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This paper contains preliminary findings from research work still in progress and should not be quoted without prior approval of the authors.

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BEST APPROXIMATE AGGREGATION OF INPUT-OUTPUT SYSTEMS[1]

by Glenn W. Harrison and Richard Manning

A method of constructing aggregate input-output systems is proposed. The procedure minimizes the mean-square error of aggregate predictions. If consistent aggregation is possible it gives the solution to the problem. The range of data on which the aggregate model is to be applied influences the best approximate aggregate. In a special case, W. D. Fisher's aggregated model emerges. Best approximate aggregates maximize Ijiri's coefficient of aggregation. Computations reveal that best approximate aggregation is feasible, and in realistic cases yields considerably better predictions than alternative methods.

1. INTRODUCTION

Input-output systems are the basis of most applied general equilibrium analysis. Since these systems are typically of large dimension, it is often convenient to reduce them in size by aggregating groups of industries. There is usually a natural method of aggregation based on product or process similarities. When the aggregate industries are identified, an aggregate input-output matrix is needed to analyze their behavior. Unfortunately the classic results on consistent aggregation offer little help in providing the aggregate input-output matrix. This is because these results require aggregation to be exact. In general, aggregation involves an error. If a loss function is defined on this aggregation error then the aggregate

input-output matrix can be selected by minimizing the loss function. This paper develops best approximate aggregate input-output matrices in this way. The properties of these matrices are described and their superiority over aggregate input-output matrices constructed in other ways is demonstrated for real systems.

This approach to aggregation has been made before. Walter D. Fisher applied a loss-minimizing principle to problems of aggregation of input-output systems. His work displayed great insight, but has been neglected by practitioners of input-output analysis who continue to use aggregation methods not based on theoretical principles. It is not appropriate to speculate here on the reasons for this neglect. Rather, a simplified account of this method of aggregation is given. Unlike Fisher, and others who worked on the assumption of consistent aggregation, the primary emphasis here is on the selection of the aggregate system, not on the choice of industries to aggregate.

The next section begins by reviewing the theory of consistent aggregation, which motivates the development of best approximate aggregation. Some properties of best approximate aggregate input-output systems, and of predictions based on them, are also given there. In the third section related work on aggregation is briefly reviewed. Also established there are the connections with earlier work on aggregation in input-output. Section 4 reports the results of applying best approximate aggregation to large-scale, real examples. Input-output tables for the U.S.A., Australia, and a multi-country table for the Pacific Basin, are each aggregated. The aggregate systems are shown to be markedly superior to those generated by standard

methods (including an official aggregated input-output table for Australia). Implications of these results for general equilibrium modelling, along with other applications of the method, are discussed in the final section of the paper.

2. MEAN-SQUARE ERROR MINIMIZING AGGREGATION

The classic results concerning the consistent aggregation of input-output systems are first presented. This allows the introduction of some important concepts.

An input-output matrix A (A non-negative) relates the gross output of commodities, denoted by a vector x, to the amounts of those commodities used up within the economy, Ax. Suppose that A is nXn, and that x is nX1. The net output is a vector y, also nX1, which is given by

$$(2.1) \qquad y = [I_n - A]x$$

where I_n is the nXn identity matrix.

It is convenient to call x the <u>exogenous variable</u>, y the <u>endogenous variable</u>, and $[I_n - A]$ the <u>model</u>. Although a particular interpretation is placed on these terms in the theoretical development, other interpretations are possible, and some will be made in the later application of the technique. Two alternative interpretations made are:

(i) The exogenous variable is net output (or final demand), so that the endogenous variable is gross output, and the model is the inverse of the net input-output matrix (the "Leontief Inverse"); and

(ii) The exogenous variable is primary input cost, so that the endogenous variable gives commodity prices, and the model is the inverse of the transposed net input-output matrix (the "Leontief Inverse transpose").

The exogenous and endogenous variables x and y may be reduced in dimension by an aggregator T, where T is a non-negative m×n matrix, $m < n$, rank $T = m$. This gives an aggregated exogenous variable X, and an aggregated endogenous variable Y, both m×1, where

(2.2) $\quad X = Tx, \; Y = Ty.$

T is assumed to be given. The extent of aggregation is determined by m. The industries that are to be aggregated determine the structure of T (the full rank assumption guarantees that aggregated industries are different). There is invariably some clear and intuitive criterion for the aggregation scheme chosen. For example, a disaggregated input-output system contains many agricultural and many manufacturing-industry sectors (among others): A natural aggregator would group together the inputs to (and outputs from) all agricultural sectors, and group together all manufacturing-industry sectors as a separate aggregate industry (specific instances of this are given in Section 4).

Economic analysis of the aggregate variables requires a relationship between X and Y. This suggests the usual approach to aggregation, which is based on the following definition (see Richmond [32; p. 48] for a general statement of this sort).

DEFINITION 2.1 The model $[I_n - A]$ relating the exogenous variable x to the endogenous variable y, can be consistently aggregated by the aggregator T if and only if there exists an aggregated model $[I_m - B]$ such that

(2.3) $\quad Y = [I_m - B]X,$

where $X = Tx$, for all $x \in R^n$.

Of course B is m×m, and I_m is the m×m identity matrix. Notice that the exogenous variables are not at all restricted in this definition.

PROPOSITION 2.1 The model $[I_n - A]$ can be consistently aggregated by the aggregator T if and only if the matrix equation

(2.4) $\quad BT = TA$

has a solution B. This solution gives the aggregated model.

PROOF: Substituting for y in (2.2) from (2.1) gives

(2.5) $\quad Y = T[I_n - A]x$

Substituting for X in (2.3) from (2.2) gives

(2.6) $\quad Y^* = [I_m - B]Tx$

But Definition 2.1 requires $Y = Y^*$ for all x. Therefore

(2.7) $\quad T[I_n - A] = [I_m - B]T$ ∎

Since (2.4) is equivalent to mn linear equations in m^2 unknowns (the elements of B), and $m^2 < mn$, no aggregated model exists in general. Although some rank conditions on A and T will ensure that B exists, there is no reason to expect that these conditions will be satisfied for an arbitrary model and aggregator. An alternative approach is needed. Before providing this, note that post-multiplying both sides of (2.4) by T', and then by $(TT')^{-1}$ gives

(2.8) $\quad B = (TAT')(TT')^{-1}$

if it exists. This will be used later.

The restricted usefulness of consistent aggregation clearly stems from Definition 2.1 which requires that $Y = Y^*$, where these are calculated as in (2.5) and (2.6) respectively. For a given exogenous variable x, (2.5) gives the true value of the aggregate endogenous variable Y. For the same value of x, and for an arbitrary aggregate model $[I_m - B]$, (2.6) may be regarded as giving a prediction of Y. There is no reason to suppose (but consistent aggregation does so) that the prediction coincides with the true value of the aggregated endogenous variable. That is

$$(2.9) \qquad (Y^* - Y|x) = ([I_m - B]T - T[I_n - A])x \neq 0$$

in general. This is the aggregation error[2] (conditional on the exogenous variable). While the model and the aggregator are given, the aggregate model can be chosen. The aggregation errors depend on this choice.

Although the exogenous variable can range over R^n, some values are more likely to be met than others. It is assumed that the user of the aggregated model has a subjective probability density function g(x) which gives the relative frequency with which the exogenous variable takes the value x. Call g the use density of the aggregated model. Since good predictions should be available when they are needed, the use density must be taken into account. For instance, if the aggregated model is to be used in an LDC it would be inappropriate to have it fit better to the output structure found in an advanced, rich, economy. The use density in this case would place most weight on output levels and structures near those likely to be found in the LDC.

With the use density known, the mean-square error can be calculated from

$$(2.10)\qquad M(B|A,T,g) = E_g[(Y^* - Y)'(Y^* - Y)]$$
$$= E_g[x'([I_m - B]T - T[I_n - A])'([I_m - B]T - T[I_n - A])x]$$

where E_g denotes expectation over x with distribution g.

DEFINITION 2.2 The <u>best approximate aggregate model</u> $[I_m - B^*]$, given the model $[I_n - A]$, the aggregator T, and the use density g, is given by that B^* which minimizes $M(B|A,T,g)$.

It is obvious that if consistent aggregation is possible then the resulting model is also the best approximate aggregate model (since $M(B|A,T,g)$ cannot be less than zero, which is its value when $Y = Y^*$ as required for consistent aggregation). Thus, best approximate aggregation generalizes the classical concept of consistent aggregation.

PROPOSITION 2.2 Given the model $[I_n - A]$, the aggregator T, and the use density g, the best approximate aggregate model is given by

$$(2.11)\qquad B^* = E_g[TAxx'T'][E_g[Txx'T']]^{-1}$$

if it exists and is finite.

PROOF: It is necessary for an interior minimum that B^* satisfies

$$(2.12)\qquad \frac{\partial M(B|A,T,g)}{\partial B} = 0.$$

Now

$$(2.13)\qquad \frac{\partial M(B|A,T,g)}{\partial B} = E_g\left[\frac{\partial Q(B|A,T,g)}{\partial B}\right]$$

where

$$(2.14) \qquad Q(B|A,T,g) = x'([I_m - B]T - T[I_n - A])'([I_m - B]T - T[I_n - A])x.$$

It can be shown[3] that

$$(2.15) \qquad \frac{\partial Q(B|A,T,g)}{\partial B} = 2(BTxx'T' - TAxx'T').$$

Substituting (2.15) into (2.13) and applying (2.12) gives (2.11). ■

It has already been noted that best approximate aggregation generalizes consistent aggregation. This can also be seen by comparing (2.8) and (2.11): Their form is similar. Indeed, if consistent aggregation is possible (2.4) may be used to substitute for TA in the first expectation in (2.11). But then

$$(2.16) \qquad B^* = E_g[BTxx'T'][E_g[Txx'T']]^{-1} = B.$$

Thus, if consistent aggregation is possible then (2.11) gives the corresponding aggregated model.

Several desirable features are displayed by B*, which can be expressed as a functional B*(A,T,g) to indicate its dependence on the model, aggregator and use density.

PROPOSITION 2.3 The best approximate aggregate model is given by

$B^*(A,T,g)$ which

(i) is homogeneous of degree zero in T;

(ii) for g_1, g_2 such that $g_1(x) = g_2(\lambda x)$, $\forall x$, $\lambda > 0$ constant, is such that

$$B^*(A,T,g_1) = B^*(A,T,g_2)$$

(iii) may be written as

(2.17) $$B^* = [TA(\Omega_g + \mu_g \cdot \mu_g')T'][T(\Omega_g + \mu_g \cdot \mu_g')T']^{-1}$$

where Ω_g is the covariance matrix of x, when g is its distribution, and μ_g is its mean.

(iv) depends only on the first two moments of x.

PROOF: (i) It is obvious from (2.11) that B* is unaffected if T is multiplied by a scalar.

(ii) Changing x to λx in (2.11) leaves B* unaffected.

(iii) B* in (2.11) may be written as

(2.18) $$B^* = [TA\,E_g[xx']T'][T\,E_g[xx']T']^{-1}$$

This may be simplified to (2.17) by substituting the well-known identity

(2.19) $$E_g[xx'] = \Omega_g + \mu_g \cdot \mu_g'$$

(iv) is obvious from (2.17). ∎

Results (i), (ii) and (iv) of this Proposition give situations in which the best approximate aggregate is insensitive to changes. Consideration of the sensitivity of the aggregate model to the method of aggregation, the model, and the use density, are best performed numerically.

Note that (i) says that identical changes in the units of measurement of <u>all</u> the aggregated variables leaves the best approximate aggregate model unchanged. It does not say that a change in measurement of one aggregate variable has this property. Indeed, such a change is equivalent to giving a different weight to the squared prediction error of that aggregate sector. One straightforward extension of the method given here is to assign weights to the prediction errors of the sectors in a way that reflects the relative

seriousness of these errors. (This extension is left to the reader.) Stated in (ii) is the invariance with respect to changes in the scale of the economy of the best approximate aggregate input-output matrix. That is, merely expanding the size of the economy, without altering its output structure, will not affect the best aggregate representation of the economy.

The formula (2.17) is used in Section 4. It relies (as does (iv)) on the use of the mean-squared error loss function. Alternative loss functions might be appropriate (where the errors of over-prediction differ from those of under-prediction an asymmetric loss function would be better, for instance. This extension is also left to the reader).

Proposition 2.1 assumes that there is a best approximate aggregate, that it is finite, and that (2.11) minimizes (2.10). These assumptions are justified if the use density has finite second-order moments. That is, if

(2.20) $E_g[xx']$ is finite

then $M(B|A,T,g)$ defined in (2.10) is finite, so it can be minimized and B^* specified in (2.11) is finite, while

(2.21) $$\frac{\partial^2 M(B|A,T,g)}{\partial B^2} = 2T E_g[xx']T'$$

is positive semi-definite so that B^* is indeed a minimum. Of course (2.20) is no limitation for practical purposes.

A major reason for aggregation is that it is simpler to make aggregate predictions using an aggregate model than it is to operate with a detailed,

disaggregated system. Such aggregate predictions involve an intrinsic, essentially random, error however. Using the best approximate aggregate matrix B*, the aggregate prediction error conditional on a value of the aggregate exogenous variable is given by

$$(2.22) \qquad (Y^* - Y|X) = [I_m - B^*]X - (Y|X).$$

Now the term $[I_m - B^*]X$ is determinate, but the term $(Y|X)$ is not. This is because many values of the exogenous variable aggregate to the given value of the aggregate exogenous variable. Formally,

$$(2.23) \qquad (Y|X) \in \{Y|Y = T[I_n - A]x, \ \forall x \in V(X)\} \equiv Z(X)$$

where $V(X) = \{x \in R^n | Tx = X\}$. That is, conditional on a value of the aggregate exogenous variable, the <u>actual</u> value of the aggregate endogenous variable belongs to the set $Z(X)$ (which is defined, in part, by possible values of the exogenous variable). Therefore the aggregate prediction error in (2.22) is multi-valued.

Suppose now that a probability density function f_X is defined over $V(X)$. This gives the (subjective) probability that the exogenous variable takes on a particular value consistent with the value of the aggregate exogenous variable. It may be that f_X is derived from the use density g, but in some situations it will be quite different. This is so, in particular, if there is available to the analyst some detailed knowledge about the economic structure that is not included in g. In any event, f_X induces a distribution over $Z(X)$. Then $(Y|X)$ is a random variable, as is the prediction error in

(2.22). The expectation and covariance of these prediction errors are easily calculated.

PROPOSITION 2.4 The conditional aggregate prediction error is a random variable with mean

$$(2.24) \qquad E[Y^* - Y|X] = [I_m - B^*]X - T[I_n - A]E_f[x]$$

and covariance

$$(2.25) \qquad Cov[Y^* - Y|X] = T[I_n - A]\ Cov_f[x]$$

where $E_f[x]$ and $Cov_f[x]$ are the mean and covariance of x respectively.

Several features of this result deserve comment. First, the random prediction error arises even though the model and aggregation procedures are deterministic. The explanation of this is intuitive. Information is lost when aggregation occurs. This loss of information shows up as an inability to make precise predictions. Secondly, the expected prediction error is not zero. In other words, the best approximate aggregate model generally gives "biased" predictions. This could only be avoided by imposing unwarranted restrictions on f_X. Thirdly, the covariance of the aggregate prediction error may be applied to Chebychev's inequality to make statements about the accuracy of predictions.

Finally, it is natural to define

$$(2.26) \qquad R^2 = 1 - \frac{E_g[(Y^* - Y)'(Y^* - Y)]}{E_g[(Y - \bar{Y})'(Y - \bar{Y})]}$$

where $\bar{Y} = E_g[T[I_n - A]x]$, as a measure of the success of aggregation (see Ijiri [16, p. 256, eq. (20)] for this measure). This is the proportion of

the variation of the aggregate endogenous variable about its mean that is "explained" by the (best approximate) aggregate model. The measure, of course, applies to any aggregate model. Note that the best approximate aggregate model maximizes R^2, and that $R^2 = 1$ if and only if consistent aggregation is possible.

3. COMPARISONS WITH THE LITERATURE

The form of the equation for B* (in (2.11)), the uncertainty associated with predictions (Proposition 2.4), and the measure of goodness of aggregation (R^2 in (2.26)) are similar to those found in econometric estimation theory. This suggests an underlying unity between the aggregation problem and econometric estimation. This has been long recognized. In surveying the state of aggregation theory, Allen [1, p. 694] wrote:

> "The model is then based on economic theory in the form of many micro-relations between micro-variables, but expressed in terms of an economic-statistical construction of macro-relations between aggregate variables."

The general necessity of this "economic-statistical construction" can be usefully illustrated by the present case. Given the use density g on x, a multi-variate distribution is induced on (X,Y) (= $(Tx, T[I_n - A]x)$). Provided that this induced distribution is non-degenerate, the relationship of Y to X is necessarily statistical, so that predictions based on any deterministic relationship of these aggregates will be subject to error. The multi-variate distribution on (X,Y) implies the econometric-like, decision-theoretic, approach suggested in this paper. Such an approach is in contrast to that dominating the literature on aggregation.

The possibility, in one form or another, of consistent aggregation has been the focus of most aggregation theory, in general, and in the input-output application in particular. May [24;25] appeared to fix the model and method of aggregation, as is done here in Definition 2.1, and ran into the problem Proposition 2.1 makes plain: There is usually no consistent (or exact) aggregate model. In the case of input-output, this conclusion is found in Fei [8] and Hatanaka [15]. An apparently more useful approach was suggested by Klein [19] and by Leontief in a celebrated paper [20]. The requirement of consistent aggregation is maintained, but the method of aggregation is the subject of choice. For input-output, this becomes a question of whether or not there are both B and T satisfying (2.4). Conditions on A ensuring this are given by Ara [2], McManus [22; 23], and Morimoto [27; 28]. Not surprisingly, consistent aggregation is more likely to be arranged when the aggregator can be chosen than when it cannot. However, it does not seem to be helpful to know that there is <u>some</u> way of aggregating industries which will yield exact predictions (and it is not possible in general anyway). Apart from the strange groupings that might be needed to achieve this, there is a more fundamental objection: The method of aggregation will depend on the model. As a consequence, aggregate comparisons could never be made between countries with different technologies. Most aggregate industries found in practice will be obvious groupings.

Several writers have endeavoured to replace consistent aggregation. Errors must then be expected: Aggregation can only be approximate. F. M. Fisher [9] developed one sort of result concerning this for general systems. His interest was whether there were aggregators which gave nearly

correct answers if Leontief's separability requirement was satisfied approximately. That is, he was concerned with finding an aggregator which worked reasonably well. It is natural to make this precise by specifying the costs of making errors.[4] For input-output, this was the (earlier) intuition of Walter D. Fisher [10], who sought that aggregator which minimized mean-square prediction error. According to Neudecker [29, p. 922] Fisher proposed[5] that the aggregate model be (in the notation of this paper)

$$(3.1) \qquad \hat{B} = [TA(x_o)T'][T(x_o)T']^{-1}$$

where (x_o) is the diagonal matrix made up from the vector of base-level gross outputs. The following is obvious.

PROPOSITION 3.1 If $\Omega_g = (x_o)$, $\mu_g = 0$, then $\hat{B} = B^*$.

That is, if the variance of the use density equals the base-level gross output, and the expected level of the gross output is zero, then Fisher's aggregate model coincides with the best approximate aggregate model. Therefore, best approximate aggregation generalizes Fisher's method (although, it must be stressed, he was principally interested in the method of aggregation, not in the aggregate model).

Information theory provides an alternative choice-theoretic method of aggregation. Theil [39] and Theil and Uribe [40] have developed this. Although some use has been made of the method, its emphasis is also on finding the best aggregator (see Roy, Batten and Lesse [34]). Moreover, the criterion of information-loss has relatively little economic appeal.

The prediction errors due to aggregation will be compounded if the model itself is subject to error. Recently, West [41; 42; 43], following

the earlier work of Quandt [30; 31] and Gerking [12], has given results on the distribution of prediction errors from this source.

Finally, it must also be noted that there is an extensive literature on aggregation in econometrics. Chipman [5] provides a collection of results formally close to those given here (and he uses the term "best approximate aggregate" to describe the aggregate model). However, the present problem is about aggregating theoretical relationships, while Chipman explains how observed data should be aggregated.

4. APPLICATIONS

A number of illustrative applications of the Best Approximate Aggregation (BAA) method are now presented, along with comparisons with the results of Naive Aggregation.

Three matrices are considered: the Direct Requirements matrix (I-A); the Leontief Inverse $(I-A)^{-1}$; and the Leontief Inverse Transpose $(I-A')^{-1}$. The applications of the first two matrices for interindustry forecasting and impact analysis are well known. The third matrix is widely used in the effective tariff protection literature (cf. Corden [6]), in the tax incidence literature (cf. Melvin [26]), and in the numerical general equilibrium policy literature (cf. Harrison and Kimbell [13], Kimbell and Harrison [18], Scarf and Shoven [35], and Shoven and Whalley [36]).

In each of the applications it is assumed that the distribution over the x vector is either Multinomial or Multivariate Normal (cf. De Groot [7; pp. 48-9, 51-6]). Each distribution has advantages and disadvantages for present purposes. The Multinomial is parsimonious with respect to calibration,

but does not allow independent first and second moments; the Multivariate Normal requires the specification of a large number of covariance terms about which there is relatively little intuition, but allows independent calibration of the first two moments. At this stage there is no reason to be dogmatic about the form of the use density. These two are for illustrative purposes only.

The Multinomial distribution is calibrated and interpreted as follows. Let N denote the number of observations in the sample, p_i denote the probability that the outcome belongs to the i^{th} category ($i=1,2,\ldots,n$), and x_i denote the number of these outcomes belonging to category i. Then $\mu_i = Np_i$, all i, and

$$\Omega_{ij} = \begin{cases} N\ p_i(1-p_i) & i=j\ (i,j=1,\ldots,n) \\ -N\ p_i p_j & i\neq j\ (i,j=1,\ldots,n). \end{cases}$$

A natural interpretation of the Multinomial is possible. Consider the aggregation of the Leontief Inverse $(I-A)^{-1}$, and its intended use for inter-industry impact analysis. Assume that the sectors to be subject to exogenous impacts are unknown at the time of aggregation. Further assume that the exogenous impact (the "outcome") is a given dollar amount N; in many applications this amount is simply some unit quantity, such as one dollar. If it is deemed equally likely that any one of the original sectors may be exogenously impacted, then an appropriate choice for the p vector is $p_i = 1/n$, all i. Alternatively, if it is more likely that the first sector is to be exogenously impacted, then a choice of the p_i such that $p_i < p_1$, all $i=2,\ldots,n$, would be appropriate.

The first two moments of the Multivariate Normal distribution were directly calibrated. The only requirement was that Ω_g be symmetric and positive definite. In the absence of strong intuition as to likely values for Ω_g, a series of pseudo-randomly generated matrices were used.[6] The only restriction imposed was that the selected values lie within a predetermined range, typically one-fifth of the mean value of the elements of μ_g, centered about zero. The results reported below were not qualitatively sensitive to significantly larger values for this range (e.g., ten times the mean value of the μ_g elements).

4.1 A Venerable Example

Balderston and Whitin [4] consider the Naive Aggregation of an 18X18 transactions table for the U.S. in 1939. They examine the effects of three alternative aggregation schemes on the size of particular coefficients of an aggregated 8X8 Leontief Inverse.[7] W. D. Fisher [10] subsequently compared these (essentially ad hoc) aggregation schemes with two further schemes chosen to minimize certain "general purpose" criteria of aggregation.[8] He demonstrates that his schemes lead to significantly lower values of his criteria (cf. his Table 4), concluding that "...substantially lower errors may be obtained by deliberate aggregation procedures based on minimal distance ideas than on haphazard procedures" [10, p. 259].

Table 4.1 presents several results for this example and the five aggregation schemes just discussed. The first case adopts a Multinomial distribution and the second case adopts a Multivariate distribution, each

TABLE 4.1: Aggregation Results for Venerable Example

Case	Aggregation Scheme	BAA R^2 (Naive R^2) (I-A)	$(I-A)^{-1}$	$(I-A')^{-1}$
1	BW1	.99 (.93)	.99 (.98)	.85 (.60)
	BW2	.99 (.97)	.99 (.98)	.99 (.47)
	BW3	.99 (.96)	.99 (.98)	.96 (.70)
	F8	.99 (.97)	.99 (.98)	.98 (.88)
	F13	.99 (.98)	.99 (.99)	.99 (.90)
2	BW1	.94 (.83)	.99 (.97)	.90 .61)
	BW2	.98 (.92)	.99 (.98)	.95 (.49)
	BW3	.99 (.81)	.99 (.85)	.96 (.37)
	F8	.99 (.91)	.99 (.98)	.98 (.73)
	F13	.99 (.93)	.99 (.98)	.98 (.77)

Description of Cases:

(1) Multinomial; $p_i = 1/18 = 0.0555\dot{}$, each i; N = 100.

(2) Multivariate Normal; each μ_g element drawn at random from an open interval ± 0.1 the base period value of the gross output of that sector, each Ω_g element drawn at random from an open interval ± 0.01 the base period value of the largest of the two corresponding sectoral gross output; Ω_g checked for positive definiteness; 1000 random drawings.

Description of Aggregation Schemes:

BW1: Table II-a, p. 119, in Balderston and Whitin [4].

BW2: Table III-a, p. 121, in Balderston and Whitin [4].

BW3: Table IV-a, p. 123, in Balderston and Whitin [4].

F8: Aggregation 8, Table 2, p. 275, in W. D. Fisher [10].

F13: Aggregation 13, Table 2, p. 275, in W. D. Fisher [10].

of which is described at the bottom of the table. The results for case 2 represent an average over 1000 random drawings.

Several features of the results in Table 4.1 are noteworthy. First, the BAA R^2 values are generally very high. Given the limited degree of aggregation undertaken, this result is not too surprising. Second, the BAA R^2 exceeds the Naive Aggregate R^2 for all cases and aggregation schemes considered.

Third, predictive performance varies with the aggregate model that is under study. In each case shown the BAA for the Leontief Inverse transpose, $(I-A')^{-1}$, has a lower R^2 than either of the other two aggregates. Note also that the Naive R^2 varies with the matrix in question, although not in the same qualitative way as the BAA variation.[9] Moreover, the (absolute and relative) deterioration of the Naive R^2 for the Leontief Inverse transpose is marked.

Fourth, for the distributions considered, virtually all of the three schemes proposed by Balderston and Whitin allow a BAA for (I-A) and $(I-A)^{-1}$ that has just as good an R^2 as the aggregation schemes preferred by W. D. Fisher. That is, whatever the "inefficiency" (in terms of Fisher's criteria or the R^2 measure) associated with Naive aggregation using the BW schemes, there exists a BAA for virtually all of the schemes considered such that the prediction errors from using that aggregate are "close" to zero. In this sense, and for the particular cases studied, the question of the "best" aggregation scheme for prediction purposes has a trivial but practically important answer: choose virtually any scheme. Note that noticeably different

BAA R^2 results are obtained for the alternative aggregation schemes when applied to the Leontief Inverse transpose. If schemes were selected on the basis of BAA predictive performance there would be a marginal preference for F13 or BW2 in Case 1 and F8 or F13 in Case 2. However, even for this matrix a BAA is obtained with an R^2 of at least 0.85 in Case 1 and 0.90 in Case 2 for <u>any</u> of the aggregation schemes considered.

4.2 The United States Input-Output Tables

The U.S. Department of Commerce publicly provides an input-output transactions table for 1975 with 85 sectors in Young and Loftus [44, pp. 69-78].[10] No official aggregation scheme for this table is endorsed, but one is suggested by the ten boldface groupings in Ritz [33, p. 45].[11] We adopt this aggregation scheme from an 85-sector matrix to a 10-sector matrix.

Two exercises were undertaken with the U.S. data, the results of which are reported in Table 4.2. The first exercise involved computing the 10-sector Naive aggregate and using it as the basis for further aggregation to a 3-sector set of matrices; the three sectors were Primary, Secondary and Tertiary.[12] The second exercise dealt with the aggregation from 85 sectors to 10 sectors.

The results support the general conclusions drawn from the example of the previous section. The MSE for the BAA (Naive Aggregate) in Case 1 when $n=10$ and $m=3$ are 0.000001 (9.86020), 0.00001 (1.16784) and 0.01672 (0.04443) for the three matrices $(I-A)$, $(I-A)^{-1}$ and $(I-A')^{-1}$, respectively. The Naive aggregate for the 3X3 Leontief Inverse in Case 1 is

TABLE 4.2: Aggregation Results for U.S. Data

Case	Original Number of Sectors (n)	Aggregated Number of Sectors (m)	BAA R^2 (Naive R^2) (I-A)	$(I-A)^{-1}$	$(I-A')^{-1}$
1	10	3	.999 (.995)	.999 (.997)	.927 (.805)
	85	10	.968 (.801)	.965 (.812)	.974 (.851)
2	10	3	.993 (.955)	.991 (.927)	.983 (.906)
	85	10	.931 (.744)	.944 (.713)	.966 (.659)

Description of Cases:

(1) Multinomial; $p_i = 1/n$, each i; N = 100.

(2) Multivariate Normal; μ_g and Ω_g drawn at random as in Table 4.1; 1000 random drawings for n = 10 to m = 3, and 100 random drawings for n = 85 to m = 10.

$$\begin{bmatrix} 1.32497 & .19794 & .03467 \\ .42548 & 1.76854 & .24929 \\ .24075 & .24215 & 1.21883 \end{bmatrix}$$

whereas the corresponding BAA is

$$\begin{bmatrix} 1.28780 & .15540 & .03960 \\ .40120 & 1.70830 & .28940 \\ .24030 & .24490 & 1.20660 \end{bmatrix}$$

Clearly the two matrices are similar but distinct.

One important feature of the results in Table 4.2 that is novel is the widening differences between the BAA and Naive R^2 measures as we move from aggregating the 10-sector model to aggregating the 85-sector model. One aspect of this point is the relative stability and high values of the BAA R^2 over different levels of aggregation and over the aggregation of different matrices.

4.3 The Australian Input-Output Tables

The Australian Bureau of Statistics [3] publicly provides an input-output transactions table for 1974-75 with 109 sectors. It also provides (Appendix D) an "official" aggregation scheme for a 29-sector table. Table 4.3 presents our results using this aggregation scheme.

The ability of the BAA method to find an aggregate matrix with an R^2 of at least 0.9 in each case is again evident. Note also the significant deterioration of the Naive R^2 relative to the BAA R^2, especially for the Leontief Inverse transpose relative to the Leontief Inverse. This suggests that the official aggregation scheme implicitly stresses the combination of sectors that are relatively homogeneous in terms of their interindustry output

TABLE 4.3: Aggregation Results for Australian Data

Case	BAA R^2 (Naive R^2)		
	(I-A)	$(I-A)^{-1}$	$(I-A')^{-1}$
1	.91 (.69)	.93 (.75)	.91 (.58)
2	.90 (.68)	.91 (.58)	.90 (.22)

Description of Cases:

(1) Multinomial; $p_i = 1/109 = 0.0091743$, each i; N = 100.

(2) Multivariate Normal; μ_g and Ω_g drawn at random as in Table 4.1; 50 random drawings.

structure, tending to combine sectors with heterogeneous input (or cost) structures. Of course, this only represents a problem for the eventual application of the aggregate $(I-A')^{-1}$ matrix if one relies on Naive Aggregation rather than BAA.

4.4 A Pacific Basin Input-Output Table

The Institute of Developing Economies [17, pp. 2-12] provides an international input-output table for 1975 with 8 nations (Indonesia, Malaysia, The Philippines, Singapore, Thailand, Japan, South Korea, and the United States) and with 7 sectors in each nation; thus the entire table distinguishes 56 sectors.

In Table 4.4 we consider the aggregation of the IDE Pacific Basin table over the 8 nations, resulting in a 7-sector table. Clearly the implied aggregation scheme calls for the combination of sectors in the original table that are quite different (e.g., "Agriculture in Thailand" with "Agriculture in the U.S." with "Agriculture in Japan", etc.). Nonetheless, BAA matrices exist that provide predictive R^2 in excess of 0.90, while the Naive Aggregate R^2 falls as low as 0.41 in one case.

5. CONCLUDING REMARKS

One important finding from the illustrative applications of the concept of BAA in the previous section is the marked deterioration of predictive power with the various aggregated Leontief Inverse transpose matrices. In these cases the improvement of BAA over Naive Aggregation is clearly significant. This result has some significance for recent applied

TABLE 4.4: Aggregation Results for Pacific Basin Data

	BAA R^2 (Naive R^2)		
Case	(I-A)	$(I-A)^{-1}$	$(I-A')^{-1}$
1	.91 (.68)	.93 (.75)	.90 (.61)
2	.90 (.62)	.91 (.58)	.92 (.41)

Description of Cases

(1) Multinomial; $p_i = 1/56 = 0.0178571$, each i; N = 100.

(2) Multivariate Normal; μ_g and Ω_g drawn at random as in Table 4.1; 50 random drawings.

general equilibrium (GE) modelling efforts, especially in the context of the systematic sensitivity analyses of such models developed in Harrison and Kimbell [14]. One major constraint on these sensitivity analyses is the burden of computing the number of GE solutions required (in the order of 10^5 for models distinguishing about 15 sectors). This computational burden decreases exponentially with the number of sectors identified in the model, providing a strong argument for aggregation. However, there are well-known examples of the loss in predictive power when one aggregates applied GE models (e.g., Fullerton, Henderson and Shoven [11] on the Harberger two-sector aggregation scheme). Given the policy value of examining the robustness of such models, one major application of the BAA method could be to allow a substantial reduction in the dimensionality of the models without substantial loss in predictive power. Preliminary research with the GE model developed in Harrison and Kimbell [13], employing BAA with respect to the input-output data alone (expenditure shares and factor shares were aggregated using Naive methods), suggests that dramatic improvements in predictive power are possible even with severe aggregation (from twenty to two sectors per trading bloc). As expected, GE computational speed is correspondingly improved. Compared to the GE solution using a Naive Aggregate input-output matrix, the improvement in predictive power when using the BAA matrix is dramatic: an R^2 with respect to factor usage in each sector of 0.95 as against 0.22! Interestingly, the R^2 difference for other endogenous GE variables is much smaller, although nonetheless significant; welfare impact in each trading bloc has an R^2 of 0.98 with BAA as against 0.70 with Naive Aggregation.

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FOOTNOTES

[1]For helpful discussions and suggestions Claude Autin, David Giles, Mukul Majumdar and Arthur Robson are thanked. Errors remain the authors' responsibility. Harrison thanks the Reserve Bank of Australia and the Social Sciences and Humanities Research Council of Canada for research support.

[2]It has been customary to call this the aggregation bias. However it is more natural to reserve the term "bias" for the expectation of $(Y^* - Y|X)$; See (2.24) below.

[3]David Giles provided a clear derivation of (2.15). He is thanked for this in particular.

[4]Theil [38] proposed a different approach to this problem (one which Allen [1] followed). This was to observe the aggregate variables and fit a relationship between them that explains as much as possible. This involves both a sampling error and the aggregation error discussed in Section 2, of course. On one interpretation the present approach gives the result of the Theil procedure when all conceivable observations are available.

[5]It is not exactly clear where Fisher made this proposal.

[6]The simulation methodology used is consistent with the literature on the probabilistic analysis of input-output coefficients: See McCamley, Schreiner and Muncrief [21], Quandt [30, 31], and West [41, 42, 43].

[7]The transactions table and the three aggregation schemes are published in Balderston and Whitin [4; pp. 116, 119, 121, 123].

[8]Specifically, see his aggregations 8 and 13, defined in his Table 2. Fisher's search over all feasible aggregation schemes for those that minimize his criteria is not exhaustive, but this is of little consequence for present purposes.

[9]Consider, for example, the Leontief Inverse Transpose. In Case 1, changing from BW1 to BW2 leads to a higher BAA R^2 but a decline in the Naive R^2. Similarly for Case 2.

[10]An Errata to this table has been issued, and was adopted in our data.

[11]Young and Loftus [44, p. 7] repeat the same groupings.

[12]Using the ten sectors listed in Young and Loftus [44, p. 7], "Primary" is an aggregate of "Agriculture, Forestry and Fishing" and "Mining", "Secondary" is an aggregate of "Construction", "Manufacturing" and "Transportation, Communication, and Utilities", and "Tertiary" aggregates the remaining five sectors.

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