



## AN ORIGINAL MODEL OF INFRASTRUCTURE SYSTEM RESILIENCE

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### ABSTRACT

Infrastructure systems of transportation, water supply, telecommunications, power supply, etc. are not isolated but highly interconnected and mutually coupled. Infrastructure interdependences can increase system vulnerability and produce cascading failures at the regional or national scales. Taking the advantage of network theory structure analysis, this paper models street, water supply network, power grid and information infrastructure as network layers that are integrated into a multilayer network. The infrastructure interdependences are detailed using five basic dependence patterns of network fundamental elements. Definitions of dynamic cascading failures and recovery mechanisms of infrastructure systems are also established. The main focus of the paper is introduction of a new infrastructure network resilience measure capable of addressing infrastructure system as well as network component (layer) interdependences. The new measure is based on infrastructure network performance, proactive infrastructure network resistance capacity and reactive infrastructure network recovery capacity. With three resilience features and corresponding network properties, this paper develops the new quantitative measure of dynamic space-time resilience and a resilience simulation model use three dimensions of resilience and network properties for infrastructure network assessments. The resilience model is applicable to any type of infrastructure and its application can improve the infrastructure planning, design and maintenance decision making.

Keywords: resilience, infrastructure system, multilayer network, infrastructure interdependence, adaptive capacity

### 1. INTRODUCTION

Due to the rising cost of infrastructure upkeep and increasing frequency of extreme events affecting its functioning, Canada's infrastructure systems have become more vulnerable to natural disasters. Recent examples include Alberta and Toronto floods of 2013. Infrastructure systems consists of diverse infrastructure elements, including telecommunications, power supply, natural gas and oil, transportation, water supply, etc. Interdependencies among different infrastructure elements can produce cascading failures throughout the whole infrastructure system at regional and national scales (Ouyang 2014). So the infrastructure system resilience is often overestimated (Cutter et al. 2008) and corresponding protection and recovery strategies don't always provide desired results.

Infrastructure system resilience refers to the ability of system to resist possible hazards, absorb the initial damage, and recover to normal operation (Ouyang and Dueñas-Orsorio 2012, Francis and Bekera 2014). Multidisciplinary Center for Earthquake Engineering Research (MCEER) provides a general resilience framework for definition and quantification of the physical and organizational systems resilience to earthquakes (Bruneau et al. 2003). As a follow up of this work, many studies emerged on the quantification of performance and resilience assessment of utility systems, such as water supply networks (Li and Lence 2007), electric infrastructure systems (Maliszewski and Perrings 2012, Ouyang and Dueñas-Orsorio 2014), telecommunications cable systems (Omer et al. 2009) and underground transportation (D'Lima and Medda 2015). Most of the reported research efforts use for resilience quantification metrics of system robustness or system recovery rapidity of individual infrastructure system.

Taking the advantage of network theory structure analysis, infrastructure systems can be described as complex networks, where nodes represent infrastructure components (such as water pumps and electric transformers), and links mimic the physical and relational connections among different infrastructure component (such as electric tie lines and water pipes) (Dudenhoeffer 2006, Johansson and Hassel 2010). Cascading failures across diverse infrastructure systems can be simulated using topology-based or flow-based methods (Ouyang 2014) that lead to the estimation of multi-infrastructure system vulnerability. Some of the published research is clearly emphasising recovery processes to evaluate for example, gas and electric infrastructure system resilience (Filippini and Silva 2014, Ouyang and Dueñas-Ororio 2012). However, the infrastructure system resilience should address both, proactive adjustment capacity and reactive recovery capacity (Manyena 2006).

Simonovic and Peck (2013) point out that continued operation and rapid restoration of the systems affected by a disturbance are essential for resilience. Resilient infrastructure system is a sustainable network of critical lifelines that “possess the capacity to survive, cope, recover, learn and transform from disturbances”. So based on the Space-Time Dynamic Resilience Measure of Simonovic and Peck (2013), this paper integrates multi-infrastructure network properties and defines a genetic infrastructure system resilience model for quantifying both, dynamic proactive adjustment capacity and reactive recovery capacity.

The remainder of this article is organized as follows. Section 2 presents the infrastructure system model, including street, water supply, power supply and information infrastructure components, as a network of networks, or a multilayer infrastructure network model. Basic dependence patterns of individual infrastructure components for establishing system dynamic cascading failures and recovery mechanisms are also provided. Section 3 provides a definition of a new multilayer infrastructure network resilience, and presents dynamic resilience metric under sequential disturbances. Finally, the potential resilience model applications are discussed.

## 2. INFRASTRUCTURE NETWORK FORMALIZATION

### 2.1 Infrastructure Network Representation

The infrastructure network model is based on the network theory, where two basic components, nodes and edges, build up the model of a system. A network is always represented by  $G$ , the nodes set and edges set are represented by  $N$  and  $E$  respectively. This paper focuses on the main urban infrastructure system networks, including streets, power grid, water supply network, and information infrastructure.

Street network is represented as  $G^S(N^S, E^S)$ , where  $N^S$  is the set of street junctions and end points, and  $E^S$  is the set of street segments (Cavallaro 2014). The edges are undirected and homogeneous. Generally, the street network is fully connected. Water supply network is represented as  $G^W(N^W, E^W)$ , where waterworks, storage facilities and pump stations are represented as nodes with different attributes, and water distribution pipes are denoted by edges (Shuang 2014, Wei and Li 2015). The edges of water supply network are directed as the water flow from waterworks to pump stations and storage facilities through distribution pipes. Generally, water supply networks are represented as trees without circle and redundant edges. The downstream nodes and edges could not operate unless all the upstream nodes and edges function normally. Power grid is represented as  $G^P(N^P, E^P)$ , where power plants, distribution and transmission substations are represented by nodes with different attributes, and power lines are represented by directed edges (Albert 2004, Kinney 2005). Same as for water supply networks, edges of power grid are directed as the electricity is transmitted from power plants to transmission substations, and then to distributing stations through power lines. The downstream nodes and edges could not operate unless all the upstream nodes and edges function normally. Information infrastructure is represented as  $G^I(N^I, E^I)$ , where Internet service providers are represented by nodes, and cable connections are denoted as undirected edges (Omer 2009). Since these networks provide bidirectional exchange of information, the edges are undirected. According to scale, population and structure of a city, information network structure could be represented as a star, chain or circle shape, and so on. A node or edge operate normally if there is an existing path connecting to the source node.

All individual infrastructure networks introduced above can be illustrated as individual infrastructure layers. Infrastructure system model is a network of networks integrating all of the layers, as illustrated in Figure 1 (Dudenhoeffer 2006). Nodes and edges in the same layer are belong to the same kind of infrastructure (intra-infrastructure connection, which denoted by solid lines within the single layer network in Figure 1). Edges crossing different layers denote dependence of different kinds of infrastructures, illustrating physical and cyber connections

between different kinds of infrastructures (inter-infrastructure connections, which denoted by dotted lines connecting different layer networks in Figure 1). Such as the red dotted lines between nodes belong to power grid and water supply network illustrate electric transmitting from electric infrastructures to water supply infrastructures.

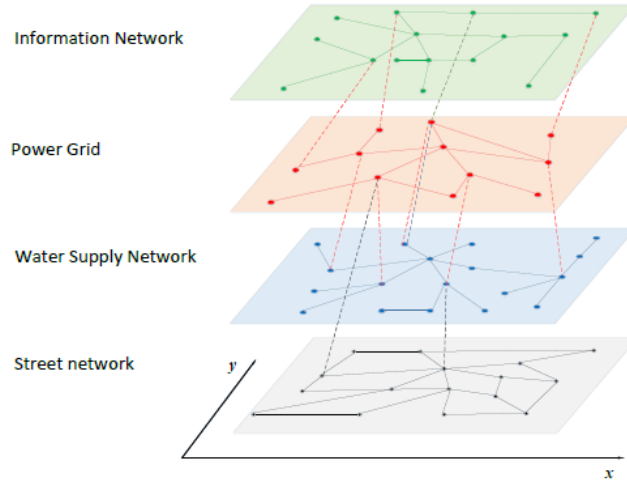


Figure1: Interdependent infrastructure system model representation

As different infrastructure components located in same area are subject to a specific disturbance (disaster), it is necessary to consider the location of infrastructure in the model description. Furthermore, location of infrastructure has important effect on topological properties and consequently on infrastructure functioning processes (Barthélemy 2011). So, the spatial attributes of nodes and edges should be included in a realistic infrastructure network model with geographical coordinates, which can be defined in a two-dimensional Euclidean coordinate system. Therefore, each node has three coordinates  $(\phi, x, y)$ , where  $\phi$  denotes the type of the infrastructure,  $(x, y)$  denotes the geographical location of the node. Edges are denoted by the two adjacent nodes.

## 2.2 Basic Infrastructure Dependence Patterns

The interdependent networks represent a complex system where emergent behaviors are rarely fully understood. Urban infrastructure components can be dependent and interdependent in multiple ways. Most of the earlier literature review interdependencies as macro-properties of coupled systems classified in different ways. For example, Dudenhofer et al. (2006) classifies interdependencies into four types: physical, geospatial, policy and informational.

Interdependence indicates the bidirectional interaction, which includes two directed dependences between two infrastructure elements (Rinaldi 2001). Generally, not any components malfunction of one infrastructure system can result in efficiency reduction, function loss or system destruction of another macro-interdependent infrastructure system. So the macro-interdependence is a function of the system attributes and status of the malfunction infrastructure systems. Therefore, micro structure, or basic pattern of infrastructure dependence, need to be considered. The focus of the proposed resilience model is direct impact of infrastructure malfunction, which is always seen as the first-order effect.

Let us consider two layer infrastructure networks  $G^{\phi_1}, G^{\phi_2}$  where  $\phi_1 \neq \phi_2$ . Every element of the infrastructure has two exclusive states: on and off. With different relations between four fundamental network structure elements: nodes, edges, paths and clusters (combinations of nodes and edges), there are five basic infrastructure dependence patterns (four patterns are illustrated in Figure 2):

(a) *Node – Node Dependence* (Figure 2a): the state of node  $n_i^{\phi_1}$  is dependent on the state of  $n_j^{\phi_2}$ , or vice versa. For example, the state of water pump depends on the state of its connecting electric transmission substation. This pattern is represented as

$$[1] \quad ID^{NN} = e_{ij}^{\phi_1 \phi_2}.$$

(b) *Node – Edge Dependence* (Figure 2b): the state of node  $n_i^{\phi_1}$  is dependent on the state of edge  $e_{ij}^{\phi_2}$ , or vice versa. For example, the state of Internet service provider depends on its connecting power supply. This pattern is represented as

$$[2] \quad ID^{NE} = n_i^{\phi_1} \times e_{ij}^{\phi_2}$$

(c) *Node/Edge – Path Dependence* (Figure 2c): the state of node  $n_i^{\phi_1}$  or edge  $e_{ik}^{\phi_1}$  is dependent on the state of the path  $p_{im}^{\phi_2}$ , which is represented as  $\{n_i^{\phi_2}, e_{ij}^{\phi_2}, n_j^{\phi_2}, e_{jk}^{\phi_2}, n_k^{\phi_2} \mid n_m^{\phi_2}\}$ . For example, the state of coal power plant is dependent on the path (transportation network) connecting the plant with coal supply locations. This pattern is represented as

$$[3] \quad ID^{NP} = n_i^{\phi_1} \times p_{im}^{\phi_2} \quad \text{or} \quad ID^{EP} = e_{ik}^{\phi_1} \times p_{im}^{\phi_2}, \quad \text{where} \quad p_{im}^{\phi_2} = \prod_i^m n_i^{\phi_2} \prod_{i,j}^{\bullet, m} e_{ij}^{\phi_2}$$

(d) *Node/Edge – Cluster dependence* (Figure 2d): the state of cluster  $c_i^{\phi_1}$ , which is a set of nodes and their edges of network  $G^{\phi_1}$ , is dependent on the state of node  $n_i^{\phi_2}$  or edge  $e_{kl}^{\phi_2}$  of network  $G^{\phi_2}$ . For example, the operations of water or power infrastructure with the same geographic or logic attributes being controlled by an Internet service provider. This pattern is represented as

$$[4] \quad ID^{NC} = n_i^{\phi_2} \times c_i^{\phi_1} \quad \text{or} \quad ID^{EC} = e_{kl}^{\phi_2} \times c_i^{\phi_1}, \quad \text{where} \quad c_i^{\phi_1} = \prod n_i^{\phi_1} e_{ij}^{\phi_1}, (n_i \in c_i^{\phi_1}, e_{ij}^{\phi_1} \in c_i^{\phi_1}).$$

(e) *Geographic Dependence*: the state of all infrastructure elements located at the same location A are affected by a disturbance simultaneously. This pattern is represented as

$$[5] \quad ID^{GL} = \{n_i^{\phi_1} \cup e_{jk}^{\phi_1} \cup e_{mn}^{\phi_1 \phi_2} \cup n_p^{\phi_2} \cup e_{qr}^{\phi_2}\}, \quad \text{where} \quad (x, y) \mid_{n_i^{\phi_1}, e_{jk}^{\phi_1}, e_{mn}^{\phi_1 \phi_2}, n_p^{\phi_2}, e_{qr}^{\phi_2}} \in A.$$

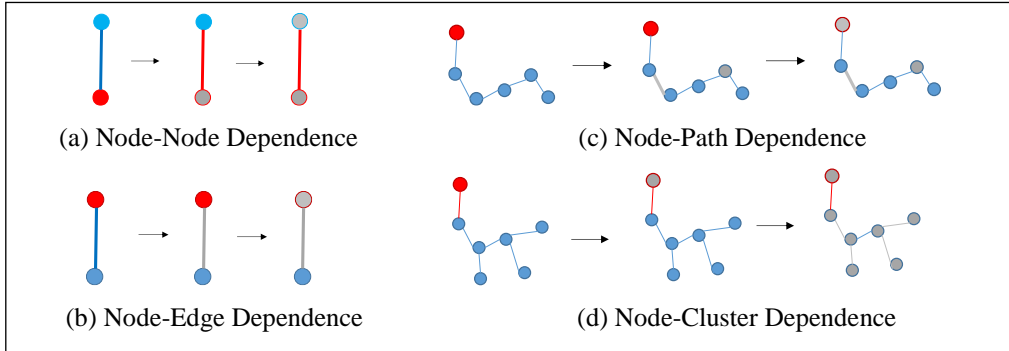


Figure 2: Basic infrastructure dependence patterns

Nodes and edges with different colors (red and blue) represent different kind of infrastructures; Grey nodes and edges represent malfunctioning infrastructures; arrows represent the time change.

In the previous discussion we looked at two infrastructure networks. The basic dependence patterns can cause cascading impacts throughout the multilayer network as time goes on. Given three infrastructure networks  $G^{\phi_1}$ ,  $G^{\phi_2}$ ,  $G^{\phi_3}$  ( $\phi_1 \neq \phi_2 \neq \phi_3$ ), there are many combinations of the five basic dependence patterns, which could form chain or cycle reaction among three single later networks and cause cascading failure spreading throughout the whole infrastructure system. On the other hand, interdependences could accelerate mitigation and be conducive to

disturbance response with repair of several components and strengthen system robustness and resilience with local protection.

### 2.3 Infrastructure System Dynamic Mechanism

Magnitude of the interrupted services and duration of the interruption are the two main characteristics of the disturbance of importance for the assessment of consequences (Johansson and Hassel 2010). In practice, an infrastructure could: (i) absorb the impacts of disturbance and minimize consequences with little effort (i.e. buffering); (ii) adjust to undesirable conditions by undergoing some changes (adaptation); and (iii) fully recover from disturbance. All three response modes define the infrastructure adaptive capacity (Francis and Bekera 2014). The response of the infrastructure system to a disturbance vary with time - adding dynamic properties to interdependent infrastructure networks (Filippini and Silva 2014).

In order to capture the dynamic character of the disturbance consequences, the change of the infrastructure performance due to various disturbances needs to be estimated. As discusses in the section 2.2, the state of an infrastructure is influenced by the disturbance as well as the state of other infrastructure. Let  $T_B$  denote buffering time of an infrastructure system,  $T_R$  its repair time; and  $T_M$  its malfunction time. The dynamic performance of an infrastructure system subject to a disturbance can be illustrated using a flow chart in Figure 3. IS in Figure 3 can be a node, an edge, a cluster and a path of an infrastructure network. Its state is decided by corresponding basic dependence patterns, and calculated using relationships [1-5].

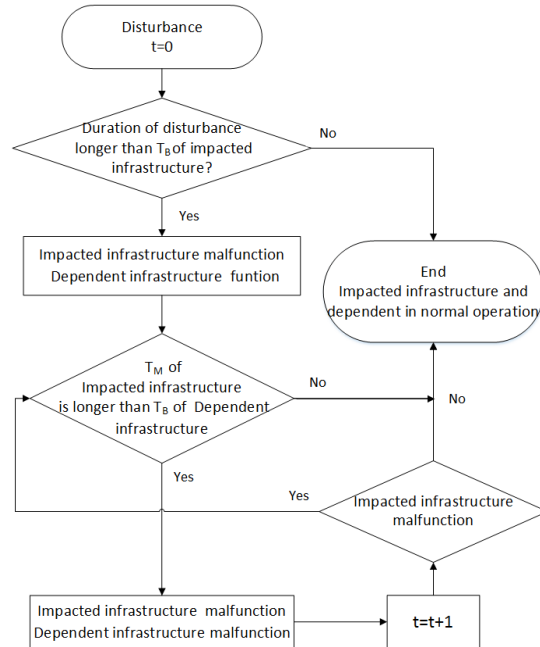


Figure 3: Dynamic process of an interdependent Infrastructure

## 3. INFRASTRUCTURE SYSTEM RESILIENCE MODEL

### 3.1 Infrastructure System Resilience Definition

The infrastructure system resilience is defined as “the ability to prepare for, and adapt to changing conditions and withstand and recover rapidly from disruptions”, including “the ability to withstand and recover from deliberate attacks, accidents, or naturally occurring threats or incidents” (Ouyang and Dueñas-Osorio 2012). Therefore infrastructure system resilience includes system performance and its adaptive capacity that can be in two different forms: proactive adjustment capacity and reactive recovery capacity. As the number of functioning infrastructure elements and the amount of resources left are the foundations for recovery, the former capacity will directly influence the latter.

Infrastructure system, a typical “*systems of systems*”, is a set of multiple and independently operational systems interacting with one another to meet specific needs (DeLaurentis and Crossley 2005). So infrastructure system resilience refers not only to the ability to resist disturbance and reorganize while undergoing change of intra-layer networks, but also the ability to retain essentially the same function, structure and feedbacks among inter-layer networks. The former capacity is concerned with an individual infrastructure system. The later need more systematic thinking and management due to potentially small unforeseen disturbances.

### 3.2 Three Features of Infrastructure System Resilience

System resilience can be represented and quantified by four key features: robustness, redundancy, resourcefulness and rapidity (Bruneau 2003). As proactive adjustment capacity is a function of network robustness, reactive recovery capacity can be represent by network resourcefulness and rapidity. Redundancy can be seen as the cause of robustness (National Infrastructure Advisory Council 2009) and therefore in this paper we analyze the three key features of resilience: robustness, resourcefulness and rapidity.

#### 3.2.1 Robustness

Robustness refers to the ability of a system to withstand a given level of stress without suffering degradation or loss function. The common measure for network robustness is the critical fraction at which the system completely collapses (Albert 2000). Similarly, this paper uses the minimum number of network components withstanding disturbance to denote infrastructure system robustness. For single layer infrastructure network, robustness  $R_{Rob}^{\zeta_1}(t_{\zeta_{IR}})$  is computed as minimum and stable ratio of operational components after a disturbance  $\zeta_1$ :

$$[6] \quad R_{Rob}^{\zeta_1}(t_{\zeta_{IR}}) = \frac{n_o^\phi(t_{\zeta_{IR}}) + e_o^\phi(t_{\zeta_{IR}})}{N^\phi + E^\phi}$$

where  $t_{\zeta_{IR}}$  is the moment when  $n_o^\phi(t_{\zeta_{IR}}) + e_o^\phi(t_{\zeta_{IR}})$  starts to be stable after the disturbance  $\zeta_1$ .  $n_o^\phi$  is the number of operational nodes,  $e_o^\phi$  is the number of operational edges,  $N^\phi$  and  $E^\phi$  are the total number of nodes and edges of undisturbed network. Multilayer infrastructure network robustness  $R_{Rob}^{\zeta_1}(t_{\zeta_{IRM}})$  is computed as the minimum ratio of operational elements:

$$[7] \quad R_{Rob}^{\zeta_1}(t_{\zeta_{IRM}}) = \frac{\sum_{\phi} (n_o^\phi(t_{\zeta_{IRM}}) + e_o^\phi(t_{\zeta_{IRM}}))}{\sum_{\phi} (N^\phi + E^\phi)}$$

where  $t_{\zeta_{IRM}}$  is the moment when the sum of  $n_o^\phi + e_o^\phi$  starts to be stable after the disturbance  $\zeta_1$ .

#### 3.2.2 Resourcefulness

Resourcefulness is the capacity to make and implement mitigation and response measures to a specific disturbance, which is limited by the ability to obtain sufficient resources, such as monetary, physical, technological, informational and human resources. This paper uses the network performance of restoration strategies to a specific disturbance  $\zeta_1$  for quantifying resourcefulness, represented as  $R_{Res}^{\phi, \zeta_1}(t)$ :

$$[8] \quad R_{Res}^{\phi, \zeta_1}(t) = f(RS^{\phi, \zeta_1}(t)) - SP_0^{\phi, \zeta_1}(t)$$

where  $RS^{\phi, \zeta_1}(t)$  is a restoration strategy (RS) of network  $\phi$  after disturbance  $\zeta_1$ .  $f(\bullet)$  is the network performance of  $RS^{\phi, \zeta_1}(t)$  at  $t$ , calculated as the ratio of operational nodes and edges to total nodes and edges of network  $\phi$  at  $t$ .  $SP_0^{\phi, \zeta_1}(t)$  is the system performance without restoration strategy (RS), which is calculated as the ratio of operational

elements to total elements of network  $\phi$  after disturbance  $\zeta_1$  at  $t$ . For multilayer infrastructure network, resourcefulness  $R_{\text{Res}}^{\zeta_1}(t)$  is measured as:

$$[9] \quad R_{\text{Res}}^{\zeta_1}(t) = \int_{\phi} f(RS^{\phi, \zeta_1}(t))$$

where  $\int_{\phi} f(RS^{\phi, \zeta_1}(t))$  is the integration all the  $R_{\text{Res}}^{\phi, \zeta_1}(t)$  of single layer networks with corresponding  $RS^{\phi, \zeta_1}(t)$ , and calculated as the ratio of normal operational nodes and edges at  $t$ .

### 3.2.3 Rapidity

Rapidity refers to the capacity to meet priorities and achieve goals in a timely manner. Duration of system recovery to normal operational levels is always used as a measure to evaluate system resilience, and can be seen as the main figure-of-merit to evaluate reactive recovery capacity. This paper uses the duration of system recovery to denote rapidity, represented as  $R_{\text{Rap}}^{\phi, \zeta_1}$ :

$$[10] \quad R_{\text{Rap}}^{\phi, \zeta_1} = t^{\phi}(R_{\text{Res}}^{\zeta_1}) - t_{\zeta_{10}}^{\phi} \quad \text{or}$$

$$[11] \quad R_{\text{Rap}}^{\phi, \zeta_1} = t_{\zeta_{20}}^{\phi} - t_{\zeta_{10}}^{\phi},$$

where  $t^{\phi}(R_{\text{Res}}^{\zeta_1})$  is the moment when single layer infrastructure network  $\phi$  recovers to normal operational level,  $t_{\zeta_{10}}^{\phi}$  is the moment disturbance  $\zeta_1$  occurs.  $t_{\zeta_{20}}^{\phi}$  is the moment disturbance  $\zeta_2$  occurs. Equation [10] is used for single disturbance or the last disturbance of a sequence of disturbances. Equation [11] is used in other situations. For multilayer infrastructure networks, rapidity  $R_{\text{Rap}}^{\zeta_1}$  is measured by the longest rapidity of single layer networks:

$$[12] \quad R_{\text{Rap}}^{\zeta_1} = \max \{R_{\text{Rap}}^{\phi, \zeta_1}(t)\}$$

### 3.3. Dynamic Infrastructure System Resilience Metric

System performance and its adaptive capacity represent dynamic system behavior in response to system disturbance and application of various adaptation measures. Original Space-Time Dynamic Resilience Measure developed by Simonovic and Peck (2013) is adapted in this research to complex network infrastructure systems. It quantifies resilience as the difference between the area under expected system performance and actual system performance (dotted shaded area in Figure 4). The introduction of system adaptation measures provides for the increase in system resilience (line shaded area in Figure 4), where the system performance without adaptation measures is shown by grey dashed line and with adaptation measures as full black line. The adaptive capacity can be achieved by: proactive adjustment measures and reactive recovery measures.

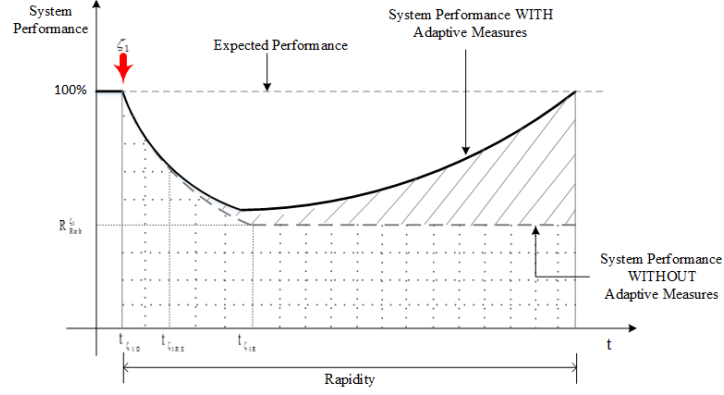


Figure 4: Typical performance process of an infrastructure system

### 3.3.1 Proactive adjustment capacity metric

With the three features above, proactive adjustment capacity of individual infrastructure network subject to a disturbance  $\zeta_1$  can be illustrated as the dotted area in Figure 4 and represented as  $\rho_{PA}^{\phi, \zeta_1}$ .

$$[13] \quad \rho_{PA}^{\phi, \zeta_1} = \frac{\int_{R_{Rap}^{\phi, \zeta_1}}^{R_{\zeta_1}^{\phi, \zeta_1}} SP_0^{\phi, \zeta_1}(t) dt}{1 \times R_{Rap}^{\phi, \zeta_1}}$$

where 1 in the denominator refers to the undisturbed system performance. For a multilayer infrastructure system network, the proactive adjustment capacity metric denote by  $\rho_{PA}^{\zeta_1}$ .

$$[14] \quad \rho_{PA}^{\zeta_1} = \frac{\int \int_{\phi R_{Rap}^{\phi, \zeta_1}}^{R_{\zeta_1}^{\phi, \zeta_1}} SP_0^{\phi, \zeta_1}(t) dt d\phi}{1 \times R_{Rap}^{\zeta_1}}$$

### 3.3.2 Reactive recovery capacity metric

Reactive recovery capacity of individual infrastructure network subject to a disturbance  $\zeta_1$  can be illustrated as the line shaded area in Figure 4 and represented as  $\rho_{RR}^{\phi, \zeta_1}$ .

$$[15] \quad \rho_{RR}^{\phi, \zeta_1} = \frac{\int_{R_{Rap}^{\phi, \zeta_1}}^{R_{\zeta_1}^{\phi, \zeta_1}} R_{Res}^{\phi, \zeta_1}(t) dt}{1 \times R_{Rap}^{\phi, \zeta_1}}$$

For a multilayer infrastructure system network, reactive recovery capacity metric is  $\rho_{RR}^{\zeta_1}$ .

$$[16] \quad \rho_{RR}^{\zeta_1} = \frac{\int \int_{\phi R_{Rap}^{\phi, \zeta_1}}^{R_{\zeta_1}^{\phi, \zeta_1}} R_{Res}^{\phi, \zeta_1}(t) dt d\phi}{1 \times R_{Rap}^{\zeta_1}}$$

### 3.3.3 Resilience metric

Resilience of a single layer infrastructure network  $\phi$  and multilayer infrastructure network to disturbance  $\zeta_1$  is represented as  $r_{\phi}^{\zeta_1}$  and  $r^{\zeta_1}$ , which are calculated as follows.



$$[17] \quad r_{\phi}^{\zeta_1} = \rho_{PA}^{\phi, \zeta_1} + \rho_{RR}^{\phi, \zeta_1} = \frac{\int \int_{\phi} SP_0^{\phi, \zeta_1}(t) R_{Rap}^{\phi, \zeta_1}}{1 \times R_{Rap}^{\phi, \zeta_1}} + \frac{\int \int_{\phi} R_{Res}^{\phi, \zeta_1}(t) R_{Rap}^{\phi, \zeta_1}}{1 \times R_{Rap}^{\phi, \zeta_1}}$$

$$[18] \quad r_{\zeta_1}^{\zeta_1} = \rho_{PA}^{\zeta_1, \zeta_1} + \rho_{RR}^{\zeta_1, \zeta_1} = \frac{\int \int_{\phi} SP_0^{\phi, \zeta_1}(t) R_{Rap}^{\phi, \zeta_1}}{1 \times R_{Rap}^{\zeta_1, \zeta_1}} + \frac{\int \int_{\phi} R_{Res}^{\phi, \zeta_1}(t) R_{Rap}^{\phi, \zeta_1}}{1 \times R_{Rap}^{\zeta_1, \zeta_1}}$$

Above metrics are derived for a single disturbance. Under a sequence of disturbances  $\{\zeta_1, \zeta_2, \dots, \zeta_d\}$ , single layer and multilayer infrastructure system resilience is the integral of the resilience under all single disturbances, which can be represented as  $r_{\phi}^{\zeta_1, \zeta_2, \dots, \zeta_d}$  and  $r_{\zeta_1, \zeta_2, \dots, \zeta_d}^{\zeta_1}$ , and calculated as follows.

$$[19] \quad r_{\phi}^{\zeta_1, \zeta_2, \dots, \zeta_d} = \rho_{PA}^{\phi, \zeta_1, \zeta_2, \dots, \zeta_d} + \rho_{RR}^{\phi, \zeta_1, \zeta_2, \dots, \zeta_d} = \frac{\int \int_{\phi} SP_0^{\phi, \zeta_1}(t) R_{Rap}^{\phi, \zeta_1, \zeta_2, \dots, \zeta_d}}{1 \times R_{Rap}^{\phi, \zeta_1, \zeta_2, \dots, \zeta_d}} + \frac{\int \int_{\phi} R_{Res}^{\phi, \zeta_1}(t) R_{Rap}^{\phi, \zeta_1, \zeta_2, \dots, \zeta_d}}{1 \times R_{Rap}^{\phi, \zeta_1, \zeta_2, \dots, \zeta_d}}$$

$$[20] \quad r_{\zeta_1, \zeta_2, \dots, \zeta_d}^{\zeta_1} = \rho_{PA}^{\zeta_1, \zeta_1, \zeta_2, \dots, \zeta_d} + \rho_{RR}^{\zeta_1, \zeta_1, \zeta_2, \dots, \zeta_d} = \frac{\int \int_{\phi} SP_0^{\phi, \zeta_1}(t) R_{Rap}^{\phi, \zeta_1, \zeta_2, \dots, \zeta_d}}{1 \times R_{Rap}^{\zeta_1, \zeta_1, \zeta_2, \dots, \zeta_d}} + \frac{\int \int_{\phi} R_{Res}^{\phi, \zeta_1}(t) R_{Rap}^{\phi, \zeta_1, \zeta_2, \dots, \zeta_d}}{1 \times R_{Rap}^{\zeta_1, \zeta_1, \zeta_2, \dots, \zeta_d}}.$$

It's worth noting that the robustness under subsequent disturbances is always weaker than of the previous disturbance. This is because the time between sequential disturbances is always shorter than the time needed for the recovery to undisturbed level.

The resilience model developed in this work needs to be enhanced for the spatial distribution of disturbances. First, a disturbance is represented as the removal of elements of the network. As natural disasters, such as severe weather conditions, earthquakes, hurricanes or floods always strike geographically confined areas, cell space method should be investigated. Second, system robustness to a specific disaster can be different as the structure of infrastructure system changes. The proactive adjustment capacity changes accordingly. Third, reactive recovery capacity can also be improved through resourcefulness and rapidity. Both of them are determined by the adaptation/restoration strategy, which is the focus of single layer infrastructure network resilience. At last, duration of infrastructure malfunction does not only depend on the repair time, but also the buffering time  $T_B$  and state of dependent infrastructure systems. So infrastructure system resilience analysis needs systematic understanding of internal infrastructure features, external disturbance attributes and overall integration platform.

#### 4. DISCUSSION

Resilience is presented as an efficient approach for the management of infrastructure systems. This paper establishes multilayer infrastructure system resilience model based on the Space-Time Dynamic Resilience Measure of Simonovic and Peck (2013) with consideration of infrastructure interdependences. By considering system performance, its adaptation capacity and consequences of specific restoration strategies, the resilience is represented as a dynamic measure to be implemented using system simulation.

Infrastructure system resilience needs systems approach. As infrastructure systems mutually interact understanding of system interdependences is essential for infrastructure system resilience analysis. This paper details macro-perspective interdependences of infrastructure systems into micro dependence patterns, and integrates them through system dynamics analyses. So the resilience metric could be used for evaluation of different kinds of infrastructure systems with cascading failures or other high-order impacts. Also the presented model could be a generic framework or a methodology for the resilience analysis of systems-of-systems.

Finally, actual infrastructure system are more complex. Integrating consequences of disaster uncertainty into infrastructure system evolution is another potential contribution of the model to be addressed in future work.

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