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by

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# Competition in Public School Districts: Charter School Entry, Student Sorting, and School Input Determination

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**Abstract**

I develop and estimate an equilibrium model of charter school entry, school input choices, and student school choices. The structural model renders a comprehensive and internally consistent picture of treatment effects when there may be general equilibrium effects of school competition. Simulations indicate that the mean effect of charter schools on attendant students varies widely across locations, and is on average 11% of a standard deviation of test scores. The mean spillover effect on public school students is marginal, but positive, and lifting caps on charter schools would more than double entry but reduce gains for attendant students.

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# 1 Introduction

The provision of school choice is often proposed as a way to improve educational outcomes for students in poorly performing public schools.[1] Charter schools are at the center of the recent debate concerning education policy reforms, such as President Obama's *Race to the Top*, which rewards states that lift restrictions on charter school growth (White [2009]).[2] In 2010, more than 1.6 million students attended almost 5,000 charter schools in 40 states (Snyder and Dillow [2012]). The number of students attending charter schools has arguably been constrained by statewide legislative caps on the number of charter schools in two-thirds of the states with charter schools. Policymakers would like to know how student achievement would be affected if they expanded the role of charter schools in public education, and how it has been affected by existing charter schools. Advocates argue that charter schools improve the performance of students attending charters ("direct effect") and students attending competing public schools ("spillover effect"). However, critics of charter schools argue that they "cream-skim" – that is, the better outcomes at charter schools represent student selection, not test score gains – and that charter schools have a negative spillover effect on students attending competing public schools.

The need to understand how charter schools affect student achievement has motivated a large body of empirical work, most of which uses one of two methods to account for potential cream-skimming.[3] In the first, authors use lottery designs which compare achievement of applicants to oversubscribed charter schools who are randomized into those charters with those randomized out of those charters (Angrist et al. [2012], Hoxby and Rockoff [2004]). The other set of studies uses student fixed-effects to estimate Value-Added models of test score growth using statewide panel data on students who switch between public and charter schools (Bettinger [2005], Bifulco and Ladd [2006], Hanushek et al. [2007], and Sass [2006]). Bifulco and Ladd [2006], Chakrabarti [2008], Imberman [2011], and Sass [2006] use a variety

[1] "School choice" is a collective term which refers to charter schools, magnet and alternative public schools, and private schools coupled with voucher schemes.

[2] Charter schools are publicly funded schools that compete with traditional public schools and which, like public schools, cannot selectively admit students. They typically have considerably more autonomy than public schools regarding personnel decisions, curricula, school hours, and pedagogical methods, and often have lower per-pupil resources due to a lack of separate capital funding streams. All students have access to a public school, but not all students have access to charter schools because charter schools enter certain areas and not others. Though charter schools are technically a type of public school, for brevity I refer to them as "charter schools" and traditional public schools as "public schools."

[3] There are many papers that have studied either parts of this problem or related problems. Glomm et al. [2005] describe determinants of charter school entry decisions. Altonji et al. [2010] develop an econometric framework to quantify the extent to which private school voucher programs cream-skim from public schools. This paper naturally complements this work by quantifying the extent to which spillover effects of charter schools on public school students are positive or negative.

of methods to estimate the spillover effect of school choice. Estimates of direct and spillover effects are widely mixed across studies. Gleason et al. [2010] use a lottery design to study charter schools operating in a number of states and find substantial heterogeneity in their impacts on attendant students, which is consistent with the mixed results from other authors.

Prior research highlights the heterogeneity of charter school effects on student achievement, but it cannot provide a comprehensive evaluation of how charter school policy affects student achievement for several reasons. First, policymakers interested in the effect of lifting caps on the number of charter schools need a way to extrapolate findings from studies of existing charter schools to new charter schools serving different populations of students. Second, the above studies do not model why charter schools open in certain places but not in others (Hanushek et al. [2007]). Understanding which public schools charters will choose to compete with and which students will be exposed to them is vital to understanding the effects of policies that might increase the number of charter schools. Third, lottery-based designs, which have recently grown in popularity, cannot quantify the effect of *all* existing charter schools on student achievement because they do not provide any mechanism for extrapolating results from oversubscribed charter schools to those that are not oversubscribed. We might think the potential for bias is large if oversubscribed charter schools are also those that households believe will deliver stronger benefits to their children. Fourth, although several authors have quantified spillover effects, none have provided a framework that uses bias from estimated spillovers to adjust their estimates of the direct effect of school choice. For example, Angrist et al. [2012] and Cullen et al. [2006] note that their findings do not take into account potential spillover effects stemming from equilibrium changes at public schools caused by the introduction of school choice programs. This could be important if a charter school improved outcomes for all students, but students remaining at the public school benefited more than did students attending the charter. In this case, a lottery-based design would find a negative direct effect of charter schools, even though the achievement of all students increased.

This paper develops and estimates an equilibrium model of competition between charter and public schools to confront these issues by modeling three key components of the debate on school choice: i) student school choices, ii) charter and public school inputs, and iii) charter school entry decisions. The model incorporates selection on student ability in two ways: charter school entry decisions take into account market-level ability distributions, and within markets, student sorting is a function of heterogeneous student ability and inputs at both charter and public schools. Modeling student school choices as a function of both student and school characteristics allows for generalization of estimates based on existing charter schools to charter schools that might enter in new markets were caps lifted, even in the presence of student sorting on unobserved ability. The model of school input choices allows charter schools to have heterogeneous treatment effects across markets through variation in input provision and also predicts what inputs for public schools would have been in the absence of charters, which is necessary to properly quantify spillover effects: how charter schools have changed public school student performance relative to the monopoly scenario. The equilibrium framework provides an internally consistent method to quantify both spillover effects and the bias introduced when one ignores equilibrium responses by public schools, unifying the two strands of the literature where authors estimate the direct effect of charter school entry or use different students and schools to estimate spillovers. By modeling charter school entry it is possible to quantify how many more charter schools would open and in which markets they would open were caps lifted, which is important if the effects of charter schools are heterogeneous across markets. This paper also estimates the extent to which contextual peer effects affect student achievement at both public and charter schools, meaning that the model allows for spillovers through both changes in student ability due to sorting and through changes in public school effort levels. Several papers have considered peer effects that operate through mean ability, but this is the first to also allow for a spillover effect due to an equilibrium change in public school effort.[4]

The model is estimated using maximum likelihood on administrative data from the North Carolina public school system. The data contain the universe of schools and students in the North Carolina public school system from 1998 to 2001 and include variables that enable estimation of the model's demand and supply sides, such as public and charter school locations, charter school entry decisions, and detailed per-pupil school resources, which enter the model as a per-pupil capital index. School attendance, average weekly hours of homework reported done, and standardized test scores are recorded for each student in each year. Weekly hours of homework done comprise the second school input to test score production. The student-level data also contain student locations, which enter the model through the distance cost of attending a school and exogenously shift the probability a student will attend a charter school. I aggregate these student-level distance data into market-level distance distributions, which affect ability sorting into charter schools and therefore provide an exclusion restriction for identifying peer effects at both charter and public schools.

This paper differs from the literature by modeling charter school entry and is the first to build and structurally estimate an equilibrium model of endogenous school inputs and

[4]This paper complements a related literature, where authors model competition between public and private schools to study student sorting due to peer effects, the effectiveness of private school vouchers, and competition between public schools and private schools (Caucutt [2002], Epple and Romano [1998], Ferreyra [2007], Nechyba [2000]). None of them explore competition between either public and charter or different public schools, which they assume are monolithic and do not make input choices or any other decisions.

student sorting. The rich data allow computation of market-level distance distributions, which help to quantify the role of peer effects at both public and charter schools. The most relevant work to this project is Ferreyra and Kosenok [2012], who develop and estimate a structural model focusing on charter school location and input decisions in Washington DC. In their model, public schools do not choose effort inputs and follow reduced-form policy rules, which means that they cannot endogenize spillover effects that operate through changes in public schools input choices. Second, in their model charter schools may draw from the citywide school district of Washington DC, which may be appropriate for their environment (a large urban school district), but is not appropriate for charter school attendance choices for households spread over the larger area examined here, the state of North Carolina. In more recent work, Walters [2012] estimates a flexible demand-side model of charter school application and attendance decisions, and then uses these estimates to quantify the distribution of treatment effects for students attending new charter schools. Unlike this paper, he does not model charter school input choices, endogenize charter school location decisions, allow public schools to respond to charter schools (creating the potential for spillovers), or include peer effects in the production technology – which may affect both internal validity (recovering the distribution of treatment effects) and external validity (understanding the effects of charter school expansion).

There are many papers studying either parts of this problem or related problems. Glomm et al. [2005] describe determinants of charter school entry decisions. Altonji et al. [2010] develop an econometric framework to quantify the extent to which private school voucher programs cream-skim from public schools, and find a small, but negative, effect on the performance of students remaining at the public school. Many authors have modeled student (or household) effort choices, as opposed to school-wide effort choices.[5] McMillan [2004] develops a theoretical model where a public school may reduce its provision of costly effort in response to competition from a private school. My paper naturally complements this work by quantifying the extent to which spillover effects of charter schools on public school students are positive or negative.

The estimated model fits the data well. Because it takes into account student sorting, charter school entry, and the equilibrium provision of school inputs, it provides a natural framework to answer several questions related to charter school policy. First, the estimated mean direct effect of charter school entry on attendant students is 11% of a standard deviation (sd) in test scores and there is a positive, though marginal mean spillover effect of 1.6% sd on public school students in those markets. These effects are driven by the more effective technology of charter schools, the relative unresponsiveness of public school effort to charter

[5] See De Fraja and Landeras [2006],Cooley [2009], and Ferreyra and Liang [2012] for example.

school entry, and lack of strong sorting on student ability. Having observed their superior technology, one might force charter schools to serve all students in markets they enter, but this would *reduce* student achievement and the number of charter schools because charters suffer from diseconomies of scale and drastically reduce effort when forced to serve many more students. Second, the direct effect of a particular charter school is increasing in demand for it, which should caution policymakers seeking to generalize results from lottery-based studies based on oversubscribed charter schools. Third, more than twice as many charter schools would enter if caps were lifted, though the larger average market size and lower average per-pupil capital in new entry markets greatly diminishes their estimated direct effects. Finally, peer effects that operate through mean school ability are substantial at public schools and small in magnitude at charter schools.

# 2 Model

An economy in the model is one market and time period. Each economy contains one traditional public school, one potential charter school entrant, and a continuum of households, each with one student. The potential charter school entrant makes an entry decision, after which the public school and charter school (if it has entered) choose effort inputs to test score production.[6] Households differ by student ability and location within the market, and maximize utility by choosing a school. In equilibrium, the charter school makes the optimal entry decision based on its expected equilibrium value of entry, neither school wishes to change its effort level, and no household wishes to change its school choice. All households are assigned to the public school if the charter school did not enter.

Denote the public school *tps* (for "traditional public school") and the potential charter school entrant *ch*. Schools and student are indexed by $s$ and $i$, respectively. Market and time subscripts are suppressed in this section to simplify exposition.

## 2.1 Students

There is a continuum of students of measure $\mu$. A student $i \in I$ has ability $a_i$, where $a_i \sim F(a_i)$ with density $f(a_i)$. Student $i$ maximizes utility by choosing a school $s_i \in S_i$, their school choice set. If there is a charter school in student $i$'s market then $S_i = \{tps, ch\}$, otherwise $S_i = \{tps\}$. Student choice-specific utility is $u_{is} = y_{is} - c_{is} + \eta_{is}$, where $y_{is}$ is the test score, $c_{is}$ is the non-pecuniary cost of attending school $s$,[7] and $\eta_{is} \sim \mathrm{N}\left(0, \sigma_\eta^2\right)$ is a

[6] Charter school "entry" indicates either entry or continued operation within a market.

[7] Charter and public schools cannot charge tuition.

choice-specific preference shock.

The test score depends on student ability,mean ability of students at school $\bar{a}_s$, school effort $e_s$ and capital $k_s$, and a productivity shock $\nu^y_{is}$, which is realized after students choose schools:[8]

$$y_{is} = a_i \omega_s \bar{a}_s^{\theta_s} \left( \alpha_s e_s^{\beta_s} + (1-\alpha_s) k_s^{\beta_s} \right)^{\tau_s/\beta_s} + \nu^y_{is} = Ey_s(a_i, \bar{a}_s, e_s, k_s) + \nu^y_{is}, \tag{1}$$

where $\omega_s$ is Hicks-neutral total factor productivity (TFP) and $\nu^y_{is} \sim i.i.d.\,\mathrm{N}\left(0, \sigma^2_{\nu^y}\right)$. Student $i$'s cost of attending school $s$ depends on school effort, distance from the student to the school, $r_{is}$, a fixed cost of attending the charter school, $c_{ch}$, and an ability-specific cost of attending the charter school $c_{ch,a}$, according to $c_{is} = c_e e_s + c_r r_{is} + (c_{ch} + a_i c_{ch,a})\, 1\{s = ch\}$.[9] The optimal school choice policy for $i$ is

$$s_i^* = \arg\max_{s \in S_i} \{\mathrm{E}_{\nu^y}[u_{is}]\}. \tag{2}$$

## 2.2 Schools

Each market is endowed with a public school and a potential charter school entrant. Each school is endowed with a capital level and location within the market, all of which are known to the potential charter school entrant before it makes its entry decision, denoted $z \in \{no\ entry, entry\}$.

The school's objective is a weighted average of average test score of students at the school $\bar{y}_s$, school size $\mu_s$, and cost function $c_s$:

$$v_s = \delta_{ys}\bar{y}_s + \mu_s - c_s = \bar{a}_s E y_s(1, \bar{a}_s, e_s, k_s) + \mu_s - c_s. \tag{3}$$

Student responses to school choices determine both $\bar{y}_s$ (through mean ability $\bar{a}_s$) and $\mu_s$ (Appendix A). The school's objective can be thought of as a reduced form of an original objective and a school budget constraint from the perspective of the school principal. If the average test score did not directly enter the school's objective function ($\delta_{ys} = 0$), the model would predict that monopoly public schools would exert no effort, because they would draw all students in their market without incurring any effort cost. If no students attend the school ($\mu_s = 0$) she may be fired for poor management.

[8]Some authors in this literature include student covariates, such as race, in the test score outcome equation (Angrist et al. [2012]) or student utility (Ferreyra and Kosenok [2012]). This paper effectively includes such covariates in student-specific ability. Note that mean peer covariates are consequently subsumed by the mean ability of students at the school.

[9]None of the parameters $c_e, c_r$, $c_{ch}$, or $c_{ch,a}$ are assumed to be "costs" in the estimation of the model in the sense that their signs are unrestricted.

The school's cost function allows for interactions between effort and both capital and school size:

$$c_s = \psi_{s1} e_s + \psi_{s2} e_s \mu_s + \psi_{s3} e_s k_s + \psi_{s4} e_s \bar{a}_s - \left(\psi_{\text{mkt.size}} \mu + \psi_{\text{fr.Black}} \mu_{\text{Black}}\right) 1\{s = ch\}, \tag{4}$$

where $\psi_{\text{mkt.size}}$ is the charter school's valuation of market size $\mu$, $\psi_{\text{fr.Black}}$ is the marginal valuation of the charter school places on being located in a market with Black students and $\mu_{\text{Black}}$ is the share of Black students in the market. This last term allows the model to capture the fact that charter schools may want to serve Black students, which is sometimes explicitly part of their mission statements (Bifulco and Ladd [2007]).

If there is a charter school in the market, each school has a policy[10]

$$e_s^* = \arg\max_{e_s} \; \mathrm{E}_{\nu^{\mathbf{e}}} \left[v_s(\mathbf{e})\right] = \arg\max_{e_s} \; \mathrm{E}_{\nu^{\mathbf{e}}} \left[v_s \left(e_s, e_{-s}\right)\right], \tag{5}$$

i.e. each school chooses its own effort $e_s$ to maximize its expected objective, given the action of the other school $e_{-s}$ and distribution of effort productivity shocks $\nu^{\mathbf{e}} = (\nu^e_{ch}, \nu^e_{tps})$. The pair of effort productivity shocks $\nu^{\mathbf{e}}$ is realized after schools choose effort. Each shock $\nu^e_s$ determines observed effort according to

$$e_s^o = e_s \nu_s^e, \tag{6}$$

where $\ln \nu_s^e \sim i.i.d.\, \mathrm{N}\left(0, \sigma^2_{\nu^e}\right)$.

Students in monopoly markets have no school choice, so the average ability for students attending the monopoly public school is the market average $\bar{a}$, and the measure of students attending is the market size, giving the monopolist objective $v_{tps}^{mono} = \delta_{y,tps} \bar{y}_{tps} + \mu - c_{tps}$. The monopolist public school has a policy

$$e_{tps}^{mono\;*} = \arg\max_{e_{tps}} \; \mathrm{E}_{\nu^e_{tps}} \left[v_{tps}^{mono}(e_{tps})\right]. \tag{7}$$

The charter school's optimal entry decision is

$$z^* = \mathit{entry} \Leftrightarrow \mathrm{E}_{\nu^{\mathbf{e}}} \left[v_{ch}(\mathbf{e}^*)\right] \geq \underline{v}, \tag{8}$$

where $\mathbf{e}^*$ is the pair of equilibrium effort levels chosen by both schools and $\underline{v}$ is a random variable known to the charter school that denotes an exogenous fixed cost of entry and/or operating.

[10] Variables in bold denote the pair of variables for both schools in a market, e.g. $\mathbf{k} = (k_{ch}, k_{tps})$.

## 2.3 Equilibrium

The solution concept is subgame perfect Nash equilibrium, so the model is solved by backwards-induction. The charter school first makes an entry decision leading to either the entry or monopoly subgame.

A *Subgame Perfect Nash Equilibrium* for the period game is an entry decision and student and school decisions such that:

- $a$: the charter school enters if and only if the entry cost shock is less than its expected payoff in the entry subgame (eq. 8),
- $b$: if the charter school enters, the ensuing subgame equilibrium is the *Entry Subgame Equilibrium*, and
- $c$: if the charter school does not enter, the ensuing subgame equilibrium is the *Monopoly Subgame Equilibrium*.

An *Entry Subgame Equilibrium* consists of chosen charter and public school effort levels, realized school effort levels, household school choices, and mean school ability levels such that:

- $b.i$: each school's chosen effort level maximizes the expected objective of the school, given the other school's chosen effort level (eq. 5),
- $b.ii$: realized school effort for each school is the chosen effort level multiplied by the effort productivity shock (eq. 6),
- $b.iii$: students choose schools to maximize utility, given realized school effort levels (eq. 2), and
- $b.iv$: mean ability at each school is consistent with realized school effort levels and student optimization (eq. 23).

A *Monopoly Subgame Equilibrium* is a public school effort level, realized public school effort level, and public school size and mean ability such that:

- $c.i$: the public school's chosen effort level solves the monopolist public school's problem (eq. 7),
- $c.ii$: realized public school effort is the chosen effort multiplied by its effort productivity shock (eq. 6),
- $c.iii$: all students attend the public school ($\mu_{tps}^{mono} = \mu$), and
- $c.iv$: mean ability at the public school is market mean ability ($\bar{a}_{tps}^{mono} = \bar{a}$).

I solve for the equilibrium value of entry by first deriving student school choices as a function of school effort, their own ability, and their distance to charter and public schools. I then find a Nash equilibrium in chosen charter and public school effort levels by iterating best responses, which requires computation of a fixed point in mean student abilities at charter and public schools within every iteration. Equilibrium effort levels are then hit with productivity shocks, after which they enter household school choice problems. The model is solved numerically (details in Appendix A).[11] I have proved existence of an equilibrium (Appendix D.1) and though I have not proven there is a unique equilibrium, an extensive search has never found more than one pair of equilibrium effort levels in the entry subgame (discussion in Appendix D.2).

The model generates rich interactions between charter and public schools that capture key parts of the debate on school choice and are key to evaluating charter school policy. Charter school entry decisions depend on market characteristics, such as student ability and locations, and public school capital. Moreover, because the effect of charter school entry on both charter and public school students varies by market, the model inherently produces heterogeneous treatment effects of charter schools so it can be used to extrapolate findings to markets charter schools have not yet entered. The model can generate direct and spillover effects of either sign. If the charter school is much more productive than the public school, high ability students will attend it with very high probability, which is attenuated if students in the market are on average far from the charter school. Depending on market characteristics and parameter values, a public school may not find it advantageous to try to retain high-ability students but rather may decrease its equilibrium effort from the monopoly level and cater to students with below-average ability.

## 2.4 Markets

Each student's school choice set (market) must be defined to solve and estimate the model. A market in the model is similar to the catchment area for a traditional public school. In reality, markets partition North Carolina, though these markets does not necessarily correspond to public school attendance zones. To make the model solution independent across markets, charter schools may compete with at most one public school, and each public school may compete with at most one charter school. Each charter school is designated to be in the market of the public school closest in distance. The assumption that each public school competes with one charter school is supported by the data. In the four instances when the same public school is the closest public school to more than one charter school, I designate the charter school closest to the public school as its competitor and exclude the other charter schools from the sample. Private schools are not part of school choice sets.[12]

[11] The model is solved using an approximate version of the schools' problems where schools do not integrate over $\nu^{\mathbf{e}}$. I have verified that this does not substantially change effort choices. All simulations incorporate $\nu^{\mathbf{e}}$.

[12] Private schools in North Carolina may target different students than charter schools. The share of Black students at charter schools in the estimation sample is 25%, while the share of Black students in private schools in the state was 7% in 2010. Source: Author's calculations from NCES [2010], `http://nces.ed.gov/surveys/pss/privateschoolsearch/`.

There are two main advantages to structuring markets this way. First, limiting the extent of competition allows me to model policy-relevant interactions between charter and public schools. This paper is the first to develop and estimate a model where both charter *and* public schools choose inputs to student achievement. Solving this model is infeasible unless there is some restriction on competition between charter and public schools. If charter schools could draw students from multiple public schools all public schools would then indirectly compete with each other, unless markets were restricted in some manner. Other models of school choice focus on distinct metropolitan areas that are much smaller than an entire state (Caucutt [2002], Epple and Romano [1998], Ferreyra [2007], Nechyba [2000]). Because this paper considers the entire state of North Carolina, it is natural to not allow every school to compete with every other school in the state.

Second, a geographic rule provides a simple mechanism to create each student's school choice set, assigns potential charter school entrants to competing public schools even in the absence of entry, which is essential to model entry, and captures observed charter school entry patterns (Appendix D.6). Because charter schools do not have geographic cut-offs for attendance, one cannot always surmise which public school a student would have attended had he not chosen the charter school, nor which charter school a student observed in a public school could have chosen to attend.[13]

# 3 Data

Model parameters are estimated using administrative data from North Carolina. The data are taken from the universe of public and charter schools, and were provided by the North Carolina Education Research Data Center (NCERDC). The data contain variables necessary to estimate student-level test score production functions using school-level inputs, and include detailed panels on teachers, students, and charter and traditional public schools in the North Carolina public school system. For teachers, the data contain years of experience and the school in which they work. For students, the data contain demographic characteristics, which school they attend, grade in school, standardized reading and math test scores for students in grades 3-8 and grade 10, self-reported weekly hours of homework done, and student household locations. School-level data are used to compute computers per pupil and

[13] An alternative way to assign charter schools to public school markets would be to link charter and public middle schools through elementary public schools feeding both. However, half of charter school students in the data are never observed in public elementary schools because they always attended charter schools. Therefore, it is impossible to know which public middle school they would have gone to based on their public elementary school attendance.

district per-pupil revenues.[14]

The test score in the model is the average of the reading and math test scores, and is normalized to have a mean of 3 and standard deviation of 1 for each grade to make it comparable across grades. The average test score is 3 to ensure that all markets have ability distributions with positive means, otherwise the model would predict that effort for any school in such a market would be zero if effort was costly for schools.[15] Student effort is measured with self-reported data on the hours of homework students say they typically do per week, which are then averaged within each school-year, to create a school-wide effort variable per year. School effort costs capture the time and resources required to develop, assign, and check that student homework, and captures broad differences in workloads across schools.[16] Both capital and effort inputs are assumed to be the same for all students at the same school. The assumption is innocuous for capital, because most school capital is applied fairly evenly to students at the schools.[17] Even if it were not, data on within-school capital expenditures are not available. By contrast, effort choices for individual students are observed. Assuming that there is only one effort level per school per year allows me to avoid solving for each student's effort choice. I lose information on the variation of effort at a school, which means that I may end up overestimating the variance of ability distributions.[18]

The estimation sample consists of middle schools (grades 6-8) from 1998-2001. Because elementary, middle, and high schools may have different test score production functions, the model is estimated on one school type. Middle school provides a natural decision-point for students because most students switch schools between grades 5 and 6. The data contain standardized test scores all years of middle school but only one year of high school and a subset of years for elementary school. Additionally, magnet schools, which have more open enrollment schemes than traditional public schools, are not a big issue at the middle school level: about 80% of students attending magnet schools are not in the grades 6-8 in the years 1999-2001. The analysis period includes the first year charter schools were allowed in North Carolina (1998), through the period of most charter school openings, until the statewide cap

[14]Charter schools are considered to be their own school districts in North Carolina. The data do not indicate whether a charter school was oversubscribed. Consequently, data on whether all charter schools held lotteries, and on which students won or lost, are not available.

[15]The estimation sample has a mean test score of 3.05 and standard deviation of 0.95, because some observations are lost while making sample restrictions.

[16]De Fraja et al. [2010] include homework assigned as a measure of school effort in their study of the relationship between child, parent, and school effort. Ferreyra and Liang [2012] use time spent doing homework as a measure of student effort.

[17]Special education and gifted and talented student programs are notable exceptions. Charter schools tend to have much smaller fractions of both types of students.

[18]See Section 4 for details.

of 100 charter schools appeared close to binding in 2002.[19] Estimating model parameters on data before the cap was binding obviates modeling the interdependence of charter school entry decisions that would be induced by the cap, meaning the same model can also be used to simulate entry decisions in the absence of caps. I called the North Carolina Department of Public Instruction to inquire about potentially oversubscribed charter schools, so that I could explore the feasibility of comparing my results to lottery-based studies, but because such data were not available I treat these schools as regular (i.e. non-oversubscribed) charter schools.[20]

The estimation sample contains markets that were stable over the sample period, students whose school choices were consistent with their market assignments, students observed for at least two years, and a random sub-sample of students from markets where charter school were never observed during the sample period.[21] This last restriction was adopted to make the ease computation of the likelihood. The estimation sample includes 78,294 public school student observations in markets without charter schools, 63,216 public school student observations in markets with charter schools, 1,984 total public-school-years, and 4,911 charter school student observations in 108 charter-school-years. Due to a potential concern that excluding students may affect parameter estimates, I re-estimated the model excluding markets where more than 5% of students were observed crossing market boundaries, and the parameter values remained substantially unchanged.

Table 1a shows the descriptive statistics for capital and effort by market and school type. Charter schools have about three-quarters of the per-pupil capital levels of public schools (0.43 versus 0.54 and 0.56 for monopolist and competitor public schools, respectively. Effort is significantly higher for both charter and public schools in entry markets than it is for public schools in non-entry markets. Table 1b shows entry and exit counts of charter schools by year, and shows that the year with the most new charter school entrants was 1998, and that charter school entry/operating decisions are quite persistent.

The following facts show patterns for charter school entry, endogenous school effort, and student outcomes. First, charter schools enter larger markets and markets in which they would have more resources (Table 1c). Average per-pupil capital for charters would be 0.431 in markets they are observed to enter, versus 0.409 in those they have not entered. Second, the amount of time spent doing homework is higher in markets in charters (Table 1d). Third,

[19]Data from 1997 are used to identify market ability distributions (Section 4).

[20]In conversations with staff at about one third of the charter schools open at some point during 1998-2001, I learned that even schools are uncertain whether or not they had waiting lists.

[21]I sample 100% of students in markets with at most 100 students, 20% of students in markets with 101-200 students, 15% of students in markets with 201-300 students, 10% of students in markets with 301-400 students, and 5% of students in markets with more than 400 students.

student choices suggest sorting on ability. In the year before a charter school enters a market, students who attend charter schools in the following year have 25% *lower* test scores than those who do not attend the charter in the following year. This fact only suggests ability sorting, which is quantified after the model has been estimated. Fourth, students in charter schools have the highest test scores, followed by students in public schools in markets charters have entered, followed by students in public schools in markets without charters (Table 1d). Finally, the last column of Table 1d shows that charter schools on average comprise only 9.15% of students in markets they enter.

# 4 Estimation

The model is estimated using maximum likelihood, which takes equilibrium outcomes from the model as inputs. The game is played in every market $m \in 1, \ldots, M$ and period $t \in 1, \ldots, T$. In every game there is a new public school and potential charter school entrant, each endowed with per-pupil capital levels and locations, and a new measure of students endowed with abilities and locations. Observed outcomes for each market and period include charter school entry decisions, school effort levels, student school choices, and student test scores. A market is linked across periods through its time-invariant ability distribution and whether a charter entered in the previous period in the market, which determines the entry cost shock distribution.

It is necessary to recover unobserved market-level ability distributions to solve and estimate the model. Ability is assumed to be normally distributed within markets to simplify the model solution (see equation (18)), so it suffices to recover its mean and variance.[22] If students can choose between schools in a market, sorting on unobservable ability may complicate recovery of market ability distributions. By using test score distributions from 1997, the year before charter school authorization, there is no such selection problem because all students attended public schools in their respective markets that period. The recovered ability distributions can then be treated as observed when integrating over student ability in school maximization problems and student likelihood statements. Because they are functions of the public school test score production function parameters, market ability distributions must be recovered jointly with the estimation of the model.

Using the production function for public schools (1), the mean test score for market $m$

[22] Market ability distributions are non-parametrically identified given the public school test score production function and test score productivity shock distribution. Details available upon request.

Figure 1: Descriptive statistics

(a) **Descriptive Statistics for Capital and Effort, by Market and School Type**

| | Capital | | | Effort | | |
|---|---|---|---|---|---|---|
| Entry | Mean | SD | N | Mean | SD | N |
| Charter | 0.43 | 0.03 | 98 | 2.66 | 0.95 | 98 |
| Public | 0.56 | 0.02 | 98 | 2.69 | 0.62 | 98 |
| No Entry | | | | | | |
| Public | 0.54 | 0.02 | 1652 | 2.43 | 0.50 | 1652 |

(b) **Charter School Entry and Exit, by Year**

| Year | Entrants | Exits | Charters Operating |
|---|---|---|---|
| 1998 | 22 | 0 | 22 |
| 1999 | 7 | 0 | 29 |
| 2000 | 8 | 3 | 34 |
| 2001 | 9 | 1 | 43 |

(c) **Fraction of Markets with Charter Schools by Market Characteristics**

| | |
|---|---|
| All markets | 0.056 |
| | |
| Charter per-pupil capital | |
| above median $k_{ch}$ | 0.095 |
| below median $k_{ch}$ | 0.017 |
| Market size | |
| above median $\mu$ | 0.067 |
| below median $\mu$ | 0.045 |

(d) **Sample Means for Effort, Test Scores, and Market Share**

| Entry | Effort | Test Scores | Market Share |
|---|---|---|---|
| Charter | 2.66 | 3.137* | 0.0915 |
| Public | 2.69 | 3.075* | 0.9085 |
| No Entry | | | |
| Public | 2.43† | 3.028* | 1.000 |

† Monopoly average hours are different from those of public and charter schools in entry markets (p-value < 0.001)

* All means are significantly different from each other (pair-wise t-tests, p-value < 0.0001)

in 1997 is

$$\bar{y}_{tps,97,m} = \int aEy_{tps}(1, \bar{a}_m, e^o_{tps,97,m}, k_{tps,97,m}) f_m(a) da + \underbrace{\int f_{\nu^y}(\nu^y_{tps}) d\nu^y_{tps}}_{0}$$
$$= Ey_{tps}(1, \bar{a}_m, e^o_{tps,97,m}, k_{tps,97,m}) \int a f_m(a) da = \bar{a}_m^{1+\theta_{tps}} Ey_{tps}(1, 1, e^o_{tps,97,m}, k_{tps,97,m}) \tag{9}$$

where $(e^o_{tps,97,m}, k_{tps,97,m})$ are observed inputs, $f_m$ is the density of ability in market $m$, and $\bar{a}_m$ is mean ability. The normalization of public school TFP $\omega_{tps} = 1$ affects the mean of recovered ability distributions, so the charter school's TFP $\omega_{ch}$ can be thought of as productivity relative to that of a public school. The second moment can be used to recover the variance of ability for each market using the recovered $\bar{a}_m$ and analogous reasoning.[23]

Let $\Phi$ and $\phi$ denote the standard normal CDF and density, respectively. The potential charter school entrant enters with probability

$$\Pr\{z^o_{tm} = entry | z^o_{t-1,m}\} = \Phi\left(\frac{\mathrm{E}_{\nu^{\mathbf{e}}}\left[v_{ch}(\mathbf{e}^*_{\mathbf{tm}})\right] - \mu_{\underline{v}, z^o_{t-1,m}}}{\sigma_{\underline{v}, z^o_{t-1,m}}}\right), \tag{10}$$

where $\mathbf{e}^*_{\mathbf{tm}}$ is equilibrium effort and $\mu_{\underline{v}, z^o_{t-1,m}}$ and $\sigma_{\underline{v}, z^o_{t-1,m}}$ depend on last period's charter school entry decision $z^o_{t-1,m}$.[24] The likelihood of observed effort at school $s$ is the density of the difference between effort predicted by the model $e^*_{stm}$ (which depends on the entry decision) and observed effort:

$$L\{e^o_{stm} | z^o_{tm}\} = \frac{1}{\sigma_{\nu^e}} \phi\left(\frac{\ln e^o_{stm} - \ln e^*_{stm}}{\sigma_{\nu^e}}\right). \tag{11}$$

The probability student $i$ attends the charter school ($s^o_{itm} = ch$) is a function of ability $a_i$, mean ability at both schools $\bar{\mathbf{a}}_{\mathbf{tm}}$, observed school inputs $(\mathbf{e}^{\mathbf{o}}_{\mathbf{tm}}, \mathbf{k}_{\mathbf{tm}})$, the pair of distances from the student to both schools $\mathbf{r}_{\mathbf{itm}}$, and own ability $a_i$:

$$\Pr\{s^o_{itm} = ch | z^o_{tm} = entry, a_i\} = \Phi\left(\frac{a_i \Delta g(\bar{\mathbf{a}}_{\mathbf{tm}}, \mathbf{e}^{\mathbf{o}}_{\mathbf{tm}}, \mathbf{k}_{\mathbf{tm}}) - \Delta c_{itm}}{\sigma_{\Delta\epsilon}}\right), \tag{12}$$

where (17) defines $\Delta g$ and $\Delta c_{itm}$. Because mean schools abilities are unobserved, (23) is used to compute the fixed point of mean abilities at charter and public schools $\bar{\mathbf{a}}_{\mathbf{tm}}$, given

[23] If only data from 1997 are used for each market, the variance of the ability distribution cannot be separated from that of the test score productivity shock, so $\sigma^2_{\nu^y}$ is set to 0.40.

[24] There was no entry before the first period of the model.

observed effort and capital.[25] The likelihood of observed test score $y^o_{istm}$ is

$$L\{y^o_{istm}|z^o_{tm}, s^o_{itm} = s, a_i\} = \frac{1}{\sigma_{\nu^y}}\phi\left(\frac{y^o_{istm} - Ey_s(a_i, \bar{a}_{stm}, e^o_{stm}, k_{stm})}{\sigma_{\nu^y}}\right). \quad (13)$$

Entry cost shocks $\underline{v}_{tm}$, effort productivity shocks $\nu^e_{stm}$, test score productivity shocks $\nu^y_{istm}$, and preference shocks $\eta_{istm}$ are assumed to be independent.

The total likelihood is in Appendix B. It combines the above school- and student-level likelihood statements, integrating the latter according to recovered market-specific ability distributions. Asymptotic standard errors are computed using the sum of the outer product of the observation-level scores. For asymptotic analysis, let the number of markets $M$ go to infinity, holding constant the average number of households within each market and number of time periods $T$.[26]

## 4.1 Identification

This section discusses how test score technologies, which include both unobservable student ability and unobservable mean ability of attendant students, are identified in the presence of potential sorting on student ability. The productivity of inputs may be estimable due to functional form restrictions, but it is preferable to identify test score production function parameters using an exclusion restriction. I follow the approach taken by Card [1993] and assume distance from a household to charter and public schools does not directly affect the gain in the outcome from attending the charter school. Though it allows for a relationship between distributions of student ability and distance *between* markets, this identification strategy assumes that distance and ability are independently distributed *within* markets.

The assumption that ability and distance distributions are independent within markets could be violated if charter schools were able to target students within markets, but it is justified for several reasons. First, the model allows charter schools to "target" students to a large degree, in the sense that markets have different ability and distance distributions and charter schools make entry decisions. Charter schools likely have less control over their specific locations within the relatively small areas inside markets because they typically locate in densely populated areas that have already been developed, so their optimal choice

[25] See Appendix A for details.

[26] I assume data are missing randomly. There are some charter school observations where homework was not reported for any students, so those schools did not contribute to the effort likelihood and affected students did not contribute to the likelihood. About 5% of observations in charter and public schools are missing test score data, so these students also do not contribute to the test score likelihood. I integrate the likelihood over market distance distributions for students missing location data. The assumption that these addresses are missing at random is justified by the fact that an indicator for whether the address is missing is not significantly associated with a student's test score when controlling for student ethnicity.

within a market (from a purely spatial perspective) is unlikely to be available. Because they do not receive capital funding streams, charter schools often use existing buildings to avoid expensive capital expenditures incurred when building a new school, meaning that even if their (spatially) optimal location within a market were available, it would unlikely trump a low-rent option close-by within the market. Cullen et al. [2005] check the plausibility of this assumption in a similar environment by checking whether observed student characteristics are related to distance, and do not find evidence of a linear relationship. I perform a similar check and also find little evidence that distance and ability within a market are related.[27]

Market-level distance distributions identify the role unobserved mean school abilities play in the production of test scores at charter and public schools by shifting mean ability at public and charter schools without directly affecting student achievement. Manski [1993] taxonomizes peer effects as *endogenous* (determined by mean test scores of attendant students), *correlated* (determined through a common shock to students in the same school), and *contextual* (operating directly through the mean ability of attendant students), and argues that separating these channels is difficult when using commonly invoked linear-in-means models of peer effects. I follow other studies in the literature studying competition between schools (such as Caucutt [2002], Epple and Romano [1998], Ferreyra [2007], and Nechyba [2000]) by assuming that neither endogenous social interaction and correlated effects affect test score production.[28]

# 5 Estimation Results

## 5.1 Parameters

Appendix C presents the estimated parameters for the model. The first nine rows are the test score production function parameters.[29] Both schools have effort shares in test score production that are higher than capital shares. Charter schools are closer to constant returns to scale than public schools $\tau_{ch} = 0.95 > \tau_{tps} = 0.06$). The elasticity of substitution between capital and effort is 0.60 for charter schools and 0.77 for public schools. Peer effects at charter schools are much less important quantitatively than those at public schools ($\theta_{ch} = 0.026 < \theta_{tps} = 0.995$). Although variance in mean ability across charter schools does

[27] A linear regression of net distance of a student from both charter and public schools on and market and time indicator variables had an $R^2$ of 0.7715 when the student test score was excluded and 0.7716 when the student test score was included.

[28] Even were there a reflection problem, the posited production technologies are non-linear and therefore in principle, both endogenous and correlated effects could also be identified.

[29] Recall that all public schools share the same test score production technology and all charter schools share (a different) test score production technology.

little to affect student test scores, charter schools have an estimated TFP that is much higher than that at public schools ($\omega_{ch} = 1.74 > \omega_{tps} = 1$).

Household preference parameters are denominated in standard deviations of test scores. The disutility of effort is negative ($c_e = -3.87$), which means households prefer attending the school where they have to work harder, even after taking into account increased test scores. The per-kilometer distance cost is about three-quarters of a standard deviation ($c_r = 0.73$) of test scores. I find little evidence of ability sorting in entry markets: mean simulated ability for students attending public schools is 1.701, while that of students attending charters is 1.70005.

In the school cost functions, charter schools have much larger diseconomies of scale from exerting effort than public schools ($\psi_{e,ch,2} = 14.14 > \psi_{e,tps,2} = -0.07$). The cost of exerting effort is mitigated by capital at both schools ($\psi_{e,ch,3} = -15.51, \psi_{e,tps,3} = -0.92$). Finally, charter schools suffer a larger effort cost for educating higher ability students ($\psi_{e,ch,4} = 7.42$).[30] Higher per-pupil capital levels may make it easier for the school to create, assign, and grade homework because there are more computers per student or if there are smaller class sizes. On the other hand, designing curricula for high ability students may be more demanding.

Finally, the mean of the entry cost shock distribution is lower when there was a charter school in the market in the previous period ($\mu_{\underline{v},entry} = 34.62 < \mu_{\underline{v},no\ entry} = 94.44$). Charter schools are also more likely to enter larger markets $\gamma_{mkt.size} = 8.58$) and in markets the higher the share of the market is Black students ($\gamma_{fr.Black} = 23.48$), though these parameters are somewhat imprecisely estimated due to the small number of markets with charter schools.

## 5.2 Model Fit

The model captures charter school entry, charter and public school effort, student choice, and student test score patterns for North Carolina. Table 1 shows the fraction of markets of certain characteristics with charter schools. The first row shows that the model does a good job of predicting the overall fraction of markets with charter schools over the estimation period. The model also captures the facts that charter schools are more likely to enter markets where they would receive higher per-pupil capital (rows 2 and 3) and that charter schools are more likely to open in larger markets (rows 4 and 5) and markets with below-median 1997 average test scores (rows 6 and 7).[31]

[30] Some of the estimated parameters are close to zero. However, they are on the interior of the parameter space because they were not constrained to be positive.

[31] Recall that per-pupil capital at charter schools is based on a prediction, so it exists even in markets where the charter school has not entered.

Table 1: Model Fit: Percent of Markets with Charter School Entry by Market Characteristics

| | Observed | Predicted |
|---|---|---|
| **All markets** | **0.056** | **0.061** |
| | | |
| Charter per-pupil capital | | |
| above median $k_{ch}$ | 0.095 | 0.080 |
| below median $k_{ch}$ | 0.017 | 0.041 |
| | | |
| Market size | | |
| above median $\mu$ | 0.067 | 0.080 |
| below median $\mu$ | 0.045 | 0.042 |
| | | |
| Average 1997 test score | | |
| above median $\bar{y}_{1997}$ | 0.050 | 0.056 |
| below median $\bar{y}_{1997}$ | 0.062 | 0.065 |

Table 2 shows that the model also reproduces patterns for school effort levels as measured by hours of homework: charter and public schools in markets in which charter schools operate exert higher levels of effort than monopolist public schools. This is because charter schools enter markets with higher per-pupil capital levels as well as the competitive effect of charter school entry. The model captures the fact that both schools in markets with charters have higher effort levels than monopolist public schools, but under-predicts effort levels of charter schools relative to their public school competitors. Table 2 also shows that the model captures the relationship between effort, school capital and market size. Charter schools in markets where the charter schools have above median per-pupil capital exert more effort than they do in markets with below median per-pupil capital. The same is true for public schools in markets with and without charter school entry. This is for two reasons: capital directly augments test score production and also makes effort exertion less costly for schools through the interaction between capital and effort in school effort cost functions. This latter effect is why public schools exert higher effort in markets where they have higher per-pupil capital in spite of the fact that the coefficient on per-pupil capital in the public school test score production function is small. Table 2 also shows that, as opposed to both entry and monopoly public schools, charter schools have much lower predicted effort levels in larger markets (above median $\mu$), which fits the data. This last fact will prove important when evaluating counterfactual policies.

The first two columns of Table 3 show that the model captures the ranking average of test scores in the estimation sample: students at charter schools have the highest average test

Table 2: Model Fit: Mean Hours of Homework by School Type by Market Characteristics

| Entry Markets | Observed | Predicted |
|---|---|---|
| Charter Schools | | |
| **Total** | **2.66** | **2.51** |
| above median $k_{ch}$ | 2.74 | 2.61 |
| below median $k_{ch}$ | 2.20 | 2.31 |
| above median $\mu$ | 2.51 | 2.37 |
| below median $\mu$ | 2.89 | 2.77 |
| | | |
| Public Schools | | |
| **Total** | **2.69** | **2.65** |
| above median $k_{tps}$ | 2.78 | 2.73 |
| below median $k_{tps}$ | 2.21 | 2.50 |
| above median $\mu$ | 2.85 | 2.76 |
| below median $\mu$ | 2.46 | 2.44 |
| | | |
| Monopoly Markets | | |
| Public Schools | | |
| **Total** | **2.43** | **2.44** |
| above median $k_{tps}$ | 2.51 | 2.51 |
| below median $k_{tps}$ | 2.35 | 2.38 |
| above median $\mu$ | 2.49 | 2.52 |
| below median $\mu$ | 2.37 | 2.37 |

Table 3: Model Fit: Test Scores and Market Share

| | Test Scores | | Market Share* | |
|---|---|---|---|---|
| Entry | Observed | Predicted | Observed | Predicted |
| Charter | 3.137 | 3.122 | 0.0915 | 0.0822 |
| Public | 3.075 | 3.062 | 0.9085 | 0.9178 |
| | | | | |
| No Entry | | | | |
| Public | 3.028 | 3.043 | 1.0000 | 1.0000 |

* Market share is the fraction of students in the market in that school type.

scores, followed by students attending public schools in markets charter schools have entered, followed by students in public schools in markets without charters. The third and fourth columns of Table 3 show model fit for the fraction of students choosing charter schools. The model fits the pattern that charter schools are smaller than the public schools in markets they have entered. The third column, 0.0915, is the observed average for all markets with charter schools. The fourth column presents the total fraction of students choosing the charter school in markets with charters for the full simulation of the model – that is, first simulating charter school entry decisions and then school effort choices, and simulating student school choices based on predicted school effort choices.

# 6 Counterfactual Simulations

Charter school policy depends critically on how they affect the achievement of attendant students, students at competing public schools, and students in markets where charter schools might enter if caps on the total number of charters were lifted. The estimated model fits the data well, and provides a framework for policy analysis. I use the developed model to simulate charter school entry decisions and school effort choices under entry and monopoly scenarios. I then simulate household school choices and test scores (i.e. potential outcomes) for each household in the market's charter school, traditional public school competitor, and monopolist public school 30 times.

## 6.1 Definitions of Treatment Effects

There are two treatments. First, the effect of being in a charter school ("direct effect") for student $i$ who lives in market $m$ is the difference between the test score the student would have received at the charter school and that she would have received at a monopolist public school in that market:

$$\begin{aligned}\Delta_{im}^{direct} &= y_{i,ch,m} - y_{i,tps,m}^{mono} \\ &= Ey_{ch}\left(a_i, \bar{a}_{ch,m}, e_{ch,m}, k_{ch,m}\right) + \nu_{i,ch}^{y} - Ey_{tps}\left(a_i, \bar{a}_m, e_{tps,m}^{mono}, k_{tps,m}\right) - \nu_{i,tps,mono}^{y},\end{aligned}$$

where $\bar{a}_{ch,m}$ is the (endogenous) mean ability of students attending the charter school, $e_{ch,m}$ is the effort level at the charter school, $\bar{a}_m$ is the average ability at the monopolist public school (which is equal to market average ability), $e_{tps,m}^{mono}$ is the effort level at the monopoly public school, and $\nu_{is}^{y}$ are ex-post test score productivity shocks.[32] Second, the effect of attending

[32]The effort levels in all simulations are those chosen by schools and then hit with effort productivity shocks. In the model section there were no market or time subscripts because the analysis was done within

a public school that is competing with a charter school ("spillover effect") for this student is the difference between the test score the student would have received at the public school competing with the charter school entrant and that she would have received at a monopolist public school in that market: $\Delta_{im}^{spill} = y_{i,tps,m} - y_{i,tps,m}^{mono}$, where $y_{i,tps,m}$ includes the relevant public school inputs.

The model is a useful tool for program evaluation because it provides potential outcomes that can be used to generate both direct and spillover effects for all students, regardless of charter school entry status or the school choice. Researchers typically focus on mean treatment effects, which are expected values of $\Delta^{direct}$ and $\Delta^{spill}$ for different (and possibly choice-based) sets of students. Consider the treatment of attending a charter school. The mean direct effect of treatment on the treated (direct TOT) is the mean effect of attending a charter school among students who would choose it. In market $m$, the mean direct TOT is

$$\overline{\Delta}_m^{direct,TOT} = \mathrm{E}\left[\Delta^{direct}|s_{im} = ch\right] = \int \Delta_{im}^{direct} f_m(a_{im}; \bar{\mathbf{a}}, \mathbf{e}, \mathbf{k}|s_{im} = ch) da_{im}, \tag{14}$$

where $f_m(a_{im}; \bar{\mathbf{a}}, \mathbf{e}, \mathbf{k}|s_{im} = ch)$ is the density of ability for students choosing the charter school in market $m$ (see (22)).[33]

The mean direct effect of treatment on the untreated (TOU) in $m$ is the mean effect of attending a charter school among students who would choose the public school, i.e.

$$\overline{\Delta}_m^{direct,TOU} = \mathrm{E}\left[\Delta^{direct}|s_{im} = tps\right] = \int \Delta_{im}^{direct} f_m(a_{im}; \bar{\mathbf{a}}, \mathbf{e}, \mathbf{k}|s_{im} = tps) da_{im}. \tag{15}$$

The mean direct average treatment effect (ATE) in $m$ averages over all students in the market $\overline{\Delta}_m^{direct,ATE} = \mathrm{E}\left[\Delta^{direct}\right] = \int \Delta_{im}^{direct} f_m(a_{im}) da_{im}$, where $f_m$ is the density of ability in market $m$. Market mean spillover effects are calculated analogously, substituting $\Delta_{im}^{spill}$ for $\Delta_{im}^{direct}$ and $tps$ for $ch$ for student school choices.

Market-level treatment effects are weighted by market size and entry status to aggregate treatment effects across markets. For example, the mean direct TOT across all entry markets is

$$\overline{\Delta}^{direct,TOT,entry} = \sum_m 1\{z_m = entry\}\mu_{ch,m} \overline{\Delta}_m^{direct,TOT} / \sum_m 1\{z_m = entry\}\mu_{ch,m},$$

where $\mu_{ch,m}$ is the measure of students choosing the charter school in $m$. Other aggregated treatment effects are calculated analogously.

Researchers who exploit lotteries among applicants to an over-subscribed charter school

one market and one time period. I drop the time subscript here to unclutter exposition, but add a market subscript to make clear comparisons across markets.

[33] I suppress the dependence of $f$ on household distance from schools. All simulations use household distance from schools.

compare test scores of students who applied to be in those charter schools and who were randomized into the charter school with those who were randomized out (therefore attending the public school competitor). Denote the treatment effect estimated for household $i$ in such a study as $\check{\Delta}_{im}^{direct}$, where

$$\check{\Delta}_{im}^{direct} = y_{i,ch,m} - y_{i,tps,m} = \Delta_{im}^{direct} - \Delta_{im}^{spill} + \left(\nu_{i,ch}^{y} - \nu_{i,tps}^{y}\right), \tag{16}$$

i.e. $\check{\Delta}_{im}^{direct}$ is the difference between the direct and spillover effects of charter school entry for $i$, plus the difference of i.i.d., mean-zero, ex-post productivity shocks. Intuitively, bigger changes in public school inputs caused by charter school entry lead to larger biases in estimated treatment effects in lottery studies. Suppose a public school drastically changed its behavior in response to charter school entry, such that $\Delta_{im}^{spill} > \Delta_{im}^{direct} > 0$. In this case, a researcher using a lottery design would *incorrectly sign* the direct effect. Theory gives us no a priori sign on the spillover effect, which means even of sign this bias cannot be determined without further structure. Even were there no spillover effect of charter school entry, researchers using lottery methods have noted that their estimates may not generalize to other existing charter schools or to charter schools that have not yet opened, if oversubscribed schools are different from those that are not oversubscribed.

## 6.2 Effects of Charter Schools on the Distribution of Test Scores, 1998-2001

Table 4 summarizes mean direct and spillover effects of charter schools on test scores for different subsets of students in the estimation sample. It also reports the mean bias on the direct effect introduced by using an estimator that ignores the spillover effect of charter school entry (such as the lottery estimator). All results are reported in percentages of a standard deviation of the average of math and reading test scores. The top half of the table ("Entry markets") reports results for markets in which charter schools are operating and the bottom half ("Monopoly markets") report what results would be in those markets in which charter schools are not operating. The row within an entry status indicates which subset of households is being considered: households who would choose charters, households who would choose traditional public schools, or all households in such markets (ATE). For example, the number associated with the first column ($\overline{\Delta}^{direct}$) and the row "Attend charter" in the top half of the table is the mean direct effect of treatment on the treated in entry markets, $\overline{\Delta}^{direct,TOT,entry}$, i.e. the mean direct effect for students who chose the charter school in markets where charter schools are present.

Charter schools have positive effects on the test scores of attendant students and neg-

Table 4: Mean Direct and Spillover Treatment Effects by School Choice in Entry and Monopoly Markets

| Entry markets | $\overline{\Delta}^{direct}$ | $\overline{\check{\Delta}}^{direct}$ | $\overline{(\Delta^{direct} - \check{\Delta}^{direct})}$ | $\overline{\Delta}^{spill}$ |
|---|---|---|---|---|
| TOT (Attend charter) | **0.10831** | 0.10355 | 0.00476 | 0.00476 |
| TOU (Attend public) | -0.03867 | -0.05449 | **0.01582** | 0.01582 |
| ATE | -0.02659 | -0.04150 | 0.01491 | 0.01491 |
| | | | | |
| Monopoly markets: TOU | | | | |
| Attend charter | 0.01774 | 0.01859 | -0.00085 | -0.00085 |
| Attend public | -0.11370 | -0.12351 | 0.00981 | 0.00981 |
| ATE | -0.10258 | -0.11148 | 0.00891 | 0.00891 |

ligible spillover effects on students attending public schools. The first column shows that the mean direct effect of charter school entry is highest for students who choose to attend charter schools in markets in which charter schools are operating, about 10.8% of a standard deviation in test scores. The third and fourth columns show the mean bias induced in the estimation of direct effects by ignoring spillover effects (column 2) and spillover effects of charter school entry, and are therefore equal. The average bias introduced by ignoring spillover effects is small, on the order of (or even less than) 2% of a standard deviation in test scores. Average treatment effects are much closer to the estimated effects for students attending public schools because the share of students attending charter schools is small (about 9% of entry markets), which causes the ATE direct to be negative.

The estimated model can be used to decompose the direct effect of charter schools. Most of the direct effect comes from charter school technology. The mean test score for students in entry markets who attend charters is 3.128. The simulated test score for the same group of students is reduced to 3.026 using charter school inputs and public school technology. The mean test score is reduced by a much smaller amount when also using monopoly public school inputs, to 3.013.

Mean treatment effects are smaller in monopoly markets. The mean direct TOT of charter schools would be about 2% of a standard deviation in monopoly markets. Mean spillover effects are slightly smaller than they would have been in entry markets. These differences are again driven mostly due to lower levels of public and charter school inputs chosen in those markets. The average difference in effective inputs, $\overline{Ey_{ch}(1, \bar{a}_{ch}, e_{ch}, k_{ch}) - Ey_{tps}(1, \bar{a}_m, e_{tps}^{mono}, k_{tps})}$, between charter schools and monopolist public school in entry markets is 0.070, while the same difference in monopoly markets is 0.016, over a four-fold difference.

Although estimated treatment effects are a combination of state-level charter school authorization laws and other institutional characteristics, market- and school-level characteristics, student characteristics, and identification strategy, one might want to compare the results presented here with those in lottery studies. Angrist et al. [2012] find using a lottery on an oversubscribed Massachusetts KIPP school that students randomized into charters have 36% of a standard deviation higher math test scores and 12% of a standard deviation higher reading test scores than applicants randomized into competing public schools. The outcome in this paper is the average of reading and math test score, so their finding of a 24% of a standard deviation increase in test scores for $\overline{\check{\Delta}}^{direct}$ is much larger than my finding of 11%. Again, they estimate the direct effect of TOT on a selected set of charter school students in one oversubscribed charter school, which they acknowledge may not generalize to students in other charter schools.

### 6.2.1 Heterogeneity of Treatment Effects by Market

Figure 2 shows how treatment effects vary across markets by plotting mean market-level direct and spillover TOT for all markets, including those where charter schools are not present. The size of each circle represents market size. It shows that the distribution of mean market-level direct TOTs is quite heterogeneous, and that this heterogeneity relates to market size. The 75th percentile mean market-level direct TOT is 10% of a standard deviation while the 25th percentile is *negative* at -21% sd. The diseconomies of scale in the effort cost function for charter schools play a key role here: mean direct effects are much more likely to be positive in smaller markets (represented by smaller circles). There is much less variation in mean spillover TOT, which is due to the relative ineffectiveness of public school inputs.

Lottery studies basing direct TOT estimates on oversubscribed charter schools consider oversubscribed charter schools, by design. These charter schools might be in highest demand by rational households interested in student achievement. Therefore, they may only recover part of the distribution of charter school treatment effects. Figure 3 plots mean direct TOT effects for each market as a function of the fraction of students within that market who would choose to attend the charter school if available. In markets with below-median demand for charter schools (i.e. those with a below-median fraction of students who would choose the charter school, were it to enter), the mean direct TOT is -18.7% of a standard deviation in test scores, compared with 11.1% for markets with above-median demand. When researchers use over-subscribed, that is, highly demanded, charter schools to infer treatment effects they draw from schools on the right side of Figure 3. This figure shows the difficulty in generalizing findings from studies of only very popular (and therefore highly demanded) charter schools.

Figure 2: Market-Level Mean TOT Direct vs. TOT Spillover

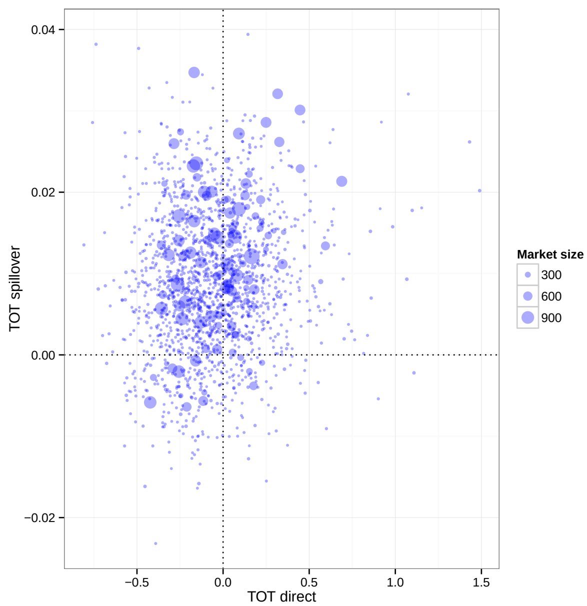
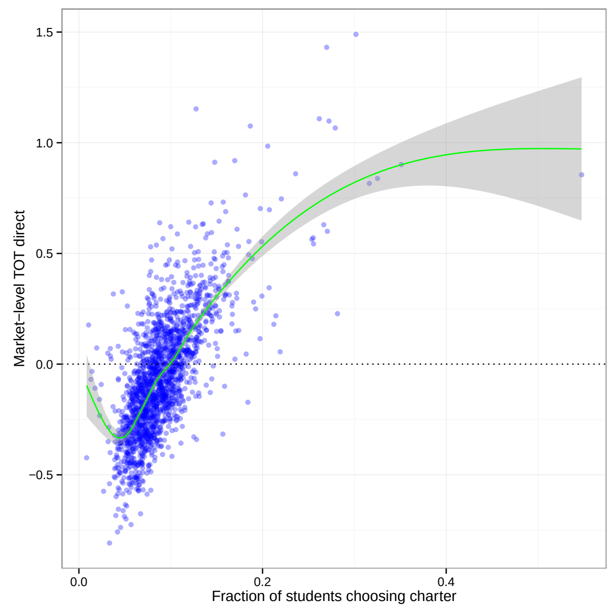



Figure 3: Market-Level Mean Direct TOT by Fraction of Students Choosing Charter School

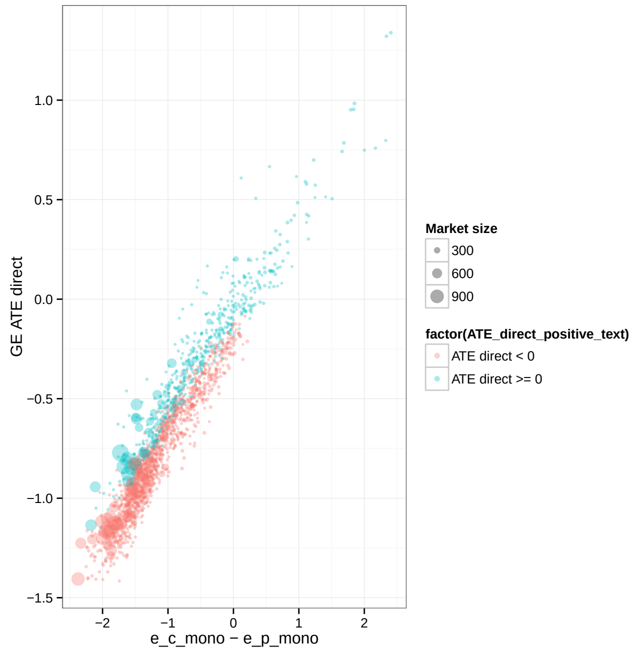



### 6.2.2 Interpreting the Direct ATE

Having observed the positive mean direct effect and negligible mean spillover effects of charter schools, policymakers may be tempted to increase the number of students in charter schools. For example, making them monopolists would increase test scores by almost 30% of a standard deviation, *assuming they had the monopolist public level of capital and exerted the monopolist public level of effort.* However, the results from Table 4 are only valid for an infinitesimal household switching from a treatment school to the monopolist public school. I therefore next explore how making charter schools monopolists, by assigning all students in a market to them after they have entered, would affect charter school entry, effort, and the distribution of student test scores. This policy is general equilibrium (GE) in the sense that a very large measure of students are moved from monopolist public to charter schools, compared with approximately 9% of the market who choose charter schools under the baseline scenario.

In this scenario, the direct effect of attending a charter school for student $i$ is $y_{i,ch,m}^{mono} - y_{i,tps,m}^{mono}$, where $y_{i,ch,m}^{mono}$ is the test score for $i$ at a monopolist charter school exerting effort $e_{ch,m}^{mono}$. When charter schools are forced to be monopolists they have a direct TOT of -32% of a standard deviation, which is drastically lower than their duopoly direct TOT. Figure 4 shows that the reason charter schools have negative average treatment effects as monopolists is that they reduce the provision of costly effort when forced to serve the entire market.

Figure 4: Market-level GE direct effect by change in effort inputs and market size




Even restricting the substitution to markets where charter schools have positive ATEs in the baseline (the blue dots) would result in negative charter monopolist ATEs. When charter schools are required to enter larger markets (the larger circles in Figure 4) they reduce their effort levels due to their diseconomies of scale in effort provision. Table 5 compares entry patterns for charter schools under the duopoly scenario and under the extreme case where they have been forced to serve all students in any markets they enter. Charter schools enter far fewer markets when forced to serve the entire market, and are much less likely to enter larger markets than they were under the duopoly scenario.

Table 5: Fraction of Markets with Entry by Market Size, Baseline vs Monopolist Charters (GE)

| | Baseline | Monopolist Charters (GE) |
|---|---|---|
| High market size | 0.07981 | 0.00354 |
| Low market size | 0.04152 | 0.01093 |
| Total | 0.06067 | 0.00724 |

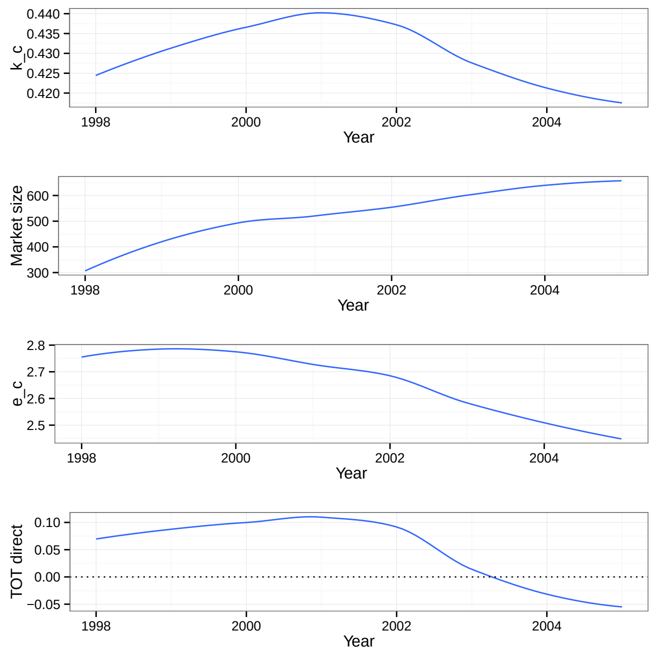
### 6.3 The Effect of Allowing Unlimited Charter School Entry on Test Scores

One of the most contentious charter school policy debates is whether caps on the number of charter schools should be loosened or even eliminated. Therefore, I next examine how lifting the cap on the total number of charter schools in North Carolina would affect the fraction of markets with charter schools and distribution of test scores in those markets. Recall that the model is estimated on data from years before the statewide cap started binding, meaning the model can be solved for the years 2002-2005 to quantify the effect of allowing unrestricted charter school entry on the distribution of test scores. The results suggest the cap on the total number of charter schools in North Carolina was binding. On average, during 2002-2005 charter schools operate in 15% of markets, up from 6% for the period 1998-2001. Figure 5 shows how entry patterns and other key variables would change. The overall effect of charter schools on attendant students is attenuated as charters enter larger markets with lower levels of capital (the top two panels of Figure 5) and subsequently reduce effort provision (the third panel), which eventually creates a slight *negative* TOT direct (-0.4% of a standard deviation in test scores). Spillover TOT impacts of charter schools in new entry markets are similar to those estimated for the first four years of charter school authorization (on average 1.2% of a standard deviation in test scores). A much larger share of students are now in markets where charter schools operate (21% up from 6%). Overall, charter schools affect more students when caps are lifted, but treatment effects are attenuated.

# 7 Conclusions

There is a long-standing and contentious debate about the effect of charter schools and school choice, more generally, on student achievement because there are many moving parts that complicate analysis. The equilibrium model developed and estimated in this paper captures key mechanisms of the debate about charter school policy, such as where charter schools choose to locate and how many would enter were entry caps lifted (by modeling which markets they choose to enter), how charter school quality is determined (by endogenizing charter school effort inputs), the potential for spillover effects on competing public schools (by endogenizing public school effort choices and allowing peer quality to enter test score production), and the potential for cream-skimming through student sorting (by modeling how students choose schools based on their unobserved ability). Student distances from schools provide a source of variation for student school choices to generate credible estimates of both charter and public school test score production technologies. The model fits patterns

Figure 5: TOT direct, 1998-2005




of charter school entry, public and charter school input provision, student school choices, and test scores quite well. I then use the structure afforded by the model to define direct and spillover treatment effects in an equilibrium framework, quantify direct and spillover effects, and simulate the effects of allowing unlimited charter school entry.

The approach developed here provides a more comprehensive picture of treatment effects than studies based on only a subset of charter schools (e.g. lottery-based designs), because there is substantial heterogeneity that such studies may not recover. Both the direct effect and spillover effects of charter school entry are on average positive for treated populations, although the direct effect is much larger than the spillover effect. One striking result is the heterogeneous treatment effects that charter schools have on attendant students across markets. Spillover effects for students attending charter schools are quite small, which suggests that lottery-based designs may have internal validity. However, the size of direct treatment effects strongly relate to demand for charter schools, casting doubt on the notion that lottery designs have external validity. I also estimate that caps on the total number of charter schools were binding, and that students attending charters in new entry markets would be affected in a manner different to those attending charters that had entered before caps started to bind. In particular, as charter schools enter less attractive (larger) markets, they exert lower average equilibrium effort levels, reducing the direct effect of charter school entry.

One limitation of this paper is that the creation of markets in which one charter and one public school compete may bias estimates of treatment effects. I address this concern by re-estimating the model excluding markets where more than 5% of students were observed attending charter schools across market boundaries, and find that parameter estimates remain largely unchanged. Moreover, I provide evidence that cross-market competition does not substantially affect charter school entry or spillover effects on public school students (Appendix D.6). Finally, I use the estimated model to quantify the extent to which sample selection assumptions might affect the direct affect of charter schools on attendant students, and find evidence that the mean direct effect would be positive, though smaller, for excluded students, and that the mean direct effect on all charter school students would change very little were excluded students included (Appendix D.7).

While quite rich, this framework could be extended. For example, intertemporal dynamics in this paper are relatively simple, and may deserve further investigation in future work. Nonetheless, the framework developed here is a useful tool for policy analysis in an environment where there may be equilibrium effects due to endogenous student school choices and school input choices, and non-random charter school entry.

# References


Joseph G Altonji, Ching-I Huang, and Christopher R Taber. Estimating the cream skimming effect of school choice. *Working paper*, 2010.

Joshua D Angrist, Susan M Dynarski, Thomas J Kane, Parag A Pathak, and Christopher R Walters. Who benefits from kipp? *Journal of Policy Analysis and Management*, 31(4): 837–860, 2012.

E.P. Bettinger. The effect of charter schools on charter students and public schools. *Economics of Education Review*, 24(2):133–147, 2005. ISSN 0272-7757.

R. Bifulco and H.F. Ladd. The impacts of charter schools on student achievement: Evidence from north carolina. *Education Finance and Policy*, 1(1):50–90, 2006.

R. Bifulco and H.F. Ladd. School choice, racial segregation, and test-score gaps: Evidence from north carolina's charter school program. *Journal of Policy Analysis and Management*, 26(1):31, 2007.

David Card. Using geographic variation in college proximity to estimate the return to schooling. *NBER Working paper*, 1993.

E.M. Caucutt. Educational vouchers when there are peer group effects – size matters. *International Economic Review*, 43(1):195–222, 2002.

R. Chakrabarti. Can increasing private school participation and monetary loss in a voucher program affect public school performance? evidence from milwaukee. *Journal of Public Economics*, 92(5-6):1371–1393, 2008. ISSN 0047-2727.

Jane Cooley. Can Achievement Peer Effect Estimates Inform Policy? A View From Inside the Black Box. 2009.

J.B. Cullen, B.A. Jacob, and S.D. Levitt. The impact of school choice on student outcomes: An analysis of the chicago public schools. *Journal of Public Economics*, 89(5-6):729–760, 2005.

J.B. Cullen, B.A. Jacob, and S. Levitt. The effect of school choice on participants: Evidence from randomized lotteries. *Econometrica*, 74(5):1191–1230, 2006.

Gianni De Fraja and Pedro Landeras. Could do better: the effectiveness of incentives and competition in schools. *Journal of Public Economics*, 90(1):189–213, 2006.

Gianni De Fraja, Tania Oliveira, and Luisa Zanchi. Must try harder: Evaluating the role of effort in educational attainment. *The Review of Economics and Statistics*, 92(3):577–597, 2010.

D. Epple and R.E. Romano. Competition between private and public schools, vouchers, and peer-group effects. *American Economic Review*, 88(1):33–62, 1998.

Maria Marta Ferreyra and Grigory Kosenok. Charter school entry and school choice: The case of washington, dc. *Working paper*, 2012.

Maria Marta Ferreyra and Pierre Jinghong Liang. Information asymmetry and equilibrium monitoring in education. *Journal of Public Economics*, 96(1):237–254, 2012.

M.M. Ferreyra. Estimating the effects of private school vouchers in multidistrict economies. *The American Economic Review*, pages 789–817, 2007.

P. Gleason, M. Clark, C.C. Tuttle, and E. Dwoyer. The evaluation of charter school impacts: Final report. ncee 2010-4029. *National Center for Education Evaluation and Regional Assistance*, page 259, 2010.

Gerhard Glomm, Douglas Harris, and Te-Fen Lo. Charter school location. *Economics of Education Review*, 24(4):451–457, 2005.

E.A. Hanushek, J.F. Kain, S.G. Rivkin, and G.F. Branch. Charter school quality and parental decision making with school choice. *Journal of Public Economics*, 91(56):823 – 848, 2007. ISSN 0047-2727.

A. Henningsen and G. Henningsen. Econometric estimation of the constant elasticity of substitution function in r: Package miceconces.

C.M. Hoxby and J.E. Rockoff. The impact of charter schools on student achievement. *Working paper*, 2004.

S.A. Imberman. The effect of charter schools on achievement and behavior of public school students. *Journal of Public Economics*, 2011.

C.F. Manski. Identification of endogenous social effects: The reflection problem. *The review of economic studies*, 60(3):531–542, 1993.

Robert McMillan. Competition, incentives, and public school productivity. *Journal of Public Economics*, 88(9):1871–1892, 2004.

NCES. Pss private school universe survey data for the 2009-2010 school year. 2010. URL `http://nces.ed.gov/surveys/pss/privateschoolsearch/`.

T.J. Nechyba. Mobility, targeting, and private-school vouchers. *American Economic Review*, 90(1):130–146, 2000.

T.R. Sass. Charter schools and student achievement in florida. *Education Finance and Policy*, 1(1):91–122, 2006.

Thomas D Snyder and Sally Dillow. Digest of education statistics 2011. 2012. URL `http://nces.ed.gov/programs/digest/d11/tables/dt11_101.asp`.

R.K. Sundaram. *A First Course in Optimization Theory*. Cambridge University Press, 1996.

Christopher Walters. A structural model of charter school choice and academic achievement. *Working paper*, 2012.

John White. States open to charters start fast in 'race to top'. Press Release, US Department of Education, `http://www2.ed.gov/news/pressreleases/2009/06/06082009a.html`, June 2009.

# A Model Solution

This section shows how to calculate school size and average test score, given an effort level pair $\mathbf{e} = (e_{ch}, e_{tps})$, which are necessary to solve for equilibrium of the entry subgame. A student with ability $a_i$ chooses a charter school if and only if

$$\begin{gathered} \mathrm{E}_{\nu^y}[y_{i,ch}] - c_{i,ch} + \eta_{i,ch} \geq \mathrm{E}_{\nu^y}[y_{i,tps}] - c_{i,tps} + \eta_{i,tps} \\ \Leftrightarrow \\ a_i \underbrace{(Ey_{ch}(1, \bar{a}_{ch}, e_{ch}, k_{ch}) - c_{ch,a} - Ey_{tps}(1, \bar{a}_{tps}, e_{tps}, k_{tps}))}_{\Delta g(\bar{\mathbf{a}}, \mathbf{e}, \mathbf{k})} + \underbrace{(\eta_{i,ch} - \eta_{i,tps})}_{\Delta\epsilon_i} \geq \underbrace{c_e(e_{ch} - e_{tps})}_{\Delta c_e} + \underbrace{c_r\,(r_{i,ch} - r_{i,tps})}_{\Delta c_{ri}} + \underbrace{c_{ch}}_{\Delta c_{ch}} \\ \Leftrightarrow \\ a_i \Delta g(\bar{\mathbf{a}}, \mathbf{e}, \mathbf{k}) + \Delta\epsilon_i \geq \underbrace{\Delta c_e + \Delta c_{ri} + \Delta c_{ch}}_{\Delta c_i} \end{gathered} , \tag{17}$$

where $\Delta\epsilon_i \sim \mathrm{N}\left(0, \sigma^2_{\Delta\epsilon}\right)$ and $\sigma^2_{\Delta\epsilon} = 2\sigma^2_\eta$. Ability within the market is distributed according to $F(a_i) = \mathrm{N}\left(\bar{a}, \sigma^2_a\right)$, making the left hand side of (17) the sum of two independent normally distributed random variables:

$$a_i \Delta g(\bar{\mathbf{a}}, \mathbf{e}, \mathbf{k}) + \Delta\epsilon_i \sim \mathrm{N}\left(\bar{a}\Delta g(\bar{\mathbf{a}}, \mathbf{e}, \mathbf{k}), \sigma^2_a \Delta g(\bar{\mathbf{a}}, \mathbf{e}, \mathbf{k})^2 + \sigma^2_{\Delta\epsilon}\right). \tag{18}$$

This provides an analytical expression for the share of students attending the charter school $\mu_{r,ch}$, given a distance difference $\Delta c_{ri}$:

$$\mu_{r,ch}(\Delta c_{ri}, \bar{\mathbf{a}}, \mathbf{e}, \mathbf{k}) = 1 - \Phi\left(\frac{\Delta c_e + \Delta c_{ri} + \Delta c_{ch} - \bar{a}\Delta g(\bar{\mathbf{a}}, \mathbf{e}, \mathbf{k})}{\sqrt{\sigma^2_a \Delta g(\bar{\mathbf{a}}, \mathbf{e}, \mathbf{k})^2 + \sigma^2_{\Delta\epsilon}}}\right), \tag{19}$$

where $\Phi$ denotes the standard cumulative normal distribution.

There are $\rho \in 1, \ldots, R$ separate distance pairs in the market, each with a measure $\mu_\rho$. The total measure of students at the charter school is the sum of the shares of students of each distance, weighted by the measure of students in each distance bin

$$\mu_{ch}(\bar{\mathbf{a}}, \mathbf{e}, \mathbf{k}) = \sum_{\rho=1}^{R} \mu_\rho \mu_{\rho,ch}(\Delta c_{\rho i}, \bar{\mathbf{a}}, \mathbf{e}, \mathbf{k}). \tag{20}$$

A student with ability $a_i$ and relative charter distance cost $\Delta c_{ri}$ will choose the charter if and only if $\Delta\epsilon_i \geq \Delta c_i - a_i \Delta g(\bar{\mathbf{a}}, \mathbf{e}, \mathbf{k})$, which happens with probability $\Phi\left(\frac{a_i \Delta g(\bar{\mathbf{a}}, \mathbf{e}, \mathbf{k}) - \Delta c_i}{\sigma_{\Delta\epsilon}}\right)$. By Bayes' Rule, the average ability of student attending the charter school is

$$\bar{a}_{r,ch}(\Delta c_{ri}, \bar{\mathbf{a}}, \mathbf{e}, \mathbf{k}) = \int a_i f_r(a_i; \bar{\mathbf{a}}, \mathbf{e}, \mathbf{k} | s_i = ch) da_i, \tag{21}$$

where the density of the ability of students at the charter school is

$$f_r(a_i; \bar{\mathbf{a}}, \mathbf{e}, \mathbf{k} | s_i = ch) = \frac{\Phi\left(\frac{a_i \Delta g(\bar{\mathbf{a}}, \mathbf{e}, \mathbf{k}) - \Delta c_i}{\sigma_{\Delta\epsilon}}\right) f(a_i)}{\mu_{r,ch}(\Delta c_{ri}, \bar{\mathbf{a}}, \mathbf{e}, \mathbf{k})}. \tag{22}$$

The average ability of students attending the charter school is the weighted average of the average abilities of attendant students from each bin:

$$\begin{aligned} \bar{a}_{ch} &= \sum_{\rho=1}^{R} \mu_\rho \mu_{\rho,ch}(\Delta c_{ri}, \bar{\mathbf{a}}, \mathbf{e}, \mathbf{k}) \bar{a}_{\rho,ch}(\Delta c_{ri}, \bar{\mathbf{a}}, \mathbf{e}, \mathbf{k}) / \mu_{ch}(\bar{\mathbf{a}}, \mathbf{e}, \mathbf{k}) \\ \bar{a}_{tps} &= \sum_{\rho=1}^{R} \mu_\rho \mu_{\rho,tps}(\Delta c_{ri}, \bar{\mathbf{a}}, \mathbf{e}, \mathbf{k}) \bar{a}_{\rho,tps}(\Delta c_{ri}, \bar{\mathbf{a}}, \mathbf{e}, \mathbf{k}) / \mu_{tps}(\bar{\mathbf{a}}, \mathbf{e}, \mathbf{k}) \end{aligned} \tag{23}$$

where $\mu_{\rho,tps}$, $\mu_{tps}$, $\bar{a}_{\rho,tps}$ and $\bar{a}_{tps}$ are defined analogously for public schools. The pair of equations (23) define a fixed point for average ability at the charter and public schools given effort level pair $\mathbf{e}$ and capital $\mathbf{k}$.

After solving for $\bar{a}_s$, the average test score at school $s$ is $\bar{y}_s = \bar{a}_s \omega_s \bar{a}_s^{\theta_s} \left(\alpha_s e_s^{\beta_s} + (1-\alpha_s) k_s^{\beta_s}\right)^{\tau_s/\beta_s} =$

$\bar{a}_s Ey_s(1, \bar{a}_s, e_s, k_s)$, which, along with $\mu_s$, is substituted into school objectives when solving for optimal school effort.

# B Likelihood

The likelihood function combines the previous probability and likelihood statements for markets and students, and integrates over the ability distribution in a market, given all the data $X$ and parameters $\theta$:

$$\begin{aligned}
L(\theta|X) = (&\prod_{m\in M}\prod_{t\in 1,\dots,T} \Pr\{z^o_{tm} = entry|z^o_{t-1,m}\}^{1\{z^o_{tm}=entry\}}(1-\Pr\{z^o_{tm} = entry|z^o_{t-1,m}\})^{1\{z^o_{tm}=no\ entry\}} \\
&(\prod_{m\in M}\prod_{s\in S_{tm}}\prod_{t\in T}\left(L(e^o_{ch,tm}|z^o_{tm}=entry)L(e^o_{tps,tm}|z^o_{tm}=entry)\right)^{1\{z^o_{tm}=entry\}} \cdot L(e^o_{tps,tm}|z^o_{tm}=no\ entry)^{1\{z^o_{tm}=no\ entry\}}) \cdot \\
(\prod_{m\in M}\int_{a_i\in A_m} &(\prod_{i\in I_{tm}}\prod_{t\in T}((\Pr\{s^o_{itm}=ch|z^o_{tm}=entry,a_i\}L(y^o_{i,ch,tm}|z^o_{tm}=entry,s^o_{itm}=ch,a_i))^{1\{s^o_{itm}=ch\}} \cdot \\
&((1-\Pr\{s^o_{itm}=ch|z^o_{tm}=entry,a_i\})L(y^o_{i,tps,tm}|z^o_{tm}=entry,s^o_{itm}=tps,a_i))^{1\{s^o_{itm}=tps\}})^{1\{z^o_{tm}=entry\}} \cdot \\
&L(y^o_{i,tps,tm}|z^o_{tm}=no\ entry,s^o_{itm}=tps,a_i)^{1\{z^o_{tm}=no\ entry\}})dF_m(a_i)) \qquad (24)
\end{aligned}$$

# C Parameter Estimates

| Parameter | Estimate | Standard error | Description |
|---|---|---|---|
| **Test score production functions** | | | |
| $\omega_{ch}$ | 1.744 | 0.114 | TFP for charter school relative to public |
| $\alpha_{ch}$ | 0.627 | 0.049 | effort share, charter school |
| $\beta_{ch}$ | -0.675 | 0.114 | substitution parameter, charter school |
| $\alpha_{tps}$ | 0.966 | 0.116 | effort share, public school |
| $\beta_{tps}$ | -0.295 | 0.991 | substitution parameter, public school |
| $\tau_{ch}$ | 0.953 | 0.052 | return to scale, charter school |
| $\tau_{tps}$ | 0.060 | 0.009 | return to scale, public school |
| $\theta_{ch}$ | 0.026 | 0.029 | peer effects, charter school |
| $\theta_{tps}$ | 0.995 | 0.012 | peer effects, public school |
| **Student cost** | | | |
| $c_{ch}$ | 15.280 | 3.662 | student charter school cost |
| $c_{ch,a}$ | -0.361 | 0.211 | student charter school cost interact ability |
| $c_e$ | -3.869 | 0.968 | student effort cost |
| $c_r$ | 0.726 | 0.179 | student distance cost |
| **School preference shock** | | | |
| $\sigma_\eta$ | 7.985 | 1.971 | st. dev. student school preference shock |
| **School valuation of test scores** | | | |
| $\delta_{y,ch}$ | 20.960 | 3.748 | value of average test score, charter school |
| $\delta_{y,tps}$ | 19.634 | 3.483 | value of average test score, public school |
| **School effort cost functions** | | | |
| $\psi_{e,ch,1}$ | 0.000 | 7.432 | disutility of effort, charter school |
| $\psi_{e,tps,1}$ | 1.930 | 0.398 | disutility of effort, public school |
| $\psi_{e,ch,2}$ | 14.137 | 3.559 | effort, school size interaction, charter school |
| $\psi_{e,tps,2}$ | -0.071 | 0.021 | effort, school size interaction, public school |
| $\psi_{e,ch,3}$ | -15.508 | 5.003 | effort, capital interaction, charter school |
| $\psi_{e,tps,3}$ | -0.917 | 0.265 | effort, capital interaction, public school |
| $\psi_{e,ch,4}$ | 7.423 | 1.980 | effort, mean ability interaction, charter school |
| $\psi_{e,tps,4}$ | 0.000 | 0.109 | effort, mean ability interaction, public school |
| **Entry cost** | | | |
| $\mu_{\underline{v},no\ entry}$ | 96.444 | 29.416 | mean entry cost distribution, no entry last period |
| $\sigma_{\underline{v},no\ entry}$ | 20.169 | 10.529 | st. dev. entry cost distribution, no entry last period |
| $\mu_{\underline{v},entry}$ | 34.621 | 18.483 | mean entry cost distribution, entry last period |
| $\sigma_{\underline{v},entry}$ | 14.053 | 17.145 | st. dev. entry cost distribution, entry last period |
| $\psi_{mkt.size}$ | 8.580 | 6.491 | weight for market size |
| $\psi_{fr.Black}$ | 23.479 | 14.077 | weight for fraction of black students in market |
| **Effort productivity shocks** | | | |
| $\sigma_{\nu_e}$ | 0.185 | 0.003 | st. dev. effort productivity shock |

# D For Online Publication

## D.1 Existence of Equilibrium

These proofs are for the case where there are no productivity shocks to the school's chosen effort level, but the results go through in the case where there are shocks. To use Brouwer's Fixed Point Theorem, the pair of best-response functions must be a continuous self-map on a compact and convex set. I prove first there is a unique best response of one school to another, then that this best response function is continuous, and finally apply Brouwer's Fixed Point Theorem. I assume both school production functions satisfy Inada conditions and have convex effort costs.

**Lemma 1.** *The solution to the charter school's effort choice problem $e^*_{ch}$ is strictly positive.*

*Proof.* $\lim_{e_{ch}\to 0} \frac{\partial v_{ch}(\mathbf{e})}{\partial e_{ch}} = \infty$ due to the Inada conditions on the test score production function, because there will always be some measure of students attending the charter school due to the preference shocks. □

Call $v^+_{ch} = \max\{v_{ch}, 0\}$. Note that $v^+_c$ is strictly quasi-concave, due to the strict concavity of $v_{ch}$ when it is above 0.

**Lemma 2.** *The effort set $E = [\underline{e}, \bar{e}]$ is compact.*

*Proof.* Let $\underline{e} = 0$. Given any allowable vector of parameters $\theta$ there exists $\bar{e}_\theta$ such that $v_{ch}(\hat{e}) < 0$, all $\hat{e} > \bar{e}_\theta$. Let $\bar{e} = \max_\theta \bar{e}_\theta$. It exists, so the set is not empty. □

**Lemma 3.** *$e^*_{ch}(e_{tps})$ is continuous.*

*Proof.* Berge's Maximum Theorem (Sundaram [1996]) requires a continuous objective $v^+_{ch}$, and compact and upper-hemicontinuous (UHC) constraint set. Note first that the constraint set, $E$, is a fixed connected interval, so it is trivially UHC. $v^+_{ch}$ is continuous, so the Maximum Theorem says the resulting correspondence which is the argmax of $v^+_{ch}$ is UHC. Because $v^+_{ch}$ is strictly quasi-concave, there is a unique argmax to $v^+_{ch}$, which means that $e^*_{ch}(e_{tps})$ is a continuous function. Analogous reasoning applies to $e^*_{tps}(e_{ch})$. □

**Lemma 4.** *There exists an equilibrium to the entry subgame.*

*Proof.* $\Gamma(e_{tps}, e_{ch}) = \left(e^*_{ch}(e_{tps}), e^*_{tps}(e_{ch})\right)$ is a continuous self map on the compact and convex domain $E^2$, there exists an equilibrium by Brouwer's Fixed Point Theorem. □

## D.2 Uniqueness of Equilibrium

I do not prove uniqueness of equilibrium in the entry subgame but can rule out multiplicity of the charter school entry decision, given a unique equilibrium in the ensuing entry subgame.

**Lemma 5.** *There is no multiplicity in the charter school entry decision given uniqueness of equilibrium in the entry subgame.*

*Proof.* The charter school only receives one shock $\underline{v}_{tm}$, which it knows. It enters if and only if $\mathrm{E}_{\nu^e}\left[v_{ch,tm}(\mathbf{e}^*)\right] \geq \underline{v}_{tm}$, where $\mathrm{E}_{\nu^e}\left[v_{ch,tm}(\mathbf{e}^*)\right]$ is known since under the assumption of the lemma there is a unique equilibrium in chosen effort levels of the entry subgame. □

Searches for more than one equilibrium for a wide range of parameter values and have never returned more than one equilibrium in a market. Intuitively, there will not be multiple equilibria in the entry subgame so long as schools are not too responsive to each other, which may be satisfied if the effort cost is sufficiently convex. I assume that in the presence of multiple equilibria for the entry subgame both schools always play the same equilibrium.

## D.3 Construction of Capital Variable

There are several measures of school resources for charter and public schools in the NCERDC data, but many are missing for many schools – especially charters. Moreover, in markets where there was no charter school entry I do not observe school-specific resources for charter schools, which are necessary for solving the model. Finally, it is not obvious how the different measures of school resources should enter into the test score production function.

The following algorithm computes a level of capital for both charter and public schools given information that is always observable for a market: i) Convert measures (computers/pupil, teachers/pupil, experienced teachers/pupil) to percentiles (using the same distribution for both charter and public schools), ii) average (unweighted) these percentiles into one index for each school. iii) regress this index on inflation-adjusted per-pupil expenditures for the public school in each market, using separate regressions for charter and public schools, iv) use the predicted value from the above regression as the capital measure for that school type in that market. This measure always exists, so long as there are data on the per-pupil expenditures for the public school in that market.[34]

The last step obviates integrating over the errors in the cost functions when solving the charter school's entry problem. Also, it precludes a role for charter schools making entry decisions based on unobservable information – that is, the predicted per-pupil capital levels are no different in expectation in entry and non-entry markets with the same level of per-pupil expenditures. Although such variation may play a role in charter school entry, it is likely second order in understanding charter school entry patterns. Finally, note that since capital is percentile-based, rank-preserving changes in the capital distribution do not effect the equilibrium. Further details are available upon request.

## D.4 Construction of Distance Distribution

The model takes into account charter school location decisions through market-level distance distributions for charter and traditional public schools. Distance distributions do not necessarily correspond to physical locations within markets, but do take into account the fact that charter schools are often further from students than traditional public schools. For computational purposes, I first discretize the distance distribution and then model what the distance distribution would be in markets without observed charter school entry, which is key for solving for an equilibrium for each market. Details are available upon request.

[34] Details available upon request.

## D.5 Sample Selection

Table 6 compares selected variables for the full sample and estimation samples. The means of most variables for public schools in the full and estimation samples are similar. In both the full and estimation samples, markets with charter schools have higher fractions of Black and Hispanic students, yet charter schools have lower fractions of both types of students relative to public schools in such markets. In both samples, female students comprise a smaller share of students at charter schools than they do for both types of public schools, and students attending charter schools are much more likely to have had at least one parent who has attended at least some college than students at either type of public school (in the estimation sample, 75% for charter schools versus 60% for public schools in markets with charters and 43% for public schools in markets without charters).

Table 7 shows how sample restrictions affect the test score distribution. In particular, removing students who attended a public school outside their market increases the average test score for students attending charter schools.[35] About 20% of excluded students are from one charter school.

[35] Note that the extent of the bias induced by using a subset of charter school students cannot be derived from simple comparisons of the mean test scores of charter and public schools in entry markets. What matters is instead the ability distribution of excluded households.

## Table 6: Summary Statistics for Full and Estimation Samples

| | Summary Statistics for Full Sample | | | | | | | | Summary Statistics for Estimation Sample | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | All Markets | | Mkts. w. Charters. | | | | Mkts. w.o. Charters | | All Markets | | Mkts. w. Charters. | | | | Mkts. w.o. Charters | |
| | % | N | % | N | % | N | % | N | % | N | % | N | % | N | % | N |
| | All Schools | | Charter | | Public | | Public | | All Schools | | Charter | | Public | | Public | |
| Ethnicity | | | | | | | | | | | | | | | | |
| White | 63.45 | 644300 | 61.75 | 4689 | 53.52 | 38214 | 64.22 | 601396 | 62.82 | 92080 | 68.40 | 3359 | 56.07 | 35443 | 67.85 | 53120 |
| Black | 29.44 | 298955 | 30.93 | 2349 | 37.89 | 27054 | 28.78 | 269552 | 30.02 | 44000 | 25.23 | 1239 | 36.47 | 23052 | 25.17 | 19709 |
| Hispanic | 3.08 | 31274 | 1.25 | 95 | 4.31 | 3077 | 3 | 28103 | 2.74 | 4016 | 1.34 | 66 | 3.26 | 2064 | 2.40 | 1882 |
| Other | 4.03 | 40925 | 6.07 | 461 | 4.28 | 3056 | 4 | 37464 | 4.43 | 6492 | 5.03 | 247 | 4.20 | 2657 | 4.58 | 3583 |
| Female | 49.09 | 1015511 | 46.93 | 7594 | 49.53 | 71402 | 49.07 | 936515 | 49.62 | 146588 | 48.61 | 4911 | 50.33 | 63216 | 49.08 | 78294 |
| % Parent College | 47.92 | 1015365 | 70.11 | 7591 | 56.92 | 71373 | 47.06 | 936401 | 51.48 | 146563 | 75.22 | 4911 | 60.07 | 63216 | 43.04 | 78276 |
| Grade | 6.97 | 1015511 | 6.81 | 7594 | 6.94 | 71402 | 6.97 | 936515 | 6.89 | 146588 | 6.80 | 4911 | 6.97 | 63216 | 6.84 | 78294 |
| % Attending Charter | 0.75 | 1015511 | 100.00 | 7594 | 0.00 | 71402 | 0.00 | 936515 | 3.37 | 146588 | 100.00 | 4911 | 0.00 | 63216 | 0.00 | 78294 |
| % With Charter in Market | 7.78 | 1015511 | 100.00 | 7594 | 100.00 | 71402 | 0.00 | 936515 | 46.59 | 146588 | 100.00 | 4911 | 100.00 | 63216 | 0.00 | 78294 |

## Table 7: Test Score Distribution for Full and Estimation Samples by Market and School Type

| | Full Sample | | | Estimation Sample | | |
|---|---|---|---|---|---|---|
| Entry | Mean | SD | N | Mean | SD | N |
| Charter | 2.975 | 0.974 | 7376 | 3.1370 | 0.938 | 4941 |
| Public | 3.032 | 0.995 | 67219 | 3.0754 | 0.990 | 63216 |
| No Entry | | | | | | |
| Public | 3.002 | 0.933 | 893413 | 3.0278 | 0.915 | 65085 |

## D.6 Robustness of Results to Market Definition

This section explains how the choice of modeling markets does not affect the validity of estimates of model parameters and inferences about charter school effects.

The first concern is that the manner in which markets are structured produces biased estimates of school technology and the distribution of treatment effects for students attending charter schools.[36] Charter school students in the estimation sample may have higher average ability than those in the overall population because the algorithm used to assign charter schools to markets excludes some students observed attending charter schools, and the excluded students on average have lower test scores. This could in principle affect the distribution of student ability at charter schools, which could affect both estimates of production function parameters and the distribution of estimated treatment effects. I recover unbiased parameters for test score production technologies for charter and public schools by developing an identification strategy that separates unobserved student ability from the effectiveness of school inputs, even when students could sort on ability due to differences in school inputs. Therefore, estimates of test score production technologies are not biased by the sample restrictions induced by the market creation algorithm. After establishing that parameters have been consistently estimated, the estimated model can be used to quantify the extent to which excluding these households might affect the simulated distribution of treatment effects – in particular the direct effect of charter school entry. I implement this by allocating excluded charter school students to a synthetic market and solving for equilibrium charter school effort levels in this synthetic market, given the ability distribution of excluded charter school students, and estimating treatment effects as in Section 6. Because individual student ability acts as TFP in the production technology at charter and public schools, lower mean ability should attenuate the direct effect of charter schools on student achievement, which is what I find.[37]

The second concern is that charter schools compete with more schools than just the competitor assigned to them through market creation. This could affect both charter school entry patterns and charter or public school effort choices.

*Entry patterns:* I test to what extent the designated public school competitors, as opposed to characteristics of the districts in which these competitors are located, drive charter school entry patterns in the data. If charter schools indeed are competing with the entire district, as opposed to the designated public school competitor, their entry decisions should depend on district-level characteristics, as opposed to those of their designated competitor. To test this, I ran a probit regression of charter school entry (or continuing operation) on both the size of the competing public school, which would enter the charter school's objective, and the overall size of the school district in which the designated competitor public school lies. The probability of charter school entry increases with the size (number of students attending) the designated competitor, but not with size of the district in which the designated competitor is located. The estimated coefficient on the size of the designated public school is positive and significantly greater than zero (z-score of 4.81), while that on overall market size is negative and not significantly different from zero (z-score of -0.98).

*School effort choices:* By restricting each charter school to only compete with one tra-

[36]See Section 2.4 for market definitions.

[37]See Appendix D.7 for details.

ditional public school, the model may overstate the effect that charter schools have on the effort levels of competing public schools and ignore potential effects on effort levels of public schools outside their market. One in principle could aggregate charter schools and report treatment effects when allowing public schools to compete against all charter schools in a county, but the estimation results indicate that public school effort is not very effective, which means that spillovers on the effort of public schools outside a charter school's market are likely to result in only negligible changes in test scores.

The third concern is that the market-based exclusion of some charter school students may affect estimated parameters of school objectives. Charter schools in the estimation sample have higher average test scores than those in the unrestricted sample, which could understate the weight attached to average student performance in the school objective. However, given that the other parameters of the school objective should adjust to fit charter school effort choices the sign of this potential bias is unclear. Excluding some charter school students may overstate how sensitive charter school effort choices are to their size. In particular, the estimates may provide a lower bound for how effective they are, especially in the counterfactual investigating treatment effects when charter schools are forced to serve all students in any markets they enter.

In sum, although there are theoretical concerns that the way markets are defined might affect this paper's findings, the definition is reasonable because i) estimates of key technology parameters are unaffected, ii) when the model is used to quantify the direct effect on excluded students, results remain substantially unchanged (Appendix D.7), iii) charter school entry patterns relate more to the characteristics of competitor public schools than larger school districts, iv) the effect of market definition on the performance of students attending public schools outside their markets is minimal, v) it is not obvious that changes to school objectives would either increase or decrease estimated treatment effects were the full sample of charter school students used, and, finally, vi) excluding a fraction of charter school students could overstate estimated diseconomies of scale in charter school objectives, therefore *understating* their effectiveness.

## D.7 Correcting for Potential Sample Selection Bias

In this section I provide an approximation for the bias induced on the direct treatment on the treated (TOT), i.e. TOT for students attending charter schools, by excluding students who attend charter schools outside their designated markets.

Recall that allowing these excluded students in the model would imply interdependence between markets, violating the assumption that the markets form a partition of North Carolina. This interdependence would render the model computationally intractable: An equilibrium where all schools were competing with all other schools would be far more complex than the two-school version used in this paper because it would require solving equilibrium outcomes for not only a large number of schools but also *for every possible configuration of charter school entry decisions.*

One concern with this restriction is that excluding charter school students who on average have lower test scores might affect mean ability of students at charter schools. As has been previously discussed, this does not in principle induce a bias in estimates of test score production parameters at charter or public schools: those parameters are consistently estimated so long as student ability can be controlled for. If school effort choices and effective school inputs to test score production have been consistently estimated, changes in the student ability distribution will change the scale of the direct TOT but not alter its direction, so long as mean ability and the difference between effective inputs at charter and monopoly public schools for students attending charter schools is positive.[38] In this scenario, a lower mean of the charter school student ability distribution would only diminish the direct treatment effect, resulting in upwards bias from excluding lower ability students.

I first approximate what the direct TOT would have been for the excluded students, and then compute an average of direct TOT for the included and excluded samples, weighed by the sample sizes. To capture the fact that excluded charter school students have lower average test scores than included charter school students, I assume that the excluded charter school students all reside in one synthetic market, denoted $m = ex$.[39] Let $I_{ex}$ denote the set of excluded students, $n_{ex} = |I_{ex}|$ the number of excluded students, and $F_{ex}$ the distribution function for ability of excluded students. The direct effect of treatment on the treated in market $ex$ is

$$\overline{\Delta}_{ex}^{direct,TOT} = \int \Delta_{i,ex}^{direct} f_{ex}(a_{i,ex}) da_{i,ex} \tag{25}$$

where $f_{ex}(a_{i,ex})$ is the density of ability for excluded students and the direct effect for $i$, $\Delta_{i,ex}^{direct}$, is the difference in test scores between the synthetic charter and monopolist public schools in market $ex$:

$$\Delta_{i,ex}^{direct} = Ey_{ch}\left(a_i, \bar{a}_{ch,ex}, e_{ch,ex}, k_{ch,ex}\right) + \nu_{i,ch,ex}^{y} - Ey_{tps}\left(a_i, \bar{a}_{ex}, e_{tps,ex}^{mono}, k_{tps,ex}\right) - \nu_{i,tps,mono,ex}^{y}$$

where $k_{s,ex}$ and $e_{s,ex}$ denote capital and effort levels for school $s$ in market $ex$. As was seen in Section 6.2, the treatment effect on the treated depends crucially on endogenous school inputs and student sorting on unobservable ability.

To evaluate the above expression one needs the ability distribution for excluded students $F_{ex}$, which can be approximated using the same method used in estimation (Section 4), but substituting in charter school *average* capital and effort inputs across all charter schools in all markets.[40] As before, assume the test score distribution is normal, which means that it is sufficient to recover the mean and standard deviation for the excluded ability distribution.[41] Mean ability for excluded students is

$$\bar{a}_{ch,ex} = \left(\frac{\overline{y}_{ch,ex}}{y_{ch}(1, 1, \overline{e}_{ch}, \overline{k}_{ch})}\right)^{\frac{1}{1+\theta_{ch}}},$$

[38] "Effective inputs" are the expected test score at a school for student with ability 1, i.e. $\omega_s \bar{a}_s^{\theta_s}\left(\alpha_s(e_s)^{\beta_s} + (1-\alpha_s)k_s^{\beta_s}\right)^{\tau_s/\beta_s}$. Differences in student ability exacerbate differences in effective inputs between schools.

[39] Observations from 1998-2001 are pooled into the same synthetic market for the current exercise.

[40] An alternative would be to posit an effort level for the charter school that enters the recovery of the ability distribution for excluded students, $\tilde{e}_{ch,ex}$, solve for the subsequent equilibrium effort level, $\tilde{e}'_{ch,ex}$, and iterate to find a fixed point in the effort level for the synthetic charter school serving excluded students.

[41] Even if market ability distributions were normal, student sorting on ability would generically induce the distribution of abilities for students at charter schools to be non-normal. Nevertheless, this method captures the fact that excluded students have lower average test scores.

where $\overline{e}_{ch}$ and $\overline{k}_{ch}$ denote average charter school capital and observed effort across all markets. The standard deviation of excluded student ability is recovered analogously. After recovering the distribution of ability for excluded students, solve for equilibrium charter and monopoly public school effort levels in market $ex$ by assuming that each school in the synthetic market has the average capital level for that school type across all markets, and that students are the same distance from both the charter and public school.

Finally, one can combine treatment effects for included and excluded students to form an estimate of the extent to which the sample selection procedure may bias estimates of the direct TOT. Let $\overline{\Delta}_{incl}^{direct,TOT}$ denote the expected direct effect of treatment on the treated for students attending charter schools who were retained in the sample, which represents $n_{incl}$ students. The estimate of the overall direct effect of treatment on the treated, taking into account excluded households, is

$$\overline{\Delta}_{all\ markets}^{direct,TOT} = \frac{\overline{\Delta}_{ex}^{direct,TOT} n_{ex} + \overline{\Delta}_{incl}^{direct,TOT} n_{incl}}{n_{ex} + n_{incl}}. \tag{26}$$

The market size of the synthetic market containing excluded students affects charter and monopoly public school inputs, so results are reported for three scenarios: i) the market serving excluded students is three times as large as an average market, ii) four times as large, and iii) five times as large. The mean direct effect on excluded students is 11.1% of a standard deviation in the first case (5.7% and 2.1% for cases ii and iii, respectively), which returns a mean direct effect on charter school students to 10.9% in the first case (9.7% and 7.9% for cases ii and iii, respectively).

Another way to characterize the extent to which excluded students might have a different treatment effect than those included in the estimation sample is to use a back of the envelope calculation applying the same effective inputs they would have received at charter schools, adjusting for their different (lower) mean ability. Using this method, I calculate that the direct TOT would be 9.4% of a standard deviation, which is similar to the result obtained using the previous method. In summary, excluding the students does not substantially affect the mean direct effect of charter school entry on student achievement.