysis we make

$$\underline{\text{Assumption (xv)}}: \quad \frac{dn^a_{i1}}{dp_{i2}} > 0, \; \frac{dn^a_{i2}}{dp_{i2}} < 0 \quad \text{if} \quad p_{i2} \in I^z_i - \{p^{aut}_{i2}\} \tag{53}$$

The set I_i of prices p_{i2} consistent with incomplete specialization obviously equals the subset of I^z_i on which (29) is satisfied, i.e., as (26) and (27) show over which the conditions

$$z_{i1} = \rho_i f'_{i1} - n^a_{i1} > 0, \quad z_{i2} = f_{i2} - \rho_{i2} f'_{i2} - n^a_{i2} > 0 \tag{54}$$

hold. A point at which z_{i1} [z_{i2}] vanishes must lie to the right [left] of p^{aut}_{i2}. There exists at most one such point for z_{i2} since $dz_{i2}/dp_{i2} > 0$. The sign of dz_{i1}/dp_{i2}, however, is indeterminate even at $z_{i1} = 0$. To avoid the complications arising from I_i possibly being a non-connected set we suppose:

$$\underline{\text{Assumption (xvi)}}: \quad \left.\frac{dz_{i1}}{dp_{i2}}\right|_{z_{i1}=0} < 0 \tag{55}$$

We denote the values of p_{i2} with $z_{i1} = 0$ or $z_{i2} = 0$, if they exist, by p^Z_{i2} and p^z_{i2}, respectively. It is easy to prove that

$$\frac{\partial p^Z_{i2}}{\partial \rho_i} < 0, \quad \frac{\partial p^z_{i2}}{\partial \rho_i} < 0, \quad \frac{\partial p^Z_{i2}}{\partial \sigma_{i1}} < 0, \quad \frac{\partial p^z_{i2}}{\partial \sigma_{i2}} > 0 \tag{56}$$

By setting

$$p^{max}_{i2} = \begin{cases} p^Z_{i2}, \text{ if defined} \\ \max[p_{i2}(\sigma_{i1}), p^{aut}_{i2}], \text{ otherwise} \end{cases} \tag{57}$$

$$p_{i2}^{min} = \begin{cases} p_{i2}^{z}, \text{ if defined} \\ \min[p_{i2}(\sigma_{i2}), p_{i2}^{aut}], \text{ otherwise} \end{cases} \quad (58)$$

we get the following delimination of the interval I_i of all values of the relative price p_{i2} of the consumption good for which a uniquely determined equilibrium with country i being incompletely specialized exists:

$$I_i = (p_{i2}^{min};\ p_{i2}^{max}) \cup \{p_{i2}^{aut}\} \quad (59)$$ [15]

The last results fit quite well into the pure theory of international trade. The world market price and the home market price of the export good in terms of the import good[16] are equal in the absence of trade impediments and are different otherwise but, as follows from (49) and (50), vary in the same direction. It was shown by Oniki and Uzawa[17] that in the first case a country will expand its exports and its imports at least as long as it is incompletely specialized if the world market price of the export good rises, while we only could prove that the same must be true in the second case if the country is nearly self-sufficient or if the production functions are of the CES type. This dissimilarity is a consequence of the existence of transport costs: An increasing import demand will cause a rising demand for transport services and, as can be seen from (32), bring about an overproportional increase in the factor inputs needed to produce these services. Therefore, the supply of the export good on the home and the world market may decline possibly even to such an extent that the actual imports the country can afford are also forced down. Since this does not necessarily mean that the actual factor inputs in the transport industry will decrease a fall first in the export volume and then also in the import volume may in fact occur if the world market price of the export good goes up. This possibility, however, was here excluded by assumption (xv). Furthermore, a variation of the total factor intensity (with prices remaining constant) has the usual consequences: According to (38) and (41) an increase in ρ_i makes the production of the capital intensive consumption good more and the production of the labour intensive capital good less advantageous for country i. This may result in a change in the pattern of specialization of this country from possibly exporting capital goods to importing them and, finally, even stopping their production. Furthermore, relation (56) shows that lower values of the transport coefficients and therefore lower transport costs allow country i to be incompletely specialized over a wider range of the home market price of the consumption good.

IV. A Single Open Economy - Complete Specialization

The analysis of the two cases of complete specialization of country i is once again based on equations (11), (12), (18)-(20) and (24)-(29) which, however, now have to be modified so that all references to industry i2 or industry i1 are to be deleted if country i produces no consumption goods (case α) or no capital goods (case β), respectively. Especially in the former case the right-hand side of (27) is $\frac{1}{p_{i2}}(f_{i1}-\rho_{i1}f'_{i1})$ and in the latter case the right-hand side of (26) reads $p_{i2}\rho_i f'_{i2}$.

It is easy to see that (18) and (20) imply:[18]

$$\frac{d\rho^{\alpha}_{ij}}{dp^{\alpha}_{i3}} < 0, \; j = 1,3, \qquad \frac{d\rho^{\beta}_{ij}}{d\pi^{\beta}_{i}} > 0, \; j = 2,3, \qquad \pi^{\beta}_{i} = \frac{p^{\beta}_{i3}}{p^{\beta}_{i2}} \tag{60}$$

In case α the modified system (24)-(28) has the solution:

$$a^{\alpha}_{i1} = \frac{\rho^{\alpha}_{i3}-\rho_i}{\rho^{\alpha}_{i3}-\rho^{\alpha}_{i1}}, \; a^{\alpha}_{i2} = 0, \quad a^{\alpha}_{i3} = \frac{\rho_i-\rho^{\alpha}_{i1}}{\rho^{\alpha}_{i3}-\rho^{\alpha}_{i1}} \tag{61}$$

$$n_{i2}^{a\alpha} = \frac{(\rho_i-\rho_{i1}^{\alpha})f_{i3}}{\sigma_{i2}(\rho_{i3}^{\alpha}-\rho_{i1}^{\alpha})} \quad (62) \qquad p_{i2}^{\alpha} = \frac{\sigma_{i2}(\rho_{i3}^{\alpha}-\rho_{i1}^{\alpha})(f_{i1}-\rho_{i1}^{\alpha}f'_{i1})}{(\rho_i-\rho_{i1}^{\alpha})f_{i3}} \quad (63)$$

$$n_{i1}^{a\alpha} = \frac{p_{i3}^{\alpha}\rho_i f_{i3} - \rho_{i3}^{\alpha}f_{i1}}{\rho_{i3}^{\alpha}-\rho_{i1}^{\alpha}} = -(p_{i2}^{\alpha}-\sigma_{i2}p_{i3}^{\alpha})n_{i2}^{a\alpha} \quad (64)$$

Conditions (11) and (29) are satisfied on the set $I^{\alpha z}(p_{i3}^{\alpha})$ on which the inequalities

$$\rho_{i3}^{\alpha} > \rho_i > \rho_{i1}^{\alpha} \quad \text{and} \quad p_{i2}^{\alpha} - \sigma_{i2}p_{i3}^{\alpha} > 0 \quad (65)$$

hold. Some further computations yield:

$$\frac{dp_{i3}^{\alpha}}{dp_{i2}^{\alpha}} < 0, \quad \frac{d\rho_{ij}^{\alpha}}{dp_{i2}^{\alpha}} > 0, \; j = 1,3, \quad \frac{dn_{ij}^{a\alpha}}{dp_{i2}^{\alpha}} < 0, \; j = 1,2 \quad (66)$$

The five variables the derivatives of which are given in (66) therefore are strictly monotonic functions of p_{i2}^{α} defined on the interval $I_i^{\alpha z}$ that is the image of $I^{\alpha z}(p_{i3}^{\alpha})$ under the transformation $p_{i2}^{\alpha} = p_{i2}^{\alpha}(p_{i3}^{\alpha})$.

Similarly we get in case β:

$$a_{i1}^{\beta} = 0, \quad a_{i2}^{\beta} = \frac{\rho_i-\rho_{i3}^{\beta}}{\rho_{i2}^{\beta}-\rho_{i3}^{\beta}}, \quad a_{i3}^{\beta} = \frac{\rho_{i2}^{\beta}-\rho_i}{\rho_{i2}^{\beta}-\rho_{i3}^{\beta}} \quad (67)$$

$$n_{i1}^{a\beta} = \frac{(\rho_{i2}^{\beta}-\rho_i)f_{i3}}{\sigma_{i1}(\rho_{i2}^{\beta}-\rho_{i3}^{\beta})} \quad (68) \qquad p_{i2}^{\beta} = \frac{(\rho_{i2}^{\beta}-\rho_i)f_{i3}}{\sigma_{i1}(\rho_{i2}^{\beta}-\rho_{i3}^{\beta})f'_{i2}} = \frac{f'_{i3}}{f'_{i2}}\,p_{i3}^{\beta} \quad (69)$$

$$n_{i2}^{a\beta} = \frac{p_{i3}^{\beta}\rho_{i2}^{\beta}f_{i3} - p_{i2}^{\beta}\rho_i f_{i2}}{p_{i2}^{\beta}(\rho_{i2}^{\beta}-\rho_{i3}^{\beta})} = -\frac{1-\sigma_{i1}p_{i3}^{\beta}}{p_{i2}^{\beta}}\,n_{i1}^{a\beta} \quad (70)$$

Conditions (12) and (29) are satisfied on the set $I^{\beta z}(\pi_i^{\beta})$ on which the inequalities

$$\rho_{i2}^{\beta} > \rho_i > \rho_{i3}^{\beta} \quad \text{and} \quad 1-\sigma_{i1}p_{i3}^{\beta} > 0 \quad (71)$$

hold. Moreover, it can be shown that[19]

$$\frac{d\pi_i^\beta}{dp_{i2}^\beta} > 0, \quad \frac{dp_{i3}^\beta}{dp_{i2}^\beta} > 0, \quad \frac{d\rho_{ij}^\beta}{dp_{i2}^\beta} > 0, \; j = 2,3, \quad \frac{dn_{ij}^{a\beta}}{dp_{i2}^\beta} > 0, \; j = 1,2 \tag{72}$$

Consequently, the last six variables are strictly monotonic functions of p_{i2}^β on the interval $I_i^{\beta z}$ which is the image of $I^{\beta z}(\pi_i^\beta)$ under the transformation $p_{i2}^\beta = p_{i2}^\beta(\pi_i^\beta)$.

It is evident that a unique equilibrium with country i producing no consumption goods or no capital goods exists if, and only if, the relative price of the consumption good belongs to the part I_i^α of $I_i^{\alpha z}$ to the left of I_i or to the part I_i^β of $I_i^{\beta z}$ to the right of I_i, respectively. We denote by I_i^ϵ the set of all prices p_{i2} associated with unique equilibria, i.e., the union of I_i^α, I_i and I_i^β, and define:

$$n_{ij}^{a\epsilon} = \begin{cases} n_{ij}^{a\alpha} & \text{for } p_{i2} \in I_i^\alpha \\ n_{ij}^{a} & \text{for } p_{i2} \in I_i \\ n_{ij}^{a\beta} & \text{for } p_{i2} \in I_i^\beta \end{cases}, \quad p_{i3}^\epsilon = \begin{cases} p_{i3}^{\alpha} & \text{for } p_{i2} \in I_i^\alpha \\ p_{i3} & \text{for } p_{i2} \in I_i \\ p_{i3}^{\beta} & \text{for } p_{i2} \in I_i^\beta \end{cases} \tag{73}$$

[20]

It was shown above that interval I_i of prices p_{i2} compatible with incomplete specialization is a subset of the interval I_i^z of prices p_{i2} for which the domestic price of the import good is not lower than the costs to ship it from abroad. We shall prove that I_i^α and I_i^β are also contained in I_i^z provided that $p_{i2}^z > p_{i2}(\sigma_{i2})$ and $p_{i2}^Z < p_{i2}(\sigma_{i1})$, respectively. As we have seen the second of these inequalities implies that p_{i2}^Z is the upper boundary point of I_i. Denote for the moment by a hat over a variable the limit it reaches if p_{i2} tends rising against p_{i2}^Z. Since $\hat{a}_{i1} = 0$ it is obvious that $\hat{a}_{i2}$, $\hat{a}_{i3}$, $\hat{n}_{i1}^a$, $\hat{n}_{i2}^a$, $\hat{p}_{i2}$ and $\hat{p}_{i3}$ form a solution of the modified system (24)-(28) in case β and therefore are described by (67)-(70).

Because $\hat{a}_{i2}$, $\hat{a}_{i3}$ and $1-\sigma_{i1}\hat{p}_{i3}$ are positive condition (71) is satisfied. This means that p_{i2}^{z} belongs to $I_i^{\beta z}$ and I_i^{β} but does not coincide with the upper boundary point $\bar{p}_{i2}^{\beta}$ of these intervals. Moreover, $n_{ij}^{a\epsilon}$ and p_{i3}^{ϵ} are continuous at p_{i2}^{z}. To prove finally $\bar{p}_{i2}^{\beta} < p_{i2}(\sigma_{i1})$ we show that

$$1-\sigma_{i1}p_{i3} > 1-\sigma_{i1}p_{i3}^{\epsilon} \quad \text{for} \quad p_{i2}^{z} < p_{i2} < p_{i2}(\sigma_{i1}) \tag{74}$$

holds. It follows from (22), (23) and (72):

$$\frac{dp_{i3}}{dp_{i2}} = \frac{p_{i3}}{p_{i2}} + \frac{(\rho_{i2}-\rho_{i3})f_{i1}}{p_{i2}(\rho_{i1}-\rho_{i2})f_{i3}} < \frac{p_{i3}}{p_{i2}} \tag{75}$$

$$\frac{dp_{i3}^{\beta}}{dp_{i2}} = \frac{d(p_{i2}\pi_i^{\beta})}{dp_{i2}} = \pi_i^{\beta} + p_{i2}\frac{d\pi_i^{\beta}}{dp_{i2}} > \pi_i^{\beta} = \frac{p_{i3}^{\beta}}{p_{i2}} \tag{76}$$

Since $p_{i3} = p_{i3}^{\beta}$ at p_{i2}^{z} (75) and (76) imply $p_{i3} < p_{i3}^{\beta}$ to the right of this point. Therefore, (74) is correct.

In a similar manner we can prove the above proposition about I_i^{α}. The crucial relation

$$p_{i2}-\sigma_{i2}p_{i3} > p_{i2}-\sigma_{i2}p_{i3}^{\epsilon} \quad \text{for} \quad p_{i2}(\sigma_{i2}) < p_{i2} < p_{i2}^{z} \tag{77}$$

is a consequence of (22) and (66).[21]

It should be noted that if the hitherto fixed value of ρ_i decreases so that p_{i2}^{z} tends rising against $p_{i2}(\sigma_{i1})$ the interval I_i^{β} moves to the right and shrinks. Eventually, if p_{i2}^{z} reaches $p_{i2}(\sigma_{i1})$ it becomes empty so that no equilibrium with country i producing no capital goods exists at all. Analogous results hold with respect to I_i^{α}.

The conclusions reached above imply that in both cases of complete specialization the world market price and the home market price of the export good in terms of the import good vary once again in the same direction but that their difference is greater than under incomplete specialization since according to (74) and (77) the transport costs for importing goods weigh more heavily if a country produces one good only. With an increase in these prices the import volume rises while at the same time because of a favourable development of the terms of trade the export volume falls. A completely specialized country will, however, be adversely affected by a growing total capital intensity since then at constant prices its imports must decrease and its exports increase.

After having established all the properties of one country's main variables we shall need below we now can carry out a static analysis of the two-country equilibria. Especially we shall ask under which circumstances either country 1 or country 2 will import capital goods and export consumption goods or both are autarkic.

V. A Static Analysis of the Two-Country Equilibria

A necessary and sufficient condition for a two-country equilibrium to prevail is that the imports, if any, of one country are the exports of the other:

$$X_1^{\epsilon}(p_{12},p_{22}) \equiv N_{11}^{a\epsilon}(p_{12}) + N_{21}^{a\epsilon}(p_{22}) = 0 \qquad (6a)$$

$$X_2^{\epsilon}(p_{12},p_{22}) \equiv N_{12}^{a\epsilon}(p_{12}) + N_{22}^{a\epsilon}(p_{22}) = 0 \qquad (6b)$$

These relations are always satisfied at the autarky point $Q^{aut} = (p_{12}^{aut}, p_{22}^{aut})$ but, as will be seen soon, this is not the equilibrium point $Q^{g} = (p_{12}^{g}, p_{22}^{g})$ if they also have another solution.

(38), (41), (64) and (70) imply that one of the equations (6a) and (6b) may be substituted by

$$v^{\epsilon}(p_{12},p_{22}) \equiv \frac{p_{12}}{1-\sigma_{11}p_{13}^{\epsilon}} - (p_{22}-\sigma_{22}p_{23}^{\epsilon}) = 0 \quad \text{for} \quad p_{12} > p_{12}^{aut} \qquad (78)$$

i.e., if country 1 imports capital goods ($N_{11}^{a\epsilon} > 0$), and by

$$w^{\varepsilon}(p_{12},p_{22}) \equiv (p_{12}-\sigma_{12}p_{13}^{\varepsilon}) - \frac{p_{22}}{1-\sigma_{21}p_{23}^{\varepsilon}} = 0 \text{ for } p_{12} < p_{12}^{aut} \tag{79}$$

i.e., if country 1 exports capital goods ($N_{11}^{a\varepsilon} < 0$).

(6a) and (6b) describe two continuous curves $gr(X_1^{\varepsilon})$ and $gr(X_2^{\varepsilon})$, respectively, which are lying in $I_1^{\varepsilon} \times I_2^{\varepsilon}$ and have end points on the boundary of this rectangle. From (53), (66) and (72) it follows that $gr(X_1^{\varepsilon})$ $[gr(X_2^{\varepsilon})]$ increases if $p_{12} \in I_1^{\alpha}$ or $p_{22} \in I_2^{\alpha}$ $[p_{12} \in I_1^{\beta}$ or $p_{22} \in I_2^{\beta}]$ and decreases elsewhere. Both these curves cross the line $p_{12} = p_{12}^{aut}$ only once at Q^{aut} since $N_{i1}^{a\varepsilon}$ and $-N_{i2}^{a\varepsilon}$ are positive to the right and negative to the left of p_{i2}^{aut}.

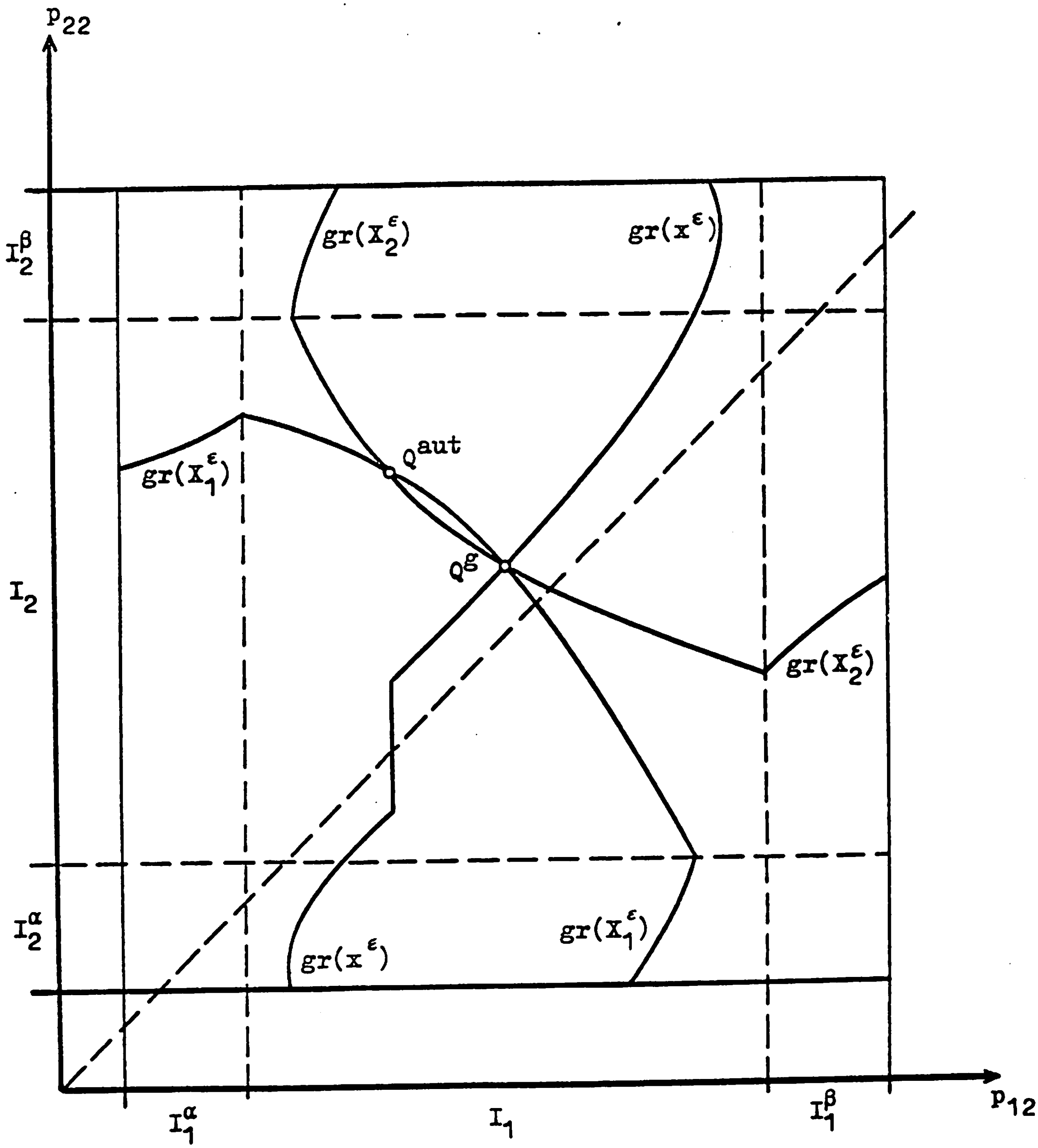
We define $gr(x^{\varepsilon})$ as the curve consisting of the solutions $gr(v^{\varepsilon})$ of (78) and $gr(w^{\varepsilon})$ of (79) and their straight line connection at $p_{12} = p_{12}^{aut}$. $gr(x^{\varepsilon})$ is also a continuous curve lying in $I_1^{\varepsilon} \times I_2^{\varepsilon}$ and ending on the boundary of this set. (49) and (50) imply that $gr(x^{\varepsilon})$ increases at least in $I_1 \times I_2^{22}$ and is connected there. Furthermore, we note that $gr(v^{\varepsilon})$ lies above and $gr(w^{\varepsilon})$ below the diagonal in the first quadrant of the (p_{12},p_{22})-plane.

It is evident that the equilibrium point Q^g must belong to $gr(x^{\varepsilon})$ as well as to $gr(X_1^{\varepsilon})$ and $gr(X_2^{\varepsilon})$. Since these three curves have, as follows from their properties stated above and from $Q^{aut} \in I_1 \times I_2$, at most one point in common the two-country equilibrium is uniquely determined if it exists. Moreover, we observe that through every point of intersection of $gr(x^{\varepsilon})$ and $gr(X_j^{\varepsilon})$ also $gr(X_k^{\varepsilon})$, $j \neq k$, must pass. (Cf. Figure 1).

To complete our static analysis we have to find and interpret the conditions under which the different possible types of equilibria will prevail.

Let Gr(v) and Gr(w) be the curves defined by

$$v(p_{12},p_{22}) \equiv \frac{p_{12}}{1-\sigma_{11}p_{13}} - (p_{22}-\sigma_{22}p_{23}) = 0 \tag{80}$$



p_{22}
I_2^β
$gr(X_2^\varepsilon)$
$gr(x^\varepsilon)$
Q^{aut}
$gr(X_1^\varepsilon)$
Q^g
I_2
$gr(X_2^\varepsilon)$
I_2^α
$gr(X_1^\varepsilon)$
$gr(x^\varepsilon)$
I_1^α
I_1
I_1^β
p_{12}

Figure 1

$$w(p_{12},p_{22}) \equiv (p_{12}-\sigma_{12}p_{13}) - \frac{p_{22}}{1-\sigma_{21}p_{23}} = 0 \tag{81}$$

respectively. They are lying in the rectangle $I(p_{12}) \times I(p_{22})$ of all admissible price points, increase because of (49) and (50) throughout and possess end points on the boundary of this set. Therefore, they divide $I(p_{12}) \times I(p_{22})$ into the three disjoint domains B_o above Gr(v), B_u below Gr(w) and the intermediate set B_m which also includes the two curves themselves. Obviously, the diagonal in the first quadrant of the (p_{12},p_{22})-plane is lying in the interior of B_m. Moreover, relations (74) and (77) in their extended form (cf. note 21) imply that the parts of $gr(v^\varepsilon)$ and $gr(w^\varepsilon)$ outside the closure of $I_1 \times I_2$ belong to B_o and B_u, respectively. (Cf. Figure 2).

Our last results lead to the following conclusions. An autarky equilibrium for both countries exists if, and only if, the autarky point is in B_m. Otherwise, there may be a trade equilibrium with country 1 importing capital goods if, and only if, $Q^{aut} \in B_o$ or importing consumption goods if, and only if, $Q^{aut} \in B_u$. This means that international trade is only possible if the autarky prices differ by more than a certain positive minimum amount which itself depends on p_{12}^{aut}. The greater the divergence between these prices becomes the more each country will cut down the output of the good in the production of which it has a comparative disadvantage until first one and then even both of them will specialize completely. Extremely high differences between the countries' autarky prices, however, may prevent the existence of a two-country equilibrium at all.

The direction of the international trade flows is as usual since, e.g., $p_{12}^{aut} < p_{22}^{aut}$ obviously means that country 1 has a comparative advantage in producing consumption goods and country 2 in producing capital goods. These

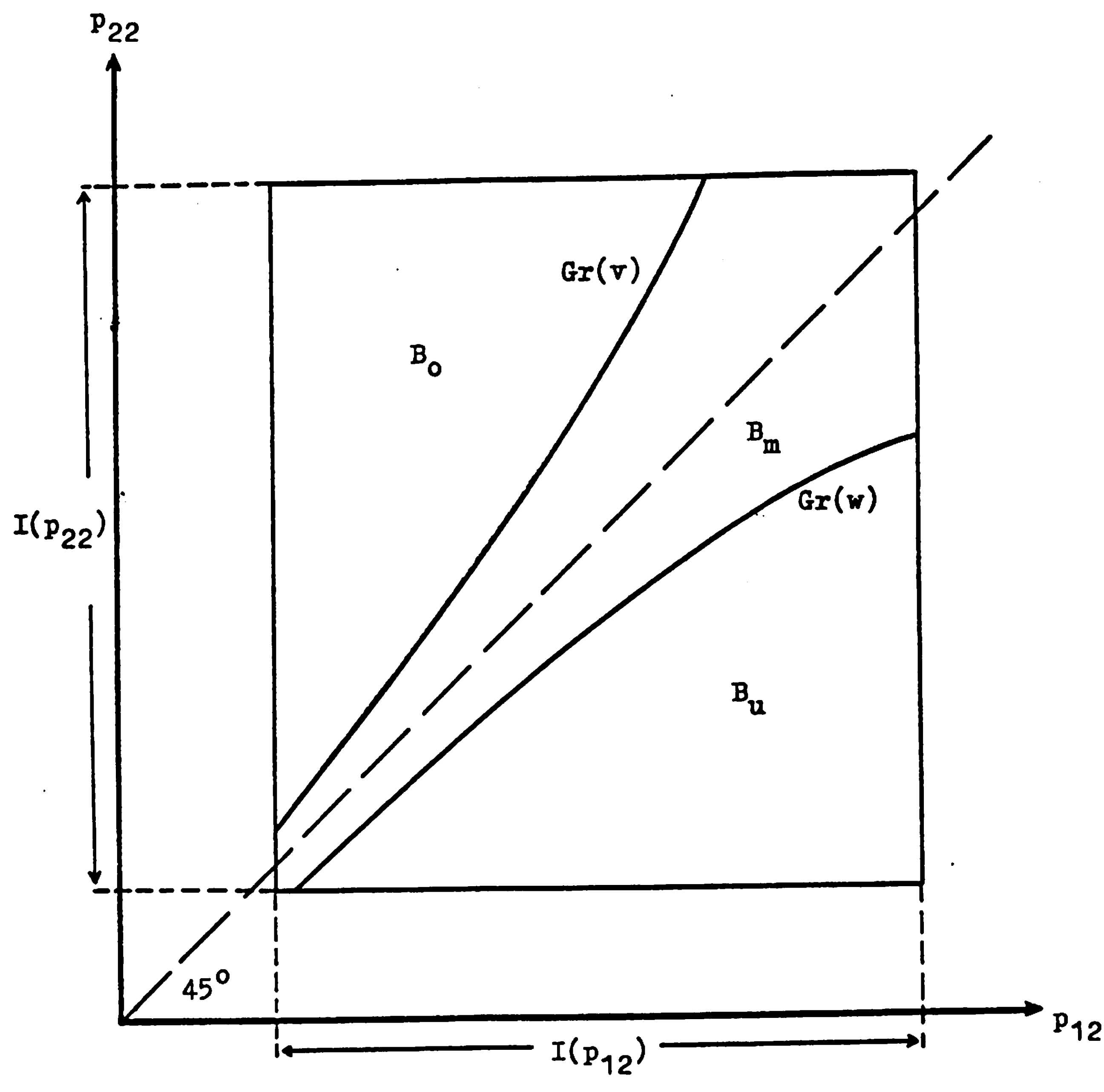


Figure 2

advantages which depend on the production techniques in the capital goods and the consumption goods industries and on the total factor intensities have, however, to be strong enough to secure that Q^{aut} belongs to B_o, i.e., to overcome the transport barrier between the countries.

We have seen earlier that to every positive, finite total capital intensity ρ_i there corresponds a unique autarky price p_{i2}^{aut} which is the lower the greater ρ_i (cf. formula (48)). On the other hand, it is easy to make sure that every admissible price p_{i2} is the autarky price associated with a certain ρ_i. Consequently, the partition of $I(p_{12}) \times I(p_{22})$ into B_o, B_m and B_u induces an equivalent partition of the whole first quadrant of the (ρ_1,ρ_2)-plane into domains B_o^ρ, B_m^ρ and B_u^ρ. The second of these sets is a "corridor" limited by two increasing curves $Gr(v^\rho)$ and $Gr(w^\rho)$ which are the images of $Gr(v)$ and $Gr(w)$, respectively, while B_o^ρ lies below and B_u^ρ above B_m^ρ. It seems worth stating that B_m^ρ contains the 45-degree line only if the sectoral production functions are internationally equal or not too different. (Cf. Figure 3).

As is now evident, it follows from $(\rho_1,\rho_2) \in B_m^\rho$ that both countries must be self-sufficient, and $(\rho_1,\rho_2) \in B_o^\rho$ $[(\rho_1,\rho_2) \in B_u^\rho]$ implies that there may be a trade equilibrium with country 1 importing [exporting] capital goods. In the latter case the degree of specialization becomes greater if the difference between ρ_1 and ρ_2 grows in absolute value. The well-known proposition, that each country will export the good in the production of which its relatively abundant factor is used intensively, remains valid in our model if we interpret, because of the existence of transport costs and the possibility of international differences in the production techniques, the term "relative abundancy" of, e.g., capital in country 1 [country 2] as meaning that (ρ_1,ρ_2) belongs to $B_o^\rho[B_u^\rho]$. Obviously, this will normally but not necessarily imply

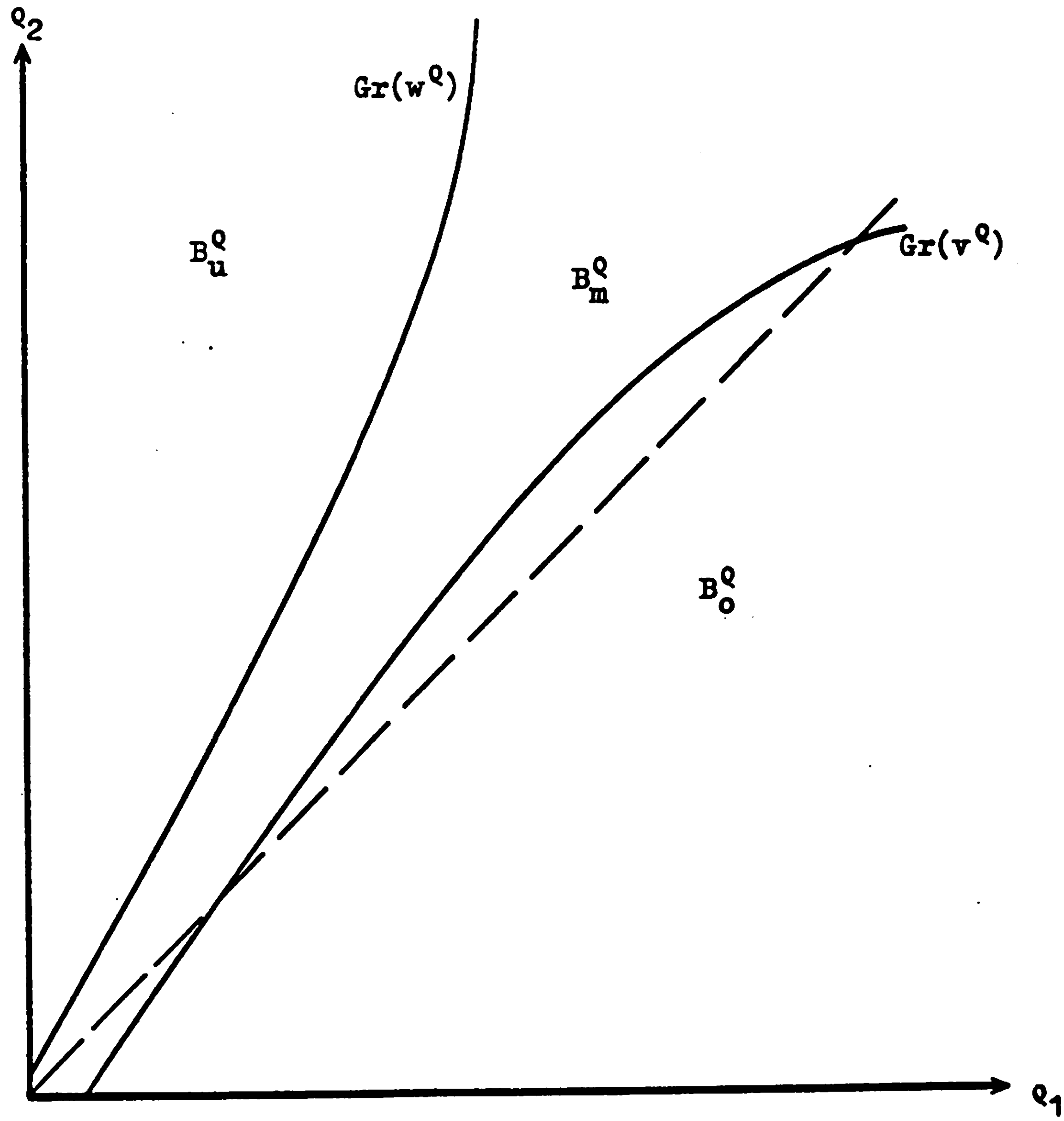


Figure 3

$\rho_1 > \rho_2 [\rho_1 < \rho_2]$.

Finally we note that the domains B_m and B_m^ρ are the narrower the smaller the values of the transport coefficients are and that in the Bardhan-Oniki-Uzawa case of zero transport costs B_m coincides with a certain part of the 45-degree line while B_m^ρ becomes an increasing curve.

VI A Dynamic Analysis of the Two-Country Equilibria

In the last part of our analysis we deal with the dynamic properties of our model, especially with the development of the total capital intensities and the associated two-country equilibria.

Equations (13) and (14) imply:

$$\frac{\dot{\rho}_i}{\rho_i} = \mu_i - \lambda_i, \text{ where } \mu_i = \begin{cases} f'_{i1} - \delta_i & \text{if } p_{i2} \in I_i \cup I_i^\alpha \\ \dfrac{n_{i1}^{a\beta}}{\rho_i} - \delta_i & \text{if } p_{i2} \in I_i^\beta \end{cases} \qquad (82)^{23}$$

μ_i is a continuous function of p_{i2} which, as follows from (22), (60), (66) and (72), is strictly decreasing over I_i^α and strictly increasing over I_i and I_i^β.

Denote by p_{i2}^μ the uniquely determined p_{i2}-value in $I(p_{i2})$, if any, with $f'_{i1}-\delta_i-\lambda_i = 0$ and by ρ_i^γ, γ = aut, z, Z, the ρ_i-value with $p_{i2}^\gamma = p_{i2}^\mu$. ($\rho_i^\gamma$ exists if p_{i2}^μ is defined). From (48) and (56) we infer that $\rho_i^z < \rho_i^{aut} < \rho_i^Z$.

$\mu_i-\lambda_i$ is positive everywhere if $\rho_i < \rho_i^z$. Otherwise this function may vanish at most twice, at $p_{i2}^{\alpha\mu} \in I_i^\alpha$ and p_{i2}^μ if $\rho_i^z \le \rho_i < \rho_i^Z$, and at $p_{i2}^{\alpha\mu} \in I_i^\alpha$ and $p_{i2}^{\beta\mu} \in I_i^\beta$ if $\rho_i \ge \rho_i^Z$. p_{i2}^μ coincides with $p_{i2}^{\alpha\mu}$ or $p_{i2}^{\beta\mu}$ provided that $\rho_i = \rho_i^z$ or $\rho_i = \rho_i^Z$, respectively. $p_{i2}^{\alpha\mu}$ and $p_{i2}^{\beta\mu}$ but not p_{i2}^μ vary with ρ_i.

The case that the labour supply increases in both countries at the same rate will be analyzed first.

We define $\varphi_i(\rho_1,\rho_2) = \mu_i - \lambda_i$ where μ_i depends on the equilibrium price p^g_{i2} associated with ρ_1 and ρ_2. The points at which φ_i vanishes from a continuous curve $gr(\varphi_i)$ lying in the set of all combinations of the total factor intensities for which a two-country equilibrium exists and ending on the boundary of this set. Obviously, steady states of growth are characterized by the points of intersection of $gr(\varphi_1)$ and $gr(\varphi_2)$.

Clearly, $gr(\varphi_1)$ is a continuous curve consisting of different segments the properties of which can be characterized as follows:

1) $Q^g = (p^{\mu}_{12}, p^g_{22})$ is a point in the interior of B_m and therefore equals Q^{aut}. The corresponding part of $gr(\varphi_1)$ is the vertical linear segment with $\rho_1 = \rho_1^{aut}$ in B_m^{ρ}.

2) $Q^g = (p^{\mu}_{12}, p^g_{22}) \in Gr(v)$. Since Q^g is fixed the relation between ρ_1 and ρ_2 defined by $\varphi_1 = 0$ is determined by equation (6a) alone. Therefore, taking also (38) and (41) into account we find that $d\rho_2/d\rho_1$, being defined for some interval $\rho_1^{aut} \leq \rho_1 \leq \bar{\rho}_1$ with $\bar{\rho}_1 \leq \rho_i^Z$, is negative and independent of ρ_1 and ρ_2. This implies that over the above interval $gr(\varphi_1)$ is a falling straight line which has the point on $Gr(v^{\rho})$ with $\rho_1 = \rho_1^{aut}$ as left-hand end point. The equilibrium corresponding to the other end point $(\bar{\rho}_1, \bar{\rho}_2)$ is characterized by complete specialization only of country 2 if, and only if, $\bar{\rho}_1 < \rho_1^Z$ and of country 1 and possibly also country 2 if, and only if $\bar{\rho}_1 = \rho_1^Z$.

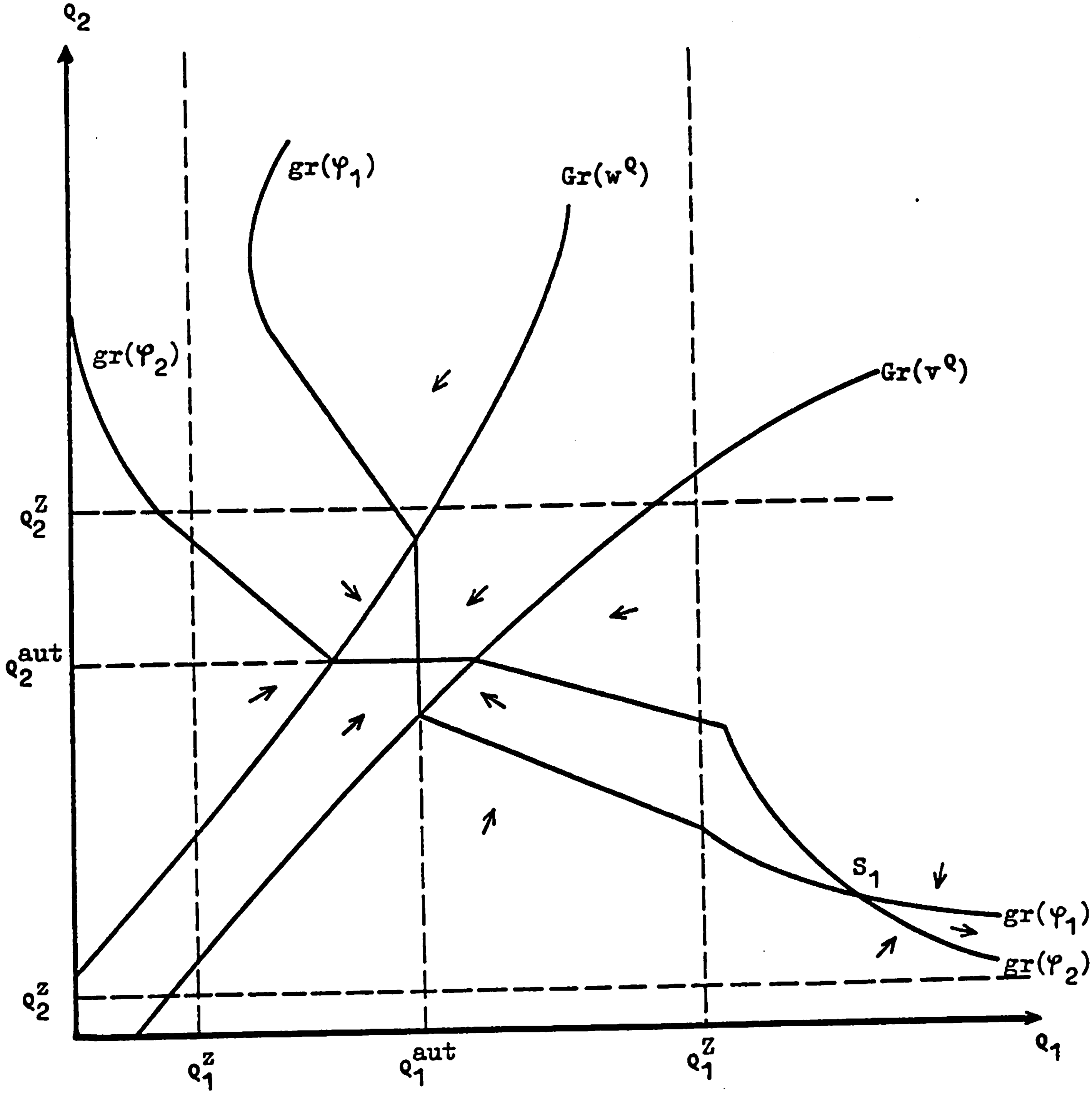
3) $Q^g = (p^{\mu}_{12}, p^g_{22}) \in Gr(w)$. As in case (2) it follows that the corresponding part of $gr(\varphi_1)$ is a downward sloping straight line segment defined for $\bar{\bar{\rho}}_1 \leq \rho_1 \leq \rho_1^{aut}$ where $\bar{\bar{\rho}}_1$ is some value not smaller than ρ_1^z. The equilibrium associated with its left-hand end point $(\bar{\bar{\rho}}_1, \bar{\bar{\rho}}_2)$ is characterized by complete specialization only of country 2 if, and only if, $\bar{\bar{\rho}}_1 > \rho_1^z$ and of country 1 and possibly also country 2 if, and only if, $\bar{\bar{\rho}}_1 = \rho_1^z$.

4) The slope of all other parts of gr(φ_1) which obviously correspond to equilibria with at least one country being completely specialized cannot be determined.

In addition it can be shown that the absolute value of the slope of this curve tends to infinity if $A_1/A_2 \to \infty$ and to some finite value if $A_1/A_2 \to 0$. Moreover, in both cases the length of the linear parts of gr(φ_1) corresponding to incomplete specialization in both countries shrinks to zero while the other parts corresponding to incomplete specialization in the smaller country only vanish totally if A_1/A_2 reaches certain extreme but finite values.

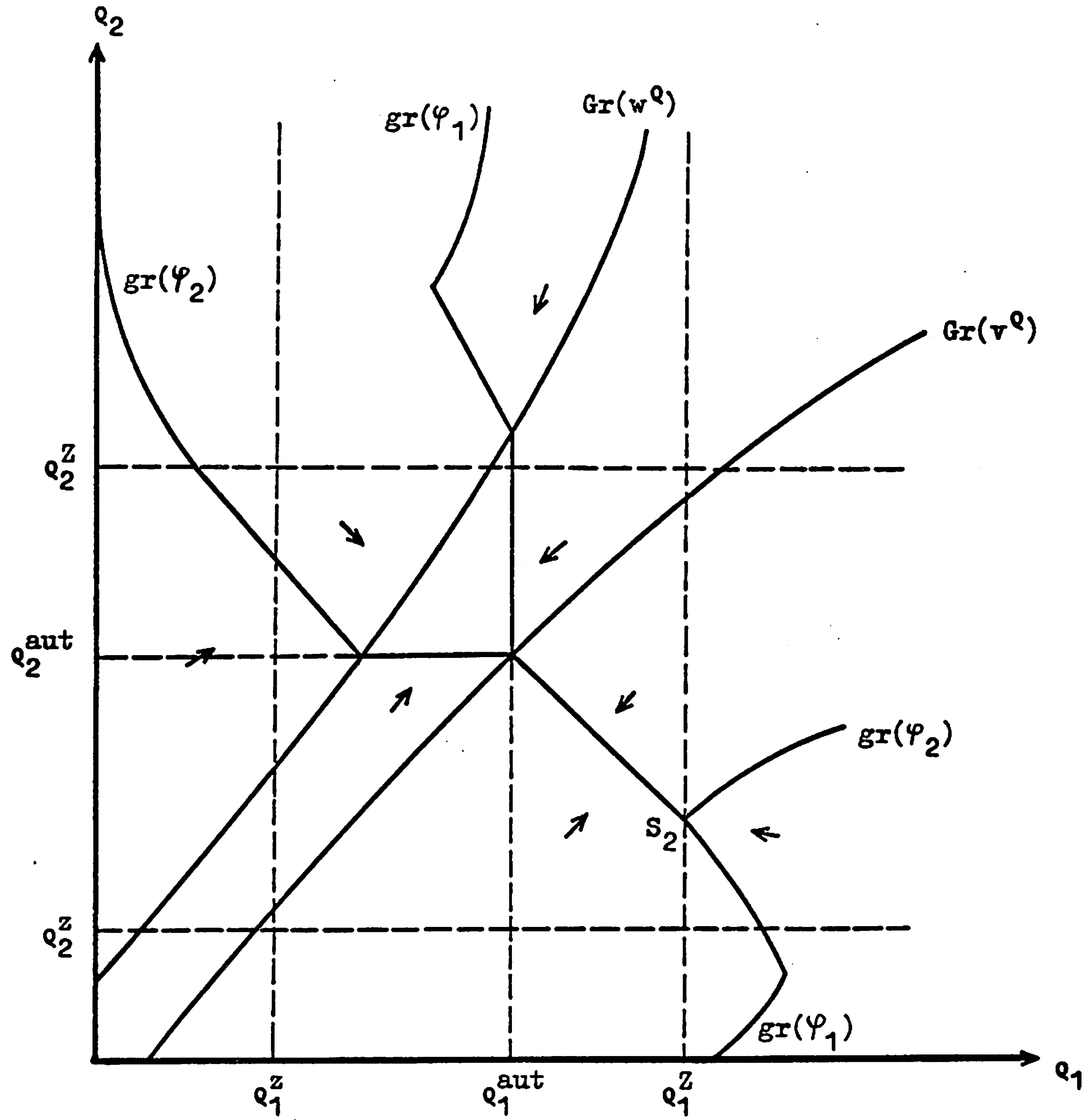
Obviously, gr(φ_2) has a similar shape as gr(φ_1). It should be observed that if $Q^{\mu} = (p_{12}^{\mu}, p_{22}^{\mu})$ belongs to Gr(v) or Gr(w) the linear segments of gr(φ_1) and gr(φ_2) in B_o^{ρ} or B_u^{ρ}, respectively, associated with the equilibrium point Q^g equal to Q^{μ} must coincide since they have a common end point on Gr(v) or Gr(w), respectively, and are both solutions of equation (6a). Furthermore, it is evident that steady states with the two countries being incompletely specialized and trading with one another exists if, and only if, Q^{μ} is lying on Gr(v) or Gr(w). This implies that in all other cases these linear parts of gr(φ_1) and gr(φ_2) cannot intersect. Therefore, the absolute value of the slope of the straight line segment of gr(φ_1) in B_o^{ρ}[B_u^{ρ}] is the higher the smaller [larger] the value of ρ_i^{aut} is. (Cf. Figures 4, 5 and 6).

Finally, it follows from (48) and $\mu_i - \lambda_i \gtreqless 0$ if $p_{i2}^{g} = p_{i2}^{aut} \gtreqless p_{i2}^{\mu}$ that $\varphi_i \lesseqgtr 0$ if $\rho_i \gtreqless \rho_i^{aut}$ and $\rho_k = \rho_k^{aut}$, $k \neq i$. Consequently, φ_1 is positive to the left and negative to the right of gr(φ_1) and φ_2 is positive below and negative above gr(φ_2).



Q2
gr(φ1)
Gr(w^Q)
gr(φ2)
Gr(v^Q)
Q2^Z
Q2^aut
S1
gr(φ1)
gr(φ2)
Q2^z
Q1^z
Q1^aut
Q1^Z
Q1

Figure 4



Q_2
gr(φ_1)
Gr(w^Q)
gr(φ_2)
Gr(v^Q)
Q_2^Z
Q_2^{aut}
gr(φ_2)
S_2
Q_2^z
gr(φ_1)
Q_1
Q_1^z
Q_1^{aut}
Q_1^Z

Figure 5

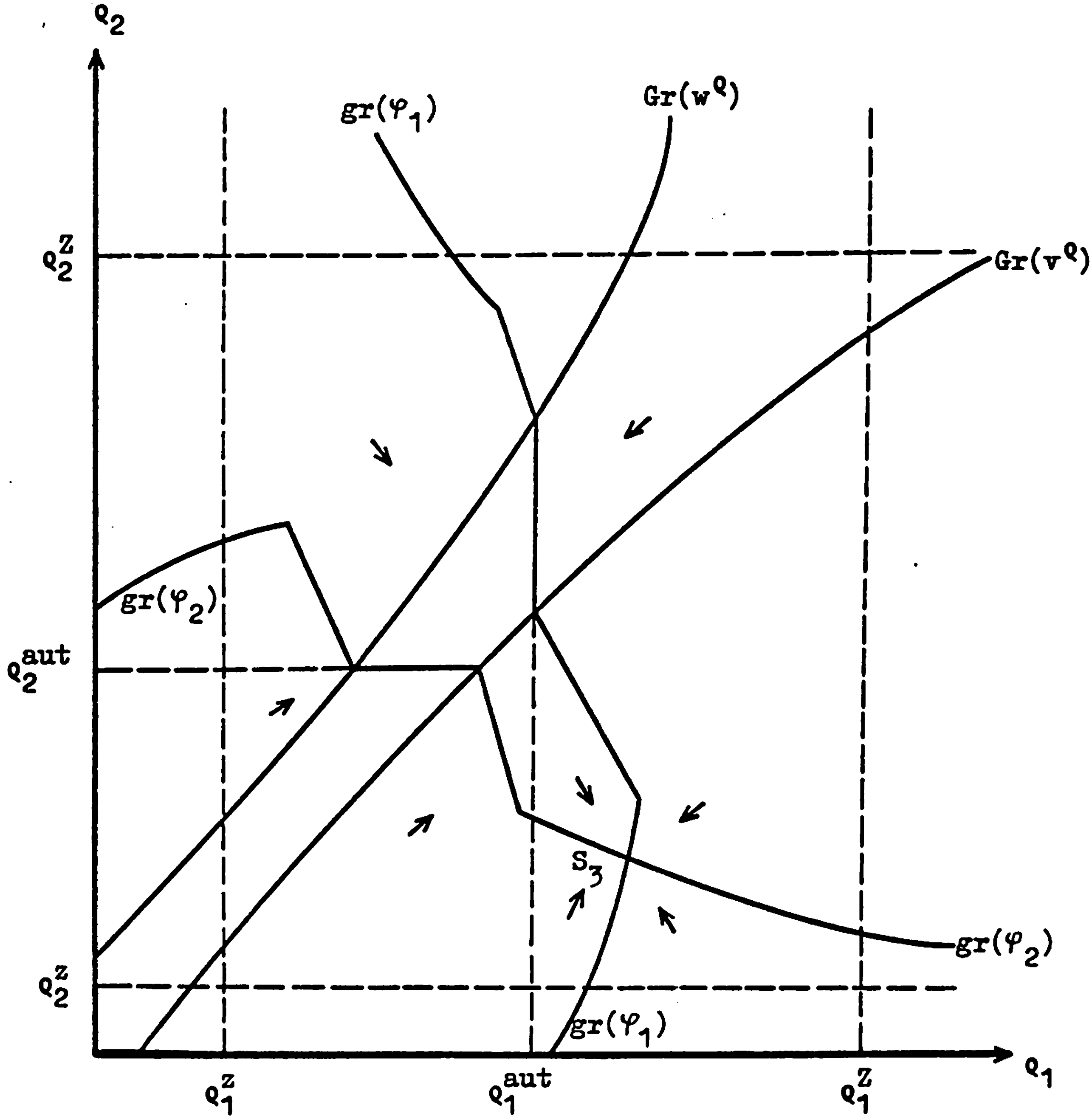

Q2
gr(φ1)
Gr(w^Q)
Q2^Z
Gr(v^Q)
gr(φ2)
Q2^aut
S3
gr(φ2)
Q2^z
gr(φ1)
Q1
Q1^z
Q1^aut
Q1^Z

Figure 6

The last results have the following consequences:

a) If Q^{μ} belongs to the interior of B_m there exists one steady state with autarky and possibly others with one or both countries being completely specialized (Figure 4).[24] The autarky state is at least locally stable.

b) Q^{μ} lying on Gr(v) or Gr(w) implies the existence of a whole range of steady states with incomplete specialization of the two countries and, in addition, of one with autarky and one or more with complete specialization of at least one country (Figure 5).

c) Finally, if Q^{μ} is a point in B_o or B_u only steady states with complete specialization of country 1 and/or country 2 are possible (Figure 6).[25]

The long-run development of the two economies will not necessarily approach a balanced growth path from every feasible initial position. For example, the system of steady states is evidently not globally stable if $gr(\varphi_1)$ and $gr(\varphi_2)$ do not intersect or if $gr(\varphi_1)$ lies above $gr(\varphi_2)$ for all ρ_1-values greater than some $\hat{\rho}_1$ (see the parts of the two curves to the right of point S_1 in Figure 4) or to the left of $gr(\varphi_2)$ for all ρ_2-values greater than some $\hat{\rho}_2$. On the other hand, global stability prevails provided that these curves have at least one point in common, that every pair of positive, finite ρ_1- and ρ_2-values is compatible with an equilibrium solution, and that, moreover, one of the following conditions is satisfied:

1) $gr(\varphi_i)$, $i = 1,2$, has a positive slope at all its points associated with complete specialization in country k, $k \neq i$. (Cf. Figure 5).

2) One country, say country 1, is so large compared with the other country, i.e., the value A_1/A_2 is so high, that $gr(\varphi_1)$ lies to the left of the vertical line $\rho_1 = \rho_1^z$ and $gr(\varphi_2)$ below the horizontal line $\rho_2 = \rho_2^z$. (Cf. Figure 6).[26]

The final case to be analyzed is that of different growth rates of the labour force in the two countries. Without loss of generality we may assume $\lambda_1 > \lambda_2$. Then only the parts of $gr(\varphi_1)$ and $gr(\varphi_2)$ in B_m^ρ remain unchanged. Outside this domain we find that, as the time t tends to infinity, $gr(\varphi_1)$ approaches the vertical line $\rho_1 = \rho_1^{aut}$ and $gr(\varphi_2)$ some limit position which has everywhere a finite slope. Furthermore, the linear parts of these curves corresponding to incomplete specialization in both countries shrink to zero length. Consequently, a unique and globally stable autarky state exists if Q^μ belongs to B_m. Otherwise, country 1 becomes more and more self-sufficient while after a finite period of time country 2 will be completely specialized, producing no consumption goods if $Q^\mu \in B_o$ or no capital goods if $Q^\mu \in B_u$. It seems interesting to note that if this development does not lead to a disequilibrium situation the total capital intensity ρ_2 tends to some positive, finite value since the limit curve of $gr(\varphi_2)$ lies between $\rho_2 = \rho_2^z$ and $\rho_2 = \rho_2^Z$. This means that country 2 will also approach a pseudo-steady state.[27]

The last results reveal the influence of the production techniques, the growth rates of the labour force and the transport costs on the growth and trade potential of the two countries in the long run. We have seen that, given the values of the transport coefficients, certain minimum dissimilarities between the countries with respect to their technology and the increase in their labour supply are necessary to overcome the transport barrier for more than a limited period of time. Under the assumption usually made in the theory of international trade that both countries use the same production techniques only a sufficiently rapidly increasing divergence in the size of their labour force, and therefore also in their capital stocks, renders international trade in the long run possible. (Clearly, even then the larger country will have a falling per-capita trade volume.) Otherwise, the countries will find themselves finally in no-trade situation unless technical progress in transportation lowers sufficiently quickly the transport barriers.

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FOOTNOTES

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2. H. Oniki, H. Uzawa, "Patterns of Trade and Investment in a Dynamic Model of International Trade," The Review of Economic Studies, vol. 32 (1965), pp. 15-38; P. K. Bardhan, "Equilibrium Growth in the International Economy," The Quarterly Journal of Economics, vol. 79 (1965), pp. 455-64; P. K. Bardhan, "On Factor Accumulation and the Pattern of International Specialization," The Review of Economic Studies, vol. 33 (1966), pp. 39-44; see also M. C. Kemp, The Pure Theory of International Trade and Investment, Englewood Cliffs, N. J., 1969, Chap. 10.

3. Foreign investment in a growth model is being analysed in, e.g., K. Inada, "International Trade, Capital Accumulation and Factor Price Equalization," Economic Record, vol. 44 (1968), pp. 322-41; M. C. Kemp, "International Trade and Investment in a Context of Growth," Economic Record, vol. 44 (1968), pp. 211-23; M. C. Kemp, op.cit., Chap. 11.

4. See also H. Herberg, Wirtschaftswachstum, Außenhandel und Transportkosten, (vol. 1 of Theorie und Politik, ed. by G. Gäfgen), Göttingen 1966.

5. Cf. H. Herberg, "Zur Möglichkeit der Einbeziehung von Transportkosten in die reine Theorie des internationalen Handels," Jahrbücher für Nationalökomie und Statistik, vol. 181 (1968), pp. 549-66.

6. This assumption seems to be acceptable if we think of transport media carrying for technical reasons goods in one direction only as, e.g., pipe lines. It is far less satisfactory with regard to media making round trips as, e.g., ships, lorries, freight trains; it would imply

that they are not used efficiently since they would be empty on every outward journey. An assumption better suited for the latter case could be adopted from a paper by G. Hadley and M. C. Kemp ("Equilibrium and Efficiency in International Trade," Metroeconomica, vol. 18 (1966), pp. 125-41). This, however, would mean greater analytical difficulties without altering the main results.

7. This assumption is only made to facilitate stating the international price differences caused by transport costs. The existence of some sort of money is not assumed.

8. This is a special case of the so-called factor intensity hypothesis that the consumption good is always more capital intensive than the capital good, a hypothesis ensuring, as is well-known, the uniqueness and global stability of the steady state in one- or two-country two-commodity models with a classical saving function. (Cf. e.g., F. H. Hahn, R. C. O. Matthews, "The Theory of Economic Growth: A Survey," The Economic Journal, vol. 74 (1964), pp. 779-902, esp. part I.8; H. Oniki, H. Uzawa, op.cit.) We do not deal in this paper with the other case that the consumption good is always more labour intensive than the capital good. It is a bit more difficult to handle as, e.g., multiple temporary equilibria may exist (cf. H. Herberg, Wirtschaftswachstum..., pp. 37 sequ.). The author doubts that the condition that all elasticities of substitution are not less than unity (see P. K. Bardhan, "Equilibrium Growth...," loc.cit.) is sufficient to eliminate these difficulties in the present model.

The assumption that the capital intensity is in the transport industry always lower than in the consumption goods industry and higher than in the capital goods industry is made since it implies that in the i^{th} country, taking the capital good as numéraire, the relative price p_{i3} of transportation services is a function of the relative price p_{i2} of the consumption good and, moreover, that the domain of definition of the variables considered, especially the quantities imported or exported, is an interval of p_{i2}-values. However interesting it would be to work out the consequences of this assumption being abandoned this is not done here.

9. If the production functions are of the CES type a necessary condition for this assumption to be satisfied is that the three elasticities of substitution are identical. See e.g., P. K. Bardhan, "Equilibrium Growth...," loc.cit., note 4, and H. Herberg, Wirtschaftswachstum..., p. 141.

10. f'_{ij} denotes the first and f''_{ij} the second derivative of f_{ij} with respect to ρ_{ij}.

11. The terms complete and incomplete specialization refer to the two goods only.

12. It can be shown that this assumption as well as the other two made later are satisfied if the production functions are of the CES variety with the same positive, finite elasticities of substitution.

13. The interval $I(p_{i2})$ may be equal to $(0, \infty)$ as in the case of Cobb-Douglas functions or to some proper subsets of $(0, \infty)$ as in the case of other CES functions.

14. The autarky price is the equilibrium price of an isolated economy. It is a well-established result that for each value of the total factor intensity at most one such equilibrium price exists provided that the capital-intensity hypothesis holds and that the saving function is of the classical type. Cf. e.g., F. H. Hahn, R. C. O. Matthews, op. cit., part I.8.

15. $(\alpha;\beta)$ denotes as usual the open interval with α as lower and β as upper boundary point. In Eq. (59) we have to add $\{p_{i2}^{aut}\}$ since the autarky price belongs to I_i but need not lie in $(p_{i2}^{min};p_{i2}^{max})$.

16. The world market price of the export good in terms of the import good is $p_{i2}/(1-\sigma_{i1}p_{i3})$ if the i^{th} country imports capital goods and $1/(p_{i2}-\sigma_{i2}p_{i3})$ if it imports consumption goods. The home market prices are p_{i2} and $1/p_{i2}$, respectively.

17. H. Oniki, H. Uzawa, op.cit.

18. The variables referring to case α [case β] have α [β] as upper index.

19. The author stated in Wirtschaftswachstum... , p. 184, that the sign of $dn_{i2}^{\alpha\beta}/dp_{i2}^{\beta}$ is indeterminate. Eqs. (275/6), pp. 183/4, however, show that this derivative is positive.

20. Below all values in I_i^{ϵ} are simply denoted by p_{i2}.

21. Eqs. (74/7) also hold for $p_{i2}(\sigma_{i2}) < p_{i2} < p_{i2}^{z}$ and $p_{i2}^{z} < p_{i2} < p_{i2}(\sigma_{i1})$, respectively.

22. In his Wirtschaftswachstum... , p. 191, the author asserted that the curves $gr(v^{\epsilon})$ and $gr(w^{\epsilon})$ are uniformly increasing. However, this need not be the case since the sign of the derivatives of $p_{i2}-\sigma_{i2}p_{i3}^{\beta}$ and $p_{i2}/(1-\sigma_{i1}p_{i3}^{\alpha})$ with respect to p_{i2} are indeterminate.

23. It is easy to see that μ_i has the dimension of a growth rate, i.e., the dimension "per unit of time".

24. This is somewhat surprising but even if both countries use the same production techniques and their labour forces grow at the same rate the possibility of steady states with complete specialization cannot be excluded without making further assumptions. For example, conditions 1) and 2), given below in another context, guarantee that in case a) only an autarky state exists, (cf. p.).

25. The points S_1 - S_3 in Figures 4 - 6 characterize the following steady states with complete specialization:

S_1 and S_2: country 1 produces only capital goods, country 2 consumption goods and possibly also capital goods.

S_3: country 2 produces only consumption goods, country 1 both goods.

26. Thus, the relative size of the countries has some influence not only on the type of the steady states which exist but also on the stability properties of the system: the larger one country is in relation to the other the more likely is it that from every initial position a balanced growth path is approached.

27. The reason for this is that the increase of the first country's labour force is counterbalanced by the fall of its per-capita trade volume to such an extent that its total trade volume eventually grows approximately with the rate λ_2.

The final equilibrium situation approached by the two countries has all the characteristics of a steady state but one: reached at some finite point of time it will be left again at once.