RESILIENT INFRASTRUCTURE
June 1–4, 2016

THE HEAT LOADS OF A TUNED MASS DAMPER

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ABSTRACT

Modern tall buildings are often susceptible to excessive wind-induced motions. Tuned mass dampers (TMDs) are used to improve occupant comfort by reducing structural accelerations during common winds, and also to reduce building drift during stronger winds. A TMD is an auxiliary mass that is connected near the top of the structure through elements that produce restoring and damping forces. If the TMD is designed to have the appropriate natural frequency and damping ratio, vibrational energy from the structure is transferred to the TMD where it is dissipated through the TMD damping. This additional source of energy dissipation increases the effective damping of the building, reducing its dynamic motion. The energy dissipating elements of the TMD, whether linear or nonlinear, will convert some of the TMD's kinetic energy into heat. It is critical that the heat generated by the TMD motion is accurately predicted, and the damping device is demonstrated to be capable of ejecting this heat. If the device cannot eject the heat it generates during operation, the device may overheat, altering its damping properties or potentially leading to device failure. This paper studies the rate of heat generation (power) of a TMD with two common forms of damping. Simple techniques are employed to calculate the mean TMD power. Nonlinear simulations are used to evaluate this simple model, and predict the peak TMD power that is expected over a short period of time. For a given structural acceleration reduction, the TMD power is independent of the form of damping.

Keywords: Structural Control; Tuned Mass Damper; Random Vibration; Damping; Heat Loads; Power

1. INTRODUCTION

Modern materials and techniques have enabled the construction of buildings that are more lightweight, tall, and slender than ever before. These characteristics, combined with the low inherent damping of these structures, have resulted in an increase in the number of buildings that are susceptible to excessive wind-induced motion. As a result, occupants on the upper floors of these high-end buildings may experience discomfort on windy days, as they can feel the building moving under them. Moreover, less common (eg. 50-year mean recurrence interval) wind events may produce interstorey deflections that exceed serviceability requirements in place to ensure the longevity of facade elements.

Tuned mass dampers (TMDs) have been successfully employed to reduce the resonant response of tall buildings with low inherent damping (Morava et al. 2012, Soto and Adeli 2013). A TMD is represented as a mass connected to the primary structure through restoring and damping elements arranged in parallel, as shown in Figure 1. As the structure begins to sway under external excitation, a properly designed TMD will also begin to oscillate out-of-phase with the structure. Energy is thereby transferred from the structure to the TMD, where it can be dissipated through the TMD's damping mechanism. This additional source of energy dissipation increases the effective damping of the primary structure, which reduces its resonant response.

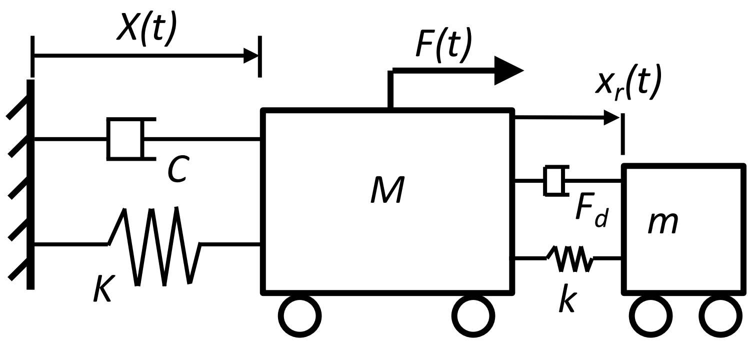

Figure 1: Schematic of structure-TMD system

Like the low inherent structural damping, the TMD damping has traditionally been represented as linear viscous damping, which allows the structure-TMD system to be modelled using well-known linear equations (Den Hartog 1985, McNamara 1977). However, it is often advantageous to utilize nonlinear TMD damping. For instance, nonlinear damping can be employed to ensure that peak TMD displacements are held within reasonable bounds during strong wind events (Love and Tait 2015). Regardless of the form of damping, TMD damping mechanisms function by converting vibrational energy from the system into heat that is then ejected into the surrounding environment. It is critical that the damping device employed is capable of ejecting heat at a sufficient, specified rate.

The rate of energy or heat ejection by the TMD is known as the TMD power. If the damping device does not have sufficient power ejection capabilities, it may overheat, leading to deterioration of its damping properties, or failure of the device. Therefore, the TMD power must be determined at the ultimate wind loading event to ensure the device continues to function as intended. Since the structure-TMD system is subjected to random excitation, both the continuous (or mean) TMD power requirements, as well as the short-term (eg. 3-minute average) power requirements must be found. The short-term average power requirements are necessary since, for brief periods of time, the power ejected by the TMD may be significantly greater than the mean power it must eject continuously.

This paper studies the rate of energy ejection of a TMD with power-law damping. Statistical linearization is employed to calculate the root-mean-square (RMS) response of the structure-TMD system. Subsequently, the mean power of the total structure-TMD system is calculated, which allows the continuous TMD power to be determined. The prediction of the continuous TMD power is evaluated using nonlinear simulations of a structure-TMD system subjected to random excitation for a long period of time. These long simulations enable a distribution of the peak hourly 3-minute average power to be determined, from which an estimate of the mean peak hourly 3-minute average power is found. The results of this study will enable rapid preliminary estimates of the power requirements of TMDs with both linear and nonlinear damping devices.

2. MODEL DEVELOPMENT

The structure-TMD system is represented by the arrangement shown in Figure 1, where M, K, and C represent the structural modal mass, stiffness, and damping constant, while m, and k represent the TMD mass and stiffness, respectively. The external force applied to the structure is F(t), while the displacement of the structure is X(t), and the relative displacement between the structure and TMD is $x_r(t)$ (henceforth the time variable, t will be omitted for brevity). Lastly, the TMD adheres to a power-law damping relationship given by:

[1] $$F_d(t) = c_\alpha m \left|\dot{x}_r\right|^\alpha sign(\dot{x}_r)$$

where c_α is the damping coefficient, α is the damping exponent, and a dot above a variable denotes a time derivative. When $\alpha = 1$, the TMD damping is linear, whereas when $\alpha = 2$, the damping is quadratic (velocity-squared). These two forms of damping are the most common for TMD applications and are therefore the ones considered in this study.

The equations of motion for the structure-TMD system shown in Figure 1 are

$$
\begin{aligned}
&(M+m)\ddot{X} + m\ddot{x}_r + C\dot{X} + KX = F \\
&m\ddot{X} + m\ddot{x}_r + F_d + kx_r = 0
\end{aligned}
\tag{2}
$$

The typical dynamic quantities of the angular natural frequency of the structure and TMD, the structural damping ratio, and the structure-TMD mass ratio are defined, respectively as:

$$
\omega_S = \sqrt{\frac{K}{M}}; \quad \omega_{TMD} = \sqrt{\frac{k}{m}}
\tag{3}
$$

$$
\zeta_S = \frac{C}{2\omega_S M}
\tag{4}
$$

$$
\mu = \frac{m}{M}
\tag{5}
$$

By substituting Eqs. [3]-[5] into Eq. [2], the equations of motion become:

$$
\begin{bmatrix} 1+\mu & \mu \\ \mu & \mu \end{bmatrix}
\begin{Bmatrix} \ddot{X} \\ \ddot{x}_r \end{Bmatrix}
+
\begin{bmatrix} 2\zeta_S\omega_S & 0 \\ 0 & 2\zeta_{TMD}(\sigma_r)\mu\omega_{TMD} \end{bmatrix}
\begin{Bmatrix} \dot{X} \\ \dot{x}_r \end{Bmatrix}
+
\begin{bmatrix} \omega_S^2 & 0 \\ 0 & \mu\omega_{TMD}^2 \end{bmatrix}
\begin{Bmatrix} X \\ x_r \end{Bmatrix}
=
\begin{Bmatrix} F/M \\ 0 \end{Bmatrix}
\tag{6}
$$

If the response distribution is Gaussian, then using statistical linearization techniques (Roberts and Spanos 1999), the equivalent amplitude-dependent viscous TMD damping is given by (Rudinger 2007):

$$
\zeta_{TMD}(\sigma_r) = \frac{1}{2\omega_{TMD}} \frac{c_\alpha \alpha \Gamma\left(\frac{\alpha}{2}\right)\left(\sqrt{2}\omega_{TMD}\sigma_r\right)^{\alpha-1}}{\sqrt{\pi}}
\tag{7}
$$

where σ_r is the RMS of the relative TMD displacement response, and $\Gamma(\ )$ denotes the gamma function. The response power spectrum of the structural and TMD responses can be calculated when the structure is subjected to white noise excitation with a power spectrum amplitude of S_0 (Crandall and Mark 1963),

$$
S_S(\omega) = S_0 \left|H_S(\omega)\right|^2
\tag{8}
$$

$$
S_r(\omega) = S_0 \left|H_r(\omega)\right|^2
\tag{9}
$$

$$
H_S(\omega) = \frac{-B_2\omega^2 + iB_1\omega + B_0}{A_4\omega^4 - iA_3\omega^3 - A_2\omega^2 + iA_1\omega + A_0}
\tag{10}
$$

$$
H_r(\omega) = \frac{-B_2\omega^2}{A_4\omega^4 - iA_3\omega^3 - A_2\omega^2 + iA_1\omega + A_0}
\tag{11}
$$

where i is the imaginary number, and

$$
\begin{aligned}
A_0 &= \omega_S^2 \omega_{TMD}^2 \\
A_1 &= 2\zeta_S \omega_S \omega_{TMD}^2 + 2\zeta_{TMD}\omega_{TMD}\omega_S^2 \\
A_2 &= \omega_S^2 + \omega_{TMD}^2(1+\mu) + 4\zeta_S \zeta_{TMD}\omega_S \omega_{TMD} \\
A_3 &= 2\zeta_S \omega_S + 2\zeta_{TMD}\omega_{TMD}(1+\mu) \\
A_4 &= 1
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
B_0 &= \omega_S^2 \omega_{TMD}^2 \\
B_1 &= 2\zeta_{TMD}\omega_S \omega_{TMD} \\
B_2 &= \omega_S^2
\end{aligned}
\tag{13}
$$

By integrating the power spectrum, the variances of the structural and TMD relative response are found (McNamara 1977):

$$
\sigma_S^2 = S_0 \pi \left(\frac{\frac{B_0^2}{A_0}(A_2A_3 - A_1A_4) + A_3(B_1^2 - 2B_0B_2) + A_1B_2^2}{A_1(A_2A_3 - A_1A_4) - A_0A_3^2} \right) \tag{14}
$$

$$
\sigma_r^2 = S_0 \pi \left[\frac{A_1B_2^2}{A_1(A_2A_3 - A_1A_4) - A_0A_3^2} \right] \tag{15}
$$

Since the TMD's RMS response amplitude, σ_r is required in Eq. [7], iteration must be used to solve Eqs. [7], [14] and [15].

3. TMD POWER

3.1 Continuous Power

For an ergodic system, the expected (mean) energy of the system is constant with time, therefore the mean rate of energy input into the system (power) must equal the mean rate of energy output from the system. This power requirement is termed the continuous power requirement, since the damping mechanisms should be capable of ejecting this power continuously during the entire loading event. For a system subjected to white noise excitation, the total mean power input into the system is given by (Newland 1966)

$$
\mu_p^{(T)} = \frac{\pi S_0}{M} \tag{16}
$$

This equation states that the total rate of energy input is only dependent upon the generalized mass of the structure, and the amplitude of the white noise excitation. Although somewhat counter-intuitive, the total system power is independent of the structural damping, or even the presence of a TMD.

The only sources of energy dissipation in the structure-TMD system are the damping elements of the structure and TMD. The total power ejected from the system is therefore the summation of the power ejected through the inherent structural damping and the TMD damping

[17] $$\mu_p^{(T)} = \mu_p^{(str)} + \mu_p^{(TMD)}$$

The continuous (or mean) power ejection associated with the inherent structural damping is

[18] $$\mu_p^{(str)} = E\left[C\dot{X}^2\right] = 2\zeta_S \omega_S^3 M \sigma_S^2$$

By rearranging Eq. [17], and utilizing Eqs. [16] and [18], the continuous power ejected by the TMD is

[19] $$\mu_p^{(TMD)} = \frac{\pi S_0}{M} - 2\zeta_S \omega_S^3 M \sigma_S^2$$

Equation [19] shows that the continuous power ejected by the TMD is independent of the form of TMD damping. Indeed, for a given level of structural motion reduction, the continuous TMD power required to achieve that reduction can be determined definitively before the TMD mass, damping, or stiffness are known.

As an alternative to Eq. [19], the continuous TMD power can be determined by directly computing the expected value of the TMD power:

[20] $$\mu_p^{(TMD)} = E\left[F_d \dot{x}_r\right]$$

By making use of Eq. [7], Eq. [20] is evaluated to produce

[21] $$\mu_P^{(TMD)} = c_\alpha m \alpha \sqrt{\frac{2^{\alpha-1}}{\pi}} \Gamma\left(\frac{\alpha}{2}\right) \omega_{TMD}^{\alpha+1} \sigma_r^{\alpha+1}$$

While not the focus of this study, numerical simulations indicate that Eqs. [19] and [21] yield the same results. Therefore, Eqs. [19] and [21] provide two simple methods to determine the continuous power requirements of the TMD.

3.2 Short-term power

The previous section outlined two simple methods by which the continuous TMD power ejection requirements can be calculated. Equally as critical to the TMD design is the power that the damping device must eject over a short period of time. Since the amplitude of the TMD displacement is a random variable, the short-term average TMD power ejection is also a random variable. As a result, the peak "a"-minute average power that is expected to occur in one hour (where "a" is much less than sixty), is greater than the continuous power ejection. Ongoing research is attempting to quantify the peak hourly a-minute average power using simple formulae, however for this paper the behaviour will be studied using numerical simulations. The peak hourly a-minute average power is determined by calculating the a-minute moving power average of the time series over one hour and selecting the maximum. Figure 2 shows a one hour time series segment, taken from the 180 hour simulation, of the power ejected by a TMD, along with the continuous power and the 3-minute average power. From Figure 2, it is clear that the 3-minute average power fluctuates considerably above and below the continuous power ejection.

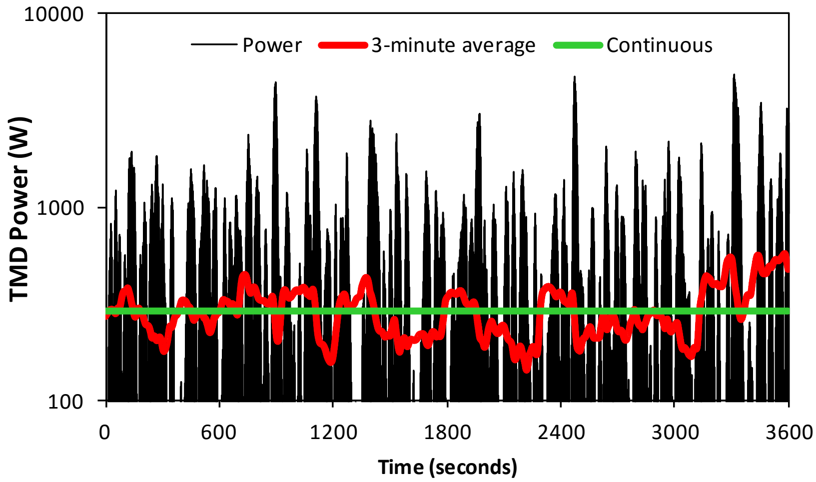

Figure 2: Typical time series of TMD power

4. NONLINEAR SIMULATIONS

Nonlinear numerical simulations are conducted on several structure-TMD systems. The structure has a generalized mass of 10,000 tonnes with a natural angular frequency of 1.26 rad/s, and an inherent damping ratio of 1%. For all simulations, the structure is subjected to 180 hours of band-limited white noise excitation, which will produce 180 peak hourly response observations from which peak response statistics can be determined.

The properties of the TMD systems studied are shown in Table 1. Structure-TMD systems are considered with a modest mass ratio of 1%, and a high mass ratio of 5%. Each system is optimally tuned using well-known formulae (Warburton 1982). The TMD has either linear or quadratic damping, which are the most common forms of TMD damping. The TMD damping coefficient, c_α is selected to provide the optimal TMD damping for the linear system, and the optimal linearized TMD damping at an excitation level of $\pi S_0/M$ = 427 W for the velocity-squared damping case. This level of excitation corresponds to the structure without a TMD experiencing a mean-peak hourly acceleration equal to 20 milli-g.

For each simulation conducted, the continuous TMD power is calculated as the mean of the entire simulation. In addition, the peak hourly 3-minute average power occurring in each of the 180 hours simulated is determined. Using this peak hourly power data, a histogram can be generated to show the variation of the peak 3-minute average power for each hour. The mean-peak hourly 3-minute average power, $\hat{P}_{3\text{-min}}$ is estimated by calculating the mean of the 180 samples. The standard deviation of these samples, $\sigma_{\hat{P}}$ is also determined to measure the spread of the distribution.

A summary of the simulation results is provided in Table 2. The continuous power is also calculated using Eq. [19]. Histograms of the peak hourly 3-minute average power (normalized by the continuous power ejection) are shown in Figures 3 and 4 for the simulations corresponding to μ = 1%, with $\pi S_0/M$ = 2668W, and μ = 5%, with $\pi S_0/M$ = 427W, respectively. The peak factor of the power, PF_p is also shown, where it is calculated as:

[22] $$PF_P = \frac{\hat{P}_{3-\min}}{\mu_P^{(TMD)}}$$

Table 1: TMD properties

μ	ω_{TMD}/ω_S	α	c_α
1%	0.993	1	0.125
		2	0.510
5%	0.964	1	0.268
		2	3.340

Table 2: TMD power ejection for structure-TMD systems considered

α	μ	Peak Accel. (milli-g)	$\pi S_0/M$ (W)	$\mu_P^{(TMD)}$ (W) Eq. (19)	$\mu_P^{(TMD)}$ (W) Sim.	$\hat{P}_{3\text{-min}}$ (W)	$\sigma_{\hat{P}}$ (W)
1	1%	20	427	297	294	633	114
		50	2668	1854	1837	3954	711
	5%	20	427	360	355	631	82
		50	2668	2250	2219	3946	510
2	1%	20	427	297	291	631	115
		50	2668	1764	1749	3753	752
	5%	20	427	360	353	632	82
		50	2668	2188	2144	3873	571

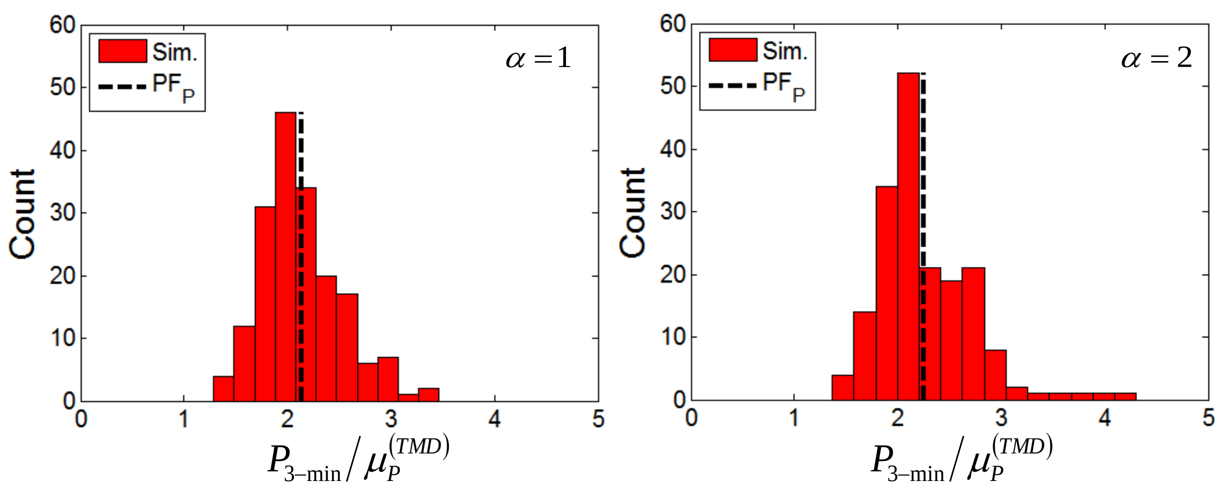

Figure 3: Histogram of normalized peak hourly 3-minute average power (μ = 1%, $\pi S_0/M$ = 2668 W)

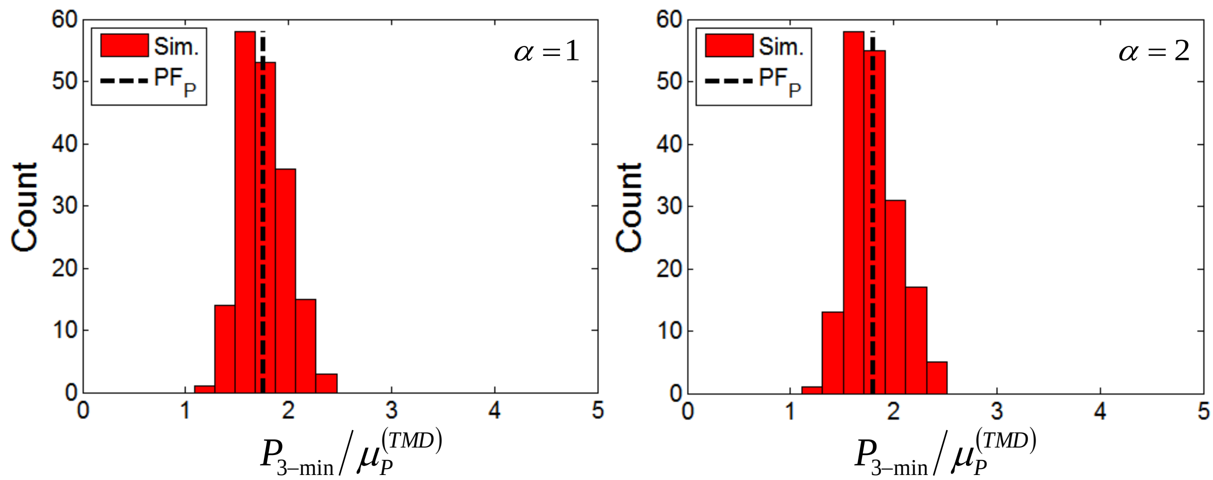

Figure 4: Histogram of normalized peak hourly 3-minute average power (μ = 5%, $\pi S_0/M$ = 427 W)

5. DISCUSSION

The continuous power ejection predicted using the simple model expressed by Eq. [19] is in excellent agreement with the simulations. The maximum relative error between the modelled and simulated results is 2%. The continuous TMD power can therefore be predicted using a linearized model for TMDs with linear or quadratic damping.

Equation [19] suggests that the continuous power of a TMD designed to achieve a specified acceleration reduction is independent of the form of TMD damping. The more power a TMD ejects, the greater the acceleration reduction it provides. From Table 2, the maximum relative difference between the continuous TMD power ejection of the linear TMD, and the corresponding TMD with quadratic damping is 1% at the targeted structural excitation amplitude of $\pi S_0/M$ = 427 W. Similarly, the maximum relative difference of the mean-peak hourly 3-minute average power from the simulations for systems with linear and quadratic is less than 1% at the targeted structural excitation amplitude of $\pi S_0/M$ = 427 W. The simulations conducted indicate that at the targeted structural response amplitude both the continuous and peak short-term power ejection requirements of the TMD are independent of the form of TMD damping that is employed. At structural response amplitudes higher than the targeted amplitude, the TMD damping for the nonlinear system will be higher than optimal, which results in a small reduction in the power ejection and effective structural damping.

For the simulations conducted, the peak factor, PF_p which is the ratio between the mean-peak hourly 3-minute average and the continuous power, is between 1.8 and 2.2. The peak factor was approximately 2.2 when the mass ratio was 1% for both linear and quadratic damping. When the larger mass ratio of 5% was studied, the peak factor was 1.8. This trend may suggest that a larger mass ratio will result in a 3-minute average power ejection that varies less with time. The coefficient of variation (ratio of the standard deviation to the mean) of the peak hourly 3-minute average power was approximately 20% when the mass ratio was 1%, and 13% when the mass ratio was 5%. This trend also suggests that there is less variability in the peak hourly 3-minute average power when the mass ratio increases. Therefore, a larger TMD mass ratio may result in a more invariant short-term power requirement with time.

The major finding of this study is that for a given level of structural acceleration reduction, the power requirements of a TMD energy dissipation mechanism will be identical regardless of the type of device employed.

CONCLUSIONS

Tuned mass dampers (TMDs) have been successfully employed to reduce the resonant response of tall buildings subjected to random wind loading. The TMD functions by transmitting vibrational energy from the structure to the TMD, where it can be dissipated through the TMD damping. The energy is dissipated in the form of heat, therefore it is critical that the damping mechanism can eject heat at a specified rate or power. This study investigated the power ejection of structure-TMD systems when the TMD possessed power-law damping.

Statistical linearization was employed to represent the nonlinear structure-TMD system. Using this representation, the RMS response of the structure and TMD were determined. Next, it was shown that the power input into the structural system by white noise excitation is constant, regardless of the level of structural damping, or the presence of a TMD. With this behaviour established, the mean or continuous power ejection of the TMD was calculated as a function of the variance of the structural response.

Since the excitation is random, the short-term average power will fluctuate with time, resulting in a short-term power requirement that is considerably greater than the continuous power requirement. Numerical simulations were conducted to evaluate the proposed model and determine the peak hourly 3-minute average power of the TMD. Several structure-TMD systems were subjected to 180 hours of random excitation, enabling the peak hourly power statistics to be computed.

The continuous power prediction provided by the simple model and simulations were in excellent agreement. For a specified structural acceleration reduction, the continuous TMD power is calculable a priori, regardless of the form of TMD damping. Moreover, the short-term average power ejection of the system was observed to be insensitive to the type of damping. A higher TMD mass ratio was observed to result in a short-term average power that deviated less from the continuous power.

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