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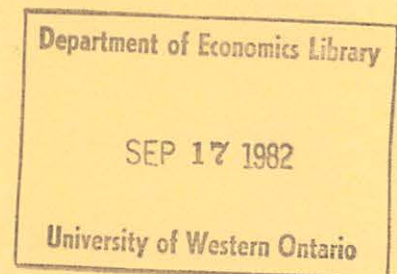
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THE GAINS FROM TRADE THEOREM WITH  
INCREASING RETURNS TO SCALE

James R. Markusen  
and  
James R. Melvin

This paper contains preliminary findings from research work still in progress and should not be quoted without prior approval of the author.

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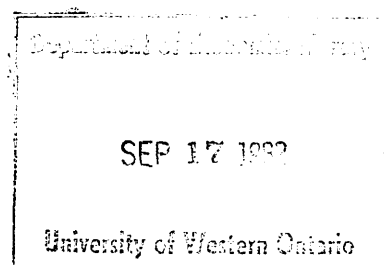
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ABSTRACT

The existing trade literature adopts a variety of approaches to increasing returns. This paper seeks a unified view of the gains from trade that covers all of the current models. Results suggest that in each case, losses from trade can be traced to the same two problems: (A) prices in excess of marginal costs; and (B) non-convexities in the production set. The paper then searches for sufficient conditions, common across models, ensuring gains from trade when either (A) alone or (A) and (B) together are present. Results are helpful in understanding how the addition of (B) to (A) complicates the conditions for gains.

The authors would like to thank Mark Bagnoli and Ignatius Horstmann for helpful discussions.

May, 1982



## I. Introduction

We have in recent years seen a strong interest in incorporating returns to scale into international trade models. Since Helpman (1982) has currently surveyed and synthesized many of the relevant contributions, we shall not repeat a literature review here, but rather simply note that a number of quite different approaches have been explored. One category of analysis, based on models with homogeneous products, can be subdivided into models with economies of scale external to the firm and models with internal economies.<sup>1</sup> Subject to correctly incorporating the externality-induced distortions, the former models can make use of competitive general-equilibrium analysis. The latter models rely either on partial equilibrium analysis based on classical duopoly/oligopoly theory, or attempt to develop simple general-equilibrium analyses which can be more easily related to traditional trade theory. A second category of analysis is based on models with differentiated goods and makes use of recent developments in the theory of monopolistic competition.<sup>2</sup>

In attempting to formulate a theory of the gains from trade in the presence of increasing returns to scale (IRS), it strikes us that there exist two problems. First, do there exist common underlying features in the classes of models just mentioned such that a unified approach to the gains from trade (GFT) might be developed? Second, many of the recent papers as surveyed by Helpman rely on very specific assumptions and functional forms. We should therefore inquire as to whether or not there exist reasonably general sufficient conditions under which GFT will be assured, or does it appear that restrictive assumptions are in fact necessary to generate such conditions.<sup>3</sup>

The purpose of this paper is to address both issues and to offer some tentative conclusions about each question. Specifics of the paper are as follows. Section II reviews the GFT theorem and offers a simple but useful dichotomy between alternative sources of breakdowns in the theorem (losses from trade). We will refer to these two sources as failures of the "tangency condition" and the "convexity condition" and will suggest that they are features that all of the models discussed above have in common. The convexity condition relates to the structure of IRS technologies while the tangency condition relates to economic pricing behavior. We will argue that for both analytical reasons and conceptual reasons these two factors deserve separate attention.

Section III reviews the possibility of losses from trade due to the failure of the tangency condition and shows that fairly simple (at least on a conceptual level) restrictions are sufficient to ensure GFT. It is pointed out that these sufficient conditions are generally well understood and that we have in fact little to contribute here outside of showing the generality of these conditions across models. We shall also try to argue that these conditions have a fairly intuitive interpretation in terms of "industry rationalization" that makes them seem less restrictive and improbable than they appear to be from a purely technical perspective.

Section IV analyzes the possibility of losses from trade due to failures of the convexity condition and argues that existing analyses are much less complete than in the case of the tangency condition. We are able to show that the same expansion of all IRS industries that ensures GFT in the presence of non-tangencies remains sufficient when there are non-convexities as well. This argument is contained in Helpman (1982) and implicitly in Kemp (1969) and we add to their findings by noting that the result does

not rely on any special functional forms or on average cost pricing. We note further that in the presence of non-convexities, this expansion effect is sometimes necessary as well as sufficient for GFT. On the other hand, we argue that the same weighted change in the outputs of IRS goods which is sufficient for GFT in the convex case is not sufficient in the case of non-convexities. Thus non-convexities do complicate the conditions when some IRS goods expand and others contract.

Section V presents a discussion of recent monopolistic competition models in light of the results of the previous section. We attempt to show how some of the restrictive assumptions used in that literature ensure that both the tangency and convexity problems are avoided and attempt to show how the weakening of some of these assumptions may lead to losses from trade. Section VI applies the analysis of the tangency condition to a multinational enterprise model and shows how a simple modification of that condition allows the GFT inequality to be expressed in terms of the distribution of profits.

In total, the paper is perhaps optimistic in the sense that the restrictions needed for GFT in the presence of IRS reduce to a fairly simple analytical condition. Further effort may well result in more robust sufficient conditions for the non-convex case. Yet even this will leave us short of our goal in the sense that the analysis does not show the circumstances under which the relevant conditions will or will not be satisfied in actual trading equilibria. Much further work is needed in this area.

## II. The Gains-from-Trade Theorem

The following notation will be used throughout the paper.  $X_i$  will denote the production of good  $i$  and  $C_i$  will denote the consumption of good  $i$ . Superscripts  $f$  and  $a$  will indicate free trade and autarky values, respectively.  $p_i$  will denote the price of good  $i$ . Since the focus here is on technology, we will simply assume that welfare can be represented by a set of community indifference curves, and let  $U$  stand for the level of national welfare or utility.  $e(p,U)$  will be the expenditure function; that is,  $e(p,U)$  gives the minimum expenditure necessary at prices  $p$  to attain utility level  $U$ .

The gains-from-trade theorem as advanced in international trade is almost elegant in its simplicity. It states that subject to certain restrictions on technology and pricing discussed below, the value of the free-trade production bundle at free-trade prices is greater than or equal to the value of any other feasible production bundle (e.g., the autarky bundle) at those free-trade prices. This proposition is given by

$$(1) \quad \sum p_i^f X_i^f \geq \sum p_i^f X_i^a.$$

Only a few additional equations are needed. These are the autarky market clearing equations and the free-trade balance-of-payments condition.

$$(2) \quad X_i^a = C_i^a ; \quad \sum p_i^f X_i^f = \sum p_i^f C_i^f.$$

Substituting (2) into (1) we have

$$(3) \quad \sum p_i^f C_i^f \geq \sum p_i^f C_i^a$$

which states that the free-trade consumption bundle is "revealed preferred" to the autarky consumption bundle. Below we will argue that if one is willing to accept the basic welfare criterion used, the analysis is perfectly applicable to situations in which the number of goods produced and consumed changes between the two equilibria.

Since the right-hand side of (3) generally exceeds the minimum expenditure necessary at  $p^f$  to attain  $U^a$ , (3) in turn implies

$$(4) \quad e(p^f, U^f) \geq e(p^f, U^a), \text{ implying that } U^f \geq U^a.$$

The theorem is illustrated in the two-good case in Figure 1 where F is the free-trade production point and A is the autarky production/consumption point. The value of F at  $p^f$  exceeds the value of A at  $p^f$  which in turn exceeds  $e(p^f, U^a)$  if there is some elasticity of substitution in consumption as shown.

In order for equation (1) and the theorem to be valid, the free-trade price vector ( $p^f$ ) must form a separating hyperplane to the production set. The price plane (price line in Figure 1) must not "cut" the production frontier at any point and no portion of the price plane may lie interior to the production set. If any portion of the price plane lies strictly interior to the production set, equation (1) will not hold since we will always be able to find a feasible X whose value at  $p^f$  exceeds  $p^f X^f$ .

Two conditions are sufficient for the separating hyperplane condition to hold. First, the free-trade price plane must not cut the production frontier at the free-trade production point. Let  $T(X_0, \dots, X_n) = 0$  be the transformation function specifying the production frontier. Assuming that T is differentiable, this condition requires that  $p^f$  be tangent to T at any interior solution, or more generally, that  $p_i^f/p_j^f = T_i^f/T_j^f$  for all  $X_i^f, X_j^f > 0$  ( $T_i = \partial T/\partial X_i$ ). Corner solutions modify the requirement to  $p_i^f/p_j^f \leq T_i^f/T_j^f$  for  $X_i = 0, X_j > 0$ . We will refer to these restrictions as the "tangency condition" even though



FIGURE 1

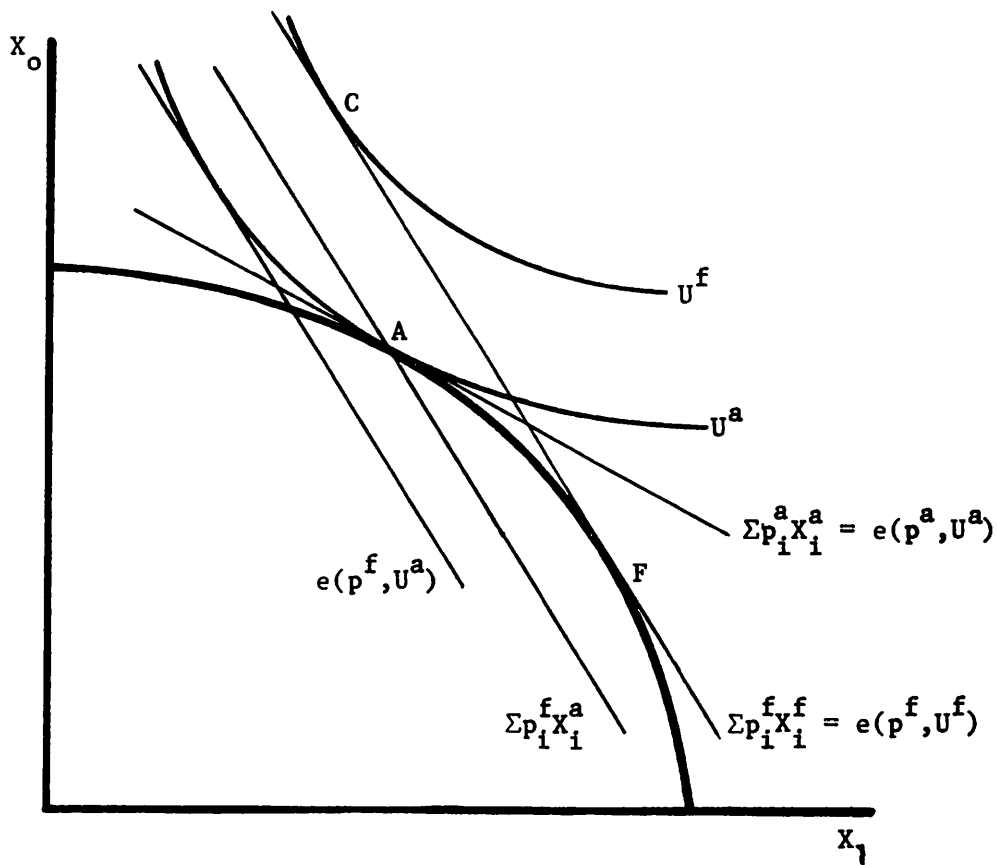
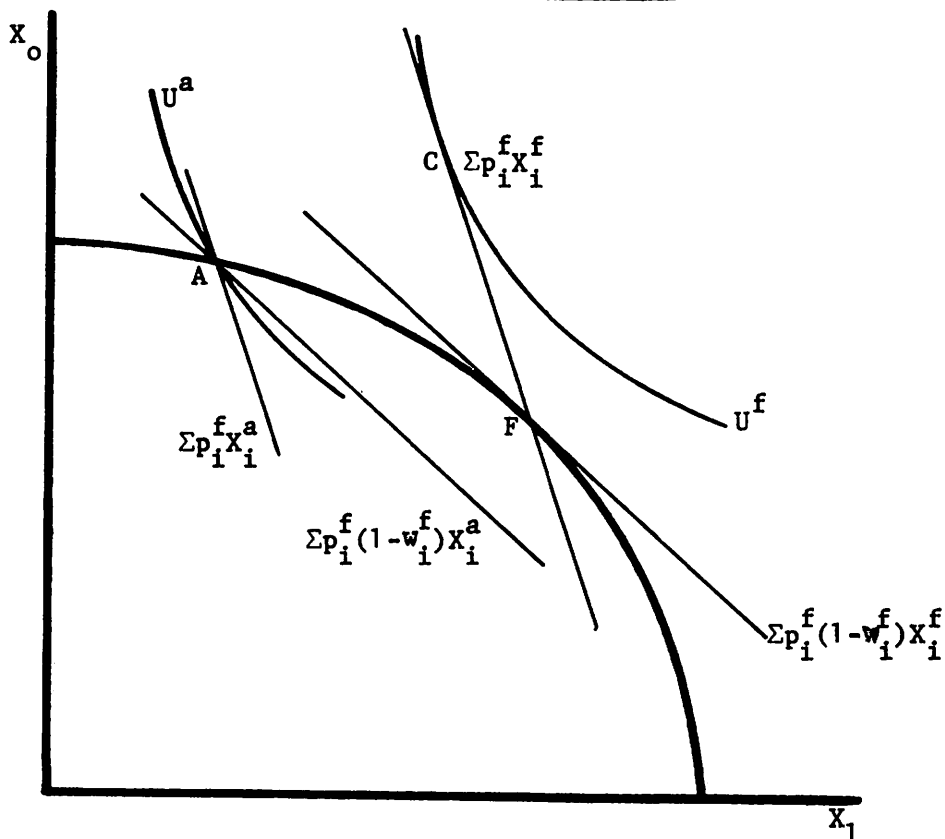


FIGURE 2



"tangency" is not an entirely appropriate term for corner solutions.

Subject to certain other restrictions, the tangency condition will be satisfied by marginal-cost pricing.

Second, the production set must be convex so that tangency ensures that the price plane does indeed form a separating hyperplane. This will be referred to as the "convexity condition". As is well known from general-equilibrium theory, the convexity condition will be satisfied if technology has non-increasing returns plus other restrictions such as distortion-free factor markets.

The final important point is that the GFT theorem can be reinterpreted such that it does not require that every good available in autarky be available in free trade or vice versa. Let  $i$  in equations (1), (2), and (3) index all of the goods that could be produced in the economy. Let  $p_i^f$  be the demand price for  $X_i$  at the free trade equilibrium (i.e., the actual price of  $X_i$  if  $X_i$  is produced at home or abroad or the price that consumers would be willing to pay for one unit if  $X_i$  is not available in free trade). If free trade production evaluated at these prices forms a separating hyperplane to the production set then equation (1) continues to hold. The balance of payments equation in (2) holds as well since for any good not available in free trade  $X_i^f = C_i^f = 0$ . The right and left-hand sides of the balance of payments constraint thus continue to give the values of actual production and consumption. Equation (3) then remains valid under this expanded interpretation. This will be relevant below in connection with monopolistic competition.

### III. The Tangency Condition

Given convex production possibilities, the absence of factor market distortions etc., the value of production at a particular set of prices is maximized by marginal cost pricing. This is embodied in the usual marginal cost pricing rule for economic efficiency. For our purposes, marginal costs form the "tangent" to the production surface ( $MC_i/MC_j = T_i/T_j = -dX_j/dX_i$ ) and thus form a separating hyperplane at the free trade production point. Let  $i$  index all goods that could be produced and  $p_i^f$  the demand prices as per the previous section. Given convexity, we have

$$(5) \quad \Sigma(MC_i^f)X_i^f \geq \Sigma(MC_i^f)X_i^a .$$

Some of the  $X_i^f$  can, of course, be zero. Marginal cost pricing implies

$$(6) \quad p_i^f = MC_i^f \quad \text{if } X_i^f > 0 ; \quad p_i^f \leq MC_i^f \quad \text{if } X_i^f = 0 .$$

Equations (5) and (6) together give us

$$(7) \quad \Sigma p_i^f X_i^f = \Sigma(MC_i^f)X_i^f \geq \Sigma(MC_i^f)X_i^a \geq \Sigma p_i^f X_i^a ,$$

which satisfies the condition in equation (1).

It is well known that marginal-cost pricing is not a characteristic of goods produced with any of the types of returns to scale mentioned in the introduction to the paper. Regardless of whether there are: (A) homogeneous products with external economies; (B) homogeneous products with internal economies; or (C) differentiated products with monopolistic competition, prices will exceed marginal costs for these goods. This must always be the case since in all categories marginal cost pricing would imply negative profits. (In category (A), prices equal firm marginal costs and industry average costs, but these costs are more than industry marginal costs which form the relevant measure.)

Let  $w_i$  be the "wedge" between  $p_i^f$  and  $MC_i^f$  expressed in an ad valorem fashion so that  $p_i^f(1-w_i^f) = MC_i^f$  for all  $X_i^f > 0$ . For any of the types of returns to scale just mentioned, we know that  $0 < w_i < 1$  ( $p_i > MC_i$ ) in equilibrium. In the case of pure monopoly or monopolistic competition, for example,  $w_i = 1/\eta_i$  where  $\eta_i$  is the elasticity of demand for  $X_i$ .  $w_i = 0$  of course for competitive industries producing goods with constant returns. The case of external diseconomies ( $w_i < 0$ ) has been dealt with by Eaton and Panagariya (1979), and Helpman (1982) and will not be treated here.

If  $X_i^f = 0$ , the definition of  $w_i^f$  is a bit more arbitrary. With  $X_i^f = 0$ , it must, however, be the case that  $p_i^f \leq AC_i^f$  where AC denotes average cost. Therefore define  $w_i^f$  for  $X_i^f = 0$  as  $(1 - w_i^f) = MC_i^f/AC_i^f$  so that we have  $p_i^f(1 - w_i^f) \leq AC_i^f(1 - w_i^f) = MC_i^f$ .  $w_i^f$  will thus continue to equal zero for CRS goods and will lie between 0 and 1 for IRS goods. Equation (6) becomes

$$(8) \quad p_i^f(1 - w_i^f) = MC_i^f \quad \text{if } X_i^f > 0 ; \quad p_i^f(1 - w_i^f) \leq MC_i^f \quad \text{if } X_i^f = 0.$$

Now assume that the convexity condition is satisfied such that equation (5) continues to hold.<sup>4</sup> Using (8) and (5) the present equivalent of equation (1) is therefore

$$(9) \quad \sum p_i^f(1-w_i^f)X_i^f \geq \sum p_i^f(1-w_i^f)X_i^a.$$

Equation (1) is of course simply a special case of (9) in which the  $w_i$  are identically zero. Equations (2) above remain unchanged and substituting (2) into (9) gives us

$$(10) \quad \sum p_i^f C_i^f \geq \sum p_i^f C_i^a + \sum p_i^f w_i^f (X_i^f - X_i^a).$$

Equation (7) gives us a condition which can be found in various forms in Kemp and Negishi (1970), Eaton and Panagariya (1979), Markusen and Melvin

(1981), Markusen (1981), and Helpman (1982). A sufficient condition for gains from trade is that trade lead to an increase in the output of every good produced with increasing returns (i.e., if  $w_i > 0$ , then  $X_1^f > X_1^a$ ).

Figures 2, 3, and 4 display the possibilities. In these diagrams and throughout the paper, it is assumed that  $X_1$  is produced with IRS and  $X_0$  with CRS unless otherwise indicated ( $0 < w_1 < 1$  and  $w_0 = 0$ ). Thus the price ratio  $p_1^f/p_0^f$  is steeper than the slope of the production frontier at the free trade production point F. Point A in each diagram continues to represent the autarky equilibrium. Equation (9) holds in each of Figures 2, 3, and 4 as shown.

Figure 2 shows the result of equation (10) that gains from trade must occur when trade expands production of the IRS good. Figure 3 shows the result that losses may occur when trade leads to a contraction of the IRS industry. Figure 4 emphasizes that expansion of the IRS industry is sufficient but not necessary by showing a case in which welfare increases despite a contraction in  $X_1$ .

It should be emphasized from equation (10) that a weighted increase in production of the IRS goods is sufficient for GFT, where the weights are  $p_i w_i = (p_i - MC_i)$ . It can be argued that such an increase is perhaps not at all unlikely in practice. Ignoring for the moment the  $p_i w_i$  weights in (10) and differences in industry size, what is required is that trade have a certain "rationalizing" effect on the IRS industries. By this we will mean that the proportion of IRS production eliminated by trade is less than the proportional increase in the output of the surviving IRS industries. Such an outcome is reasonable if, for example, trade does not decrease the total domestic resources devoted to the IRS industries. With the same total resources, the lower average costs of the remaining industries due to larger free trade production will tend to outweigh the loss of the other industries. A rigorous statement of the sufficient conditions is postponed to Section V on monopolistic competition.

FIGURE 3

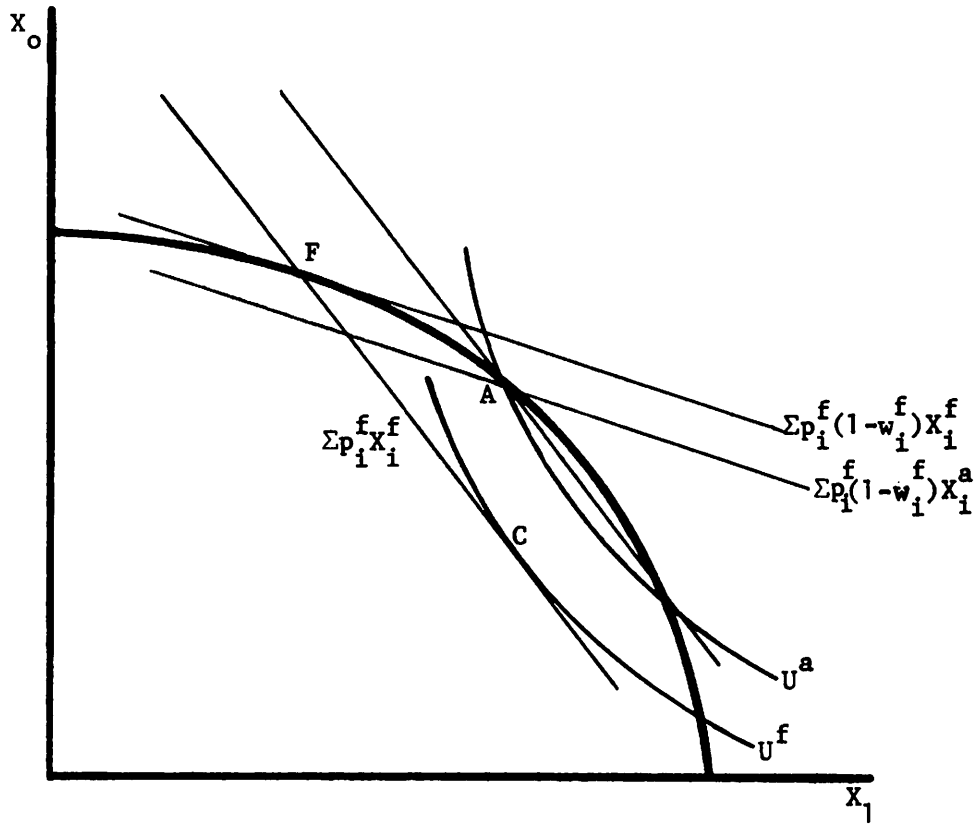
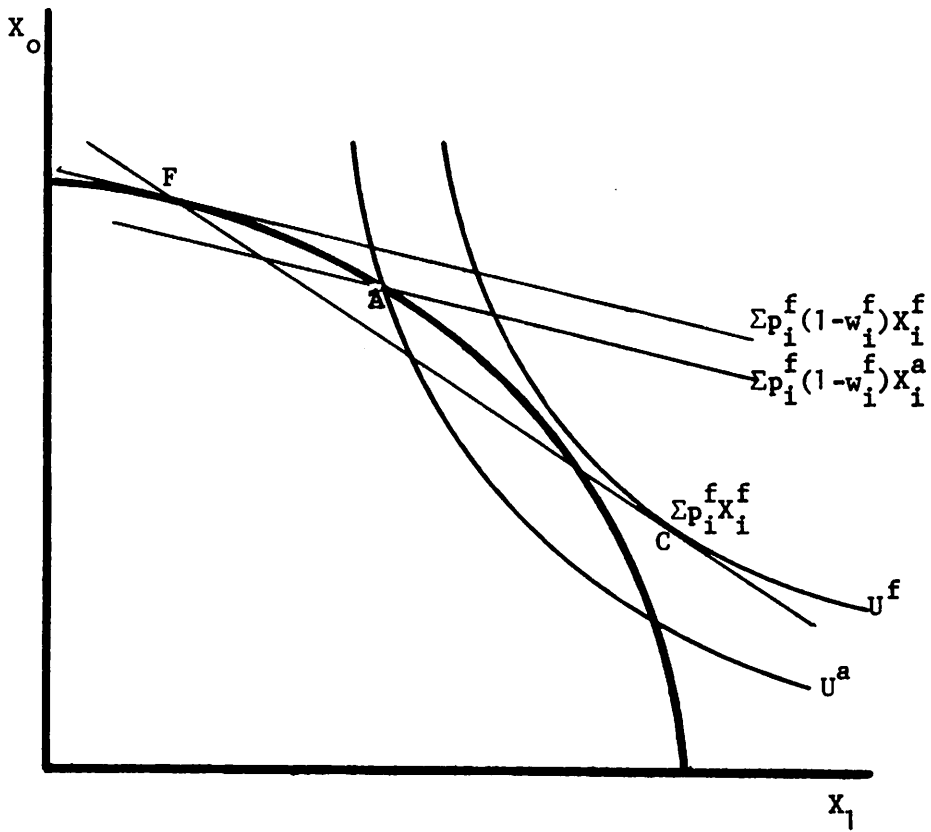


FIGURE 4



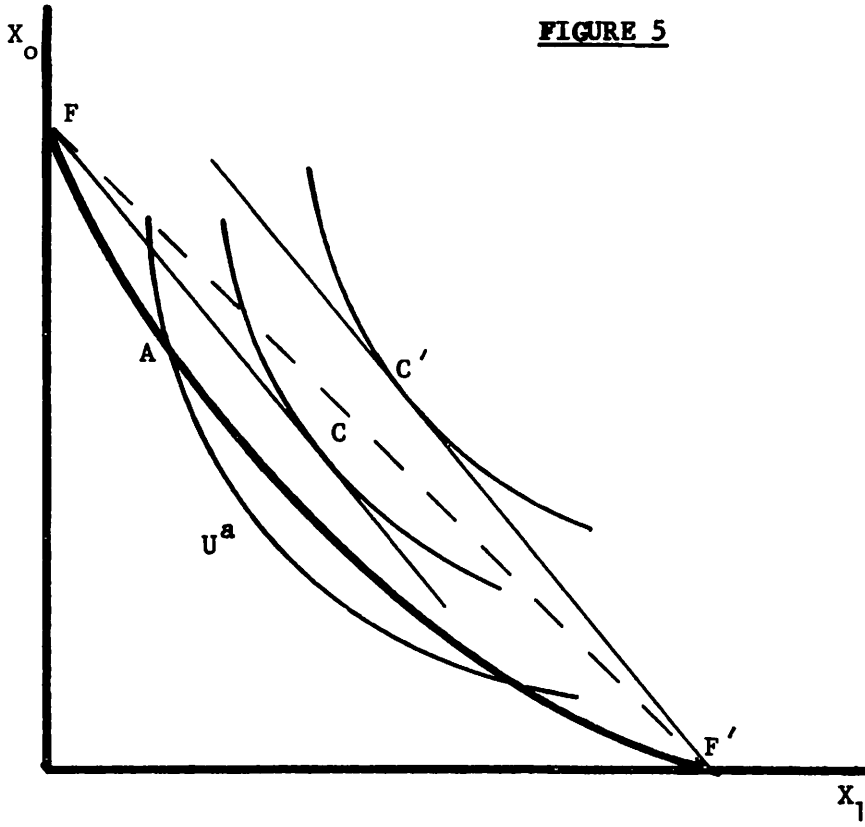
#### IV. The Convexity Condition

The fact that with IRS the production set may be non-convex (the production frontier may be convex to the origin) is probably well known and does not deserve extensive comment here. In the absence of strong factor-intensity effects, production frontiers can be everywhere convex as in Figures 5, 6 and 7. Stronger factor intensity effects can lead to complex outcomes with alternating convex and concave segments (Kemp (1969), Markusen and Melvin (1981), Section VII below).

GFT analyses in the presence of non-convexities contain a certain irony in that there are more possibilities of gains from trade yet also more things that can go wrong relative to the case of convex production possibilities. Figure 5 suggests some of the things that can go right while Figures 6 and 7 suggest some of the things that can go wrong.

Figure 5 shows the same two-good problem we have been using but with a convex production frontier ( $X_1$  has IRS,  $X_0$  has CRS). Note first that specialization in either good may produce gains at a given price ratio (Melvin (1969)). This in turn implies of course that specialization in the IRS good is not necessary. Second, note that GFT may occur even if trade decreases the price of the export good relative to autarky (e.g., specializing in  $X_1$  at  $F'$  leads to gains even though  $(p_1^f/p_0^f < p_1^a/p_0^a)$ ). Third, the cord connecting  $F$  and  $F'$  forms a portion of the boundary of the convex hull of the production set. Note that GFT may occur even if consumption occurs at a point like  $C$  interior to this convex hull, a point which will be relevant later in connection with monopolistic competition.

**FIGURE 5**



**FIGURE 6**

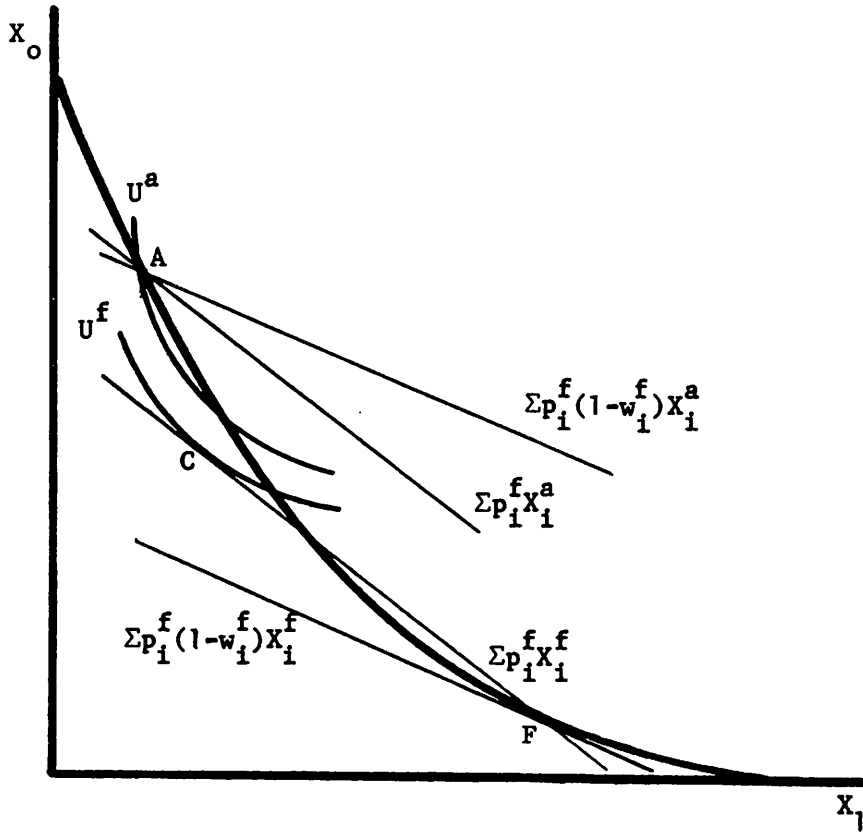




Figure 6 illustrates the possibility that expansion of the IRS good may not be sufficient for GFT. In fact convexity of the frontier combined with non-specialization at F makes expansion of the IRS good a necessary condition. Note from Figure 6 that with non-specialization, equation (9) now becomes

$$(11) \quad \sum p_i^f (1-w_i^f) X_i^f \leq \sum p_i^f (1-w_i^f) X_i^a$$

that is, the direction of the inequality in (9) is reversed. The present equivalent of (10) becomes

$$(12) \quad \sum p_i^f C_i^f \leq \sum p_i^f C_i^a + \sum p_i^f w_i^f (X_i^f - X_i^a).$$

If production is diversified at both A and F, expansion in production of the IRS good becomes a necessary condition for GFT from a revealed preference point of view. Using the expenditure function criterion,  $\sum p_i^f C_i^a \geq e(p^f, U^a)$  so expansion of the IRS good may be neither necessary nor sufficient.

Figure 7 helps illustrate the contrapositive of this result: if expansion of the IRS good is necessary for GFT using the revealed preference criterion, then contraction is sufficient for losses. The value of the autarky bundle A at free trade prices must exceed the value of F at those free trade prices in Figure 7.

Fortunately, it turns out that non-negative profit restrictions will ensure that expansion of the IRS industries is still sufficient for GFT. Thus from a revealed preference point of view, expansion is both necessary and sufficient when production is non-specialized. If  $X_0$  is a CRS good in Figure 6, the situation shown cannot occur. We can show that if the output of the IRS good expands, then the situation must be as shown in Figure 7, where the value of production at  $F'$  exceeds the value at prices  $p^{f'}$  of any other bundle with a smaller output of  $X_1$ . The proof proceeds as follows. Gains will occur if

FIGURE 7

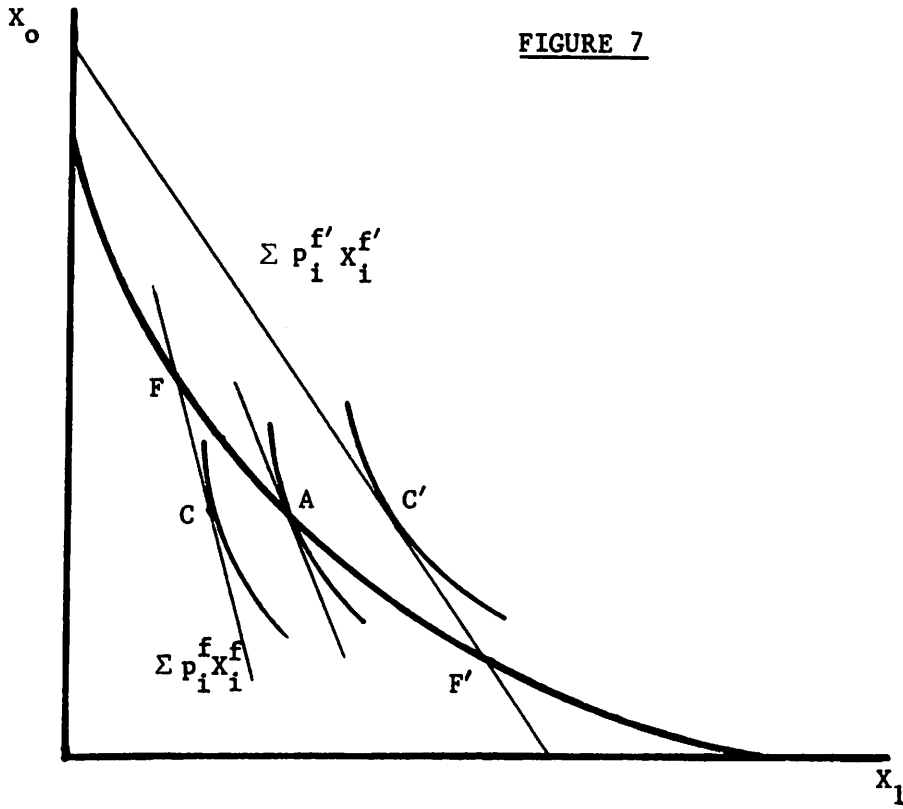
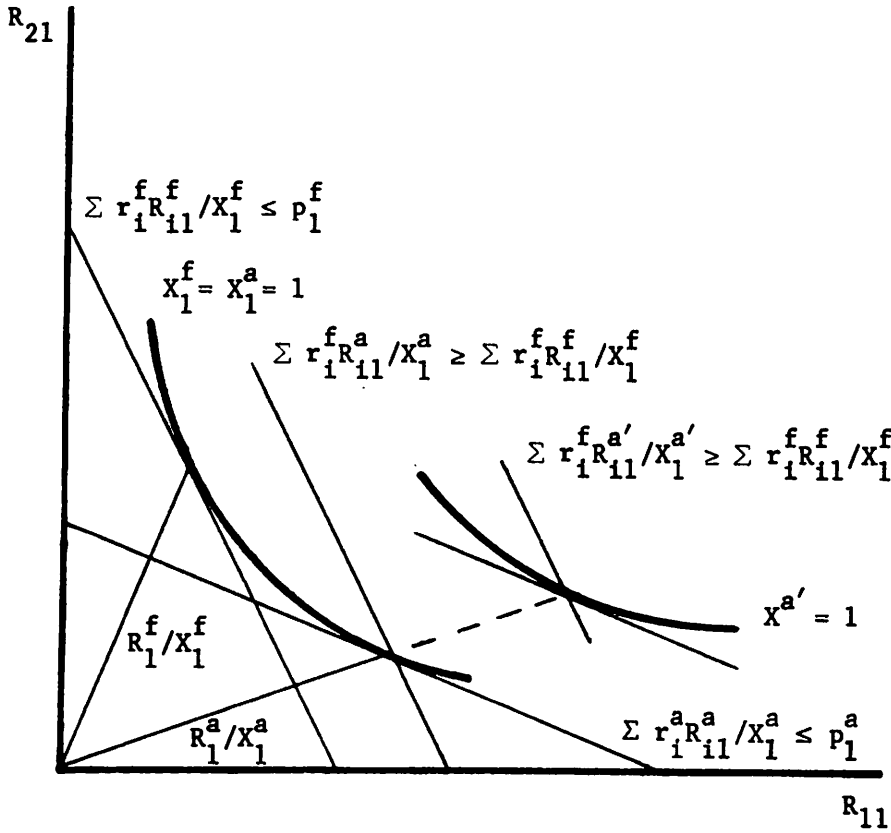


FIGURE 8



$$(13) \quad \sum p_i^f X_i^f \geq \sum p_i^a X_i^a .$$

Let  $r_i$  and  $R_{ij}$  denote the price of resource  $i$  and the quantity of the  $i^{\text{th}}$  resource used in the production of the  $j^{\text{th}}$  good respectively. To each side of equation (13) we can subtract the value of the total resource endowment at free trade prices:  $\sum \sum r_i^f R_{ij}^f = \sum r_i^f \bar{R}_i^f$ . For the moment, assume that we have only two goods ( $X_0, X_1$ ) as shown in Figures 5, 6, and 7. Suppose also that there are only two resources ( $R_1, R_2$ ). Equation (13) can be written as

$$(14) \quad (p_1^f X_1^f - \sum r_i^f R_{i1}^f) + (p_0^f X_0^f - \sum r_i^f R_{i0}^f) \geq (p_1^a X_1^a - \sum r_i^a R_{i1}^a) + (p_0^a X_0^a - \sum r_i^a R_{i0}^a) .$$

Sufficient conditions for this inequality to hold are that

$$(15) \quad (p_1^f - \sum r_i^f R_{i1}^f / X_1^f) X_1^f \geq (p_1^a - \sum r_i^a R_{i1}^a / X_1^a) X_1^a$$

$$(16) \quad (p_0^f - \sum r_i^f R_{i0}^f / X_0^f) X_0^f \geq (p_0^a - \sum r_i^a R_{i0}^a / X_0^a) X_0^a .$$

These inequalities can be analyzed using Figure 8. Suppose that  $X_1$  is a CRS industry in which case the unit isoquant is given by  $X_1^f = X_1^a = 1$  in Figure 8.  $R_1^f / X_1^f$  and  $R_1^a / X_1^a$  give the cost-minimizing unit input vectors at factor prices  $r^f$  and  $r^a$  respectively. As is clear from Figure 8, the cost of the autarky unit inputs at  $r^f$  exceed the costs of the free trade unit inputs at  $r^f$ . With reference to equation (15), we have

$$(17) \quad (p_1^f - \sum r_i^f R_{i1}^f / X_1^f) \geq (p_1^a - \sum r_i^a R_{i1}^a / X_1^a) .$$

If the left-hand side of (17) is zero due to average-cost pricing in the CRS industry, then the right-hand side is non-positive. Thus for any CRS industry, the left-hand side of (15) or (16) exceeds the right-hand side and the inequalities hold.

For IRS industries, the left-hand side of (17) may be positive (free trade profits are positive) and more to the point, the unit isoquant is in a different position for the two equilibria. If  $X_1^f > X_1^a$ , then the  $X_1^a$  unit isoquant lies further from the origin as in Figure 8 where  $X^{a'} = 1$  denotes its new position. With  $X_1^f > X_1^a$  equation (17) remains valid (illustrated in Figure 8) as does equation (15).

Thus despite non-convexities, expansion of the IRS sectors remains sufficient, although in some cases it may be necessary as well. Note especially that this result is robust with respect to the following: (A) we can add any number of goods and factors to the analysis, (B) average-cost pricing in the IRS sectors is not required, (C) no special functional forms are needed for the result, (D) the production frontier may have any number of concave and convex segments.

Unfortunately, it is not true that the same weighted change in the IRS industries that was sufficient for GFT in the convex case is sufficient here. Suppose in Figure 6 that both  $X_1$  and  $X_0$  have IRS but that at A and F  $w_1 > w_0$  so that the price ratio continues to cut the frontier in the direction shown. In this case, the situation shown in Figure 6 cannot be ruled out. Algebraically, equation (16) need not hold and thus (13) need not hold. With IRS in  $X_0$ , the unit cost of  $X_0^a$  at  $r^f$  may in fact be less than the unit cost of  $X_0^f$  at  $r^f$  since the unit isoquant for  $X_0^a$  lies closer to the origin. With  $X_0^a > X_0^f$  this would be sufficient for (16) not to hold. Thus the price ratio through F could possibly pass below A in Figure 6 and we would have

$$(18) \quad \sum p_i^f X_i^f < \sum p_i^f X_i^a \quad \text{or} \quad \frac{p_1^f}{p_0^f} < \frac{X_0^a - X_0^f}{X_1^f - X_1^a} .$$

With both industries characterized by IRS, we noted following equation (10) above that  $\sum p_i^f w_i^f (X_i^f - X_i^a) > 0$  is sufficient for gains. In the present case, this requires

$$(19) \quad \frac{p_1^f w_1^f}{p_0^f w_0^f} > \frac{X_0^a - X_0^f}{X_1^f - X_1^a}.$$

But our example in Figure 6 is constructed under the assumption that  $w_1^f > w_0^f$ . Thus (18) and (19) may be consistent. GFT are not realized despite the fact that the weighted increase condition of equation (10) is satisfied. Such a result would, of course, be consistent with our equation (12).

In Section VII below, we present an example in which (19) and (18) are consistent, thus proving that the IRS expansion condition of equation (10) is not sufficient for GFT in the presence of non-convexities. This example does rely on some restrictive assumptions such as no factor intensity effects. On the other hand, the example does not rely on non-specialization and well-behaved functional forms are used as well as average cost pricing.

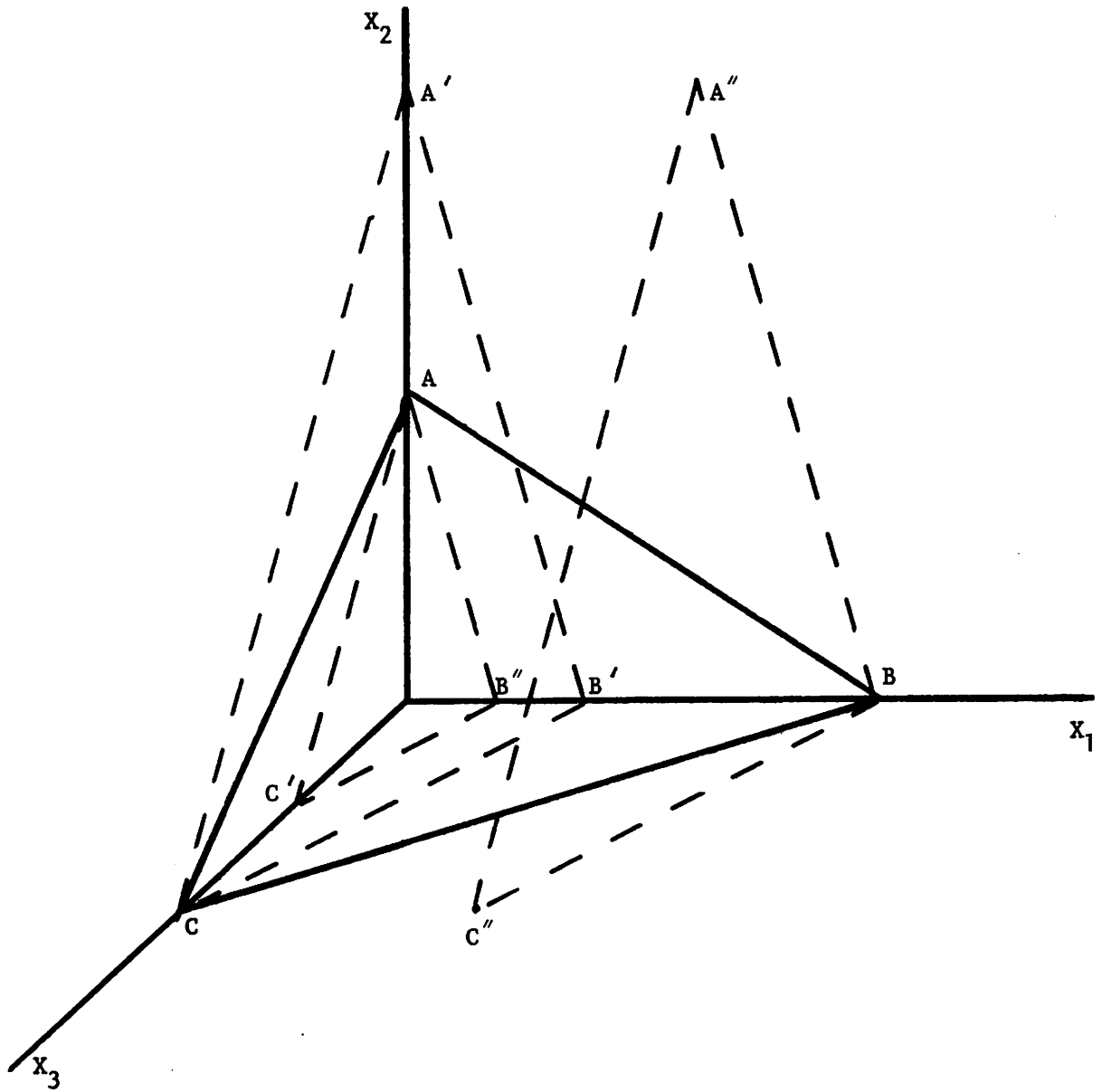
One possible way out is to show that a trading equilibrium such as that in Figure 6 is not a global maximum. It would seem that at  $p^f$  the economy should specialize in either  $X_0$  or  $X_1$ . This approach has pitfalls as well. While we should surely be able to show that profits at  $p^f$  would be greater at one or both specialized production points, it does not follow that  $p^f$  could be the price at such an equilibrium except in the very special case where the country is a price taker on world markets. More generally, a movement toward specialization in  $X_1$  beginning at F in Figure 6 should drive down the price of  $X_1$  (and vice versa for moving toward specialization in  $X_0$ ). If marginal revenue falls faster than marginal cost in such a movement, F is indeed a global profit maximum.

This general problem is illustrated in Figure 9 where it is assumed that the plane ABC represents a portion of the convex hull of the production set. The dotted triangles represent the price plane drawn through alternative points of specialization and thus represent alternative consumption possibilities sets. At these prices, specializing in  $X_2$  is the worst alternative,  $X_3$  the next best, and  $X_1$  clearly the best. The point is that with a non-convex production set, the economy could indeed get stuck at a production point like A or C. For reasons noted in the previous paragraph, A or C could indeed be a profit maximum. When they are not, a second problem arises with respect to whether or not free entry will ensure that the economy moves to the right pattern of specialization. Further discussion of this point is postponed until the following section.

#### V. An Application to Monopolistic Competition

Recent monopolistic competition models have moved the discussion of returns to scale into an interesting new area (Krugman (1979, 1980), Lancaster (1980), Helpman (1981)). But as noted by Helpman (1982), little has been done in the way of formulating a theory of the GFT in the presence of monopolistic competition. The purpose of this section is to show that the theory of the previous several sections is in fact widely applicable to monopolistic competition as well.

In Sections II and III above, we argued that the general theory is applicable to situations in which the number of goods produced and consumed changes between the free trade and autarky equilibria if one is willing to accept the basic welfare criterion used.<sup>5</sup> An important source of GFT as emphasized by the above-mentioned authors is changes in the number of goods

FIGURE 9

available for consumption. Let us therefore examine the IRS production expansion condition given in equations (10) and (12) in light of recent monopolistic competition models. (Recall that we demonstrated that this condition is generally not sufficient for GFT in the presence of non-convexities.) For simplicity, we will for the moment adopt the assumptions of Krugman (1979) and others that all differentiated products are perfectly symmetric in production and consumption such that all goods produced are produced in the same amount at the same price. If there is no other class of IRS goods, the IRS expansion condition becomes

$$(20) \quad \sum_i p_i^f w_i^f (X_i^f - X_i^a) = q^f w^f (n^f X^f - n^a X^a) \geq 0,$$

where  $q^f$  and  $w^f$  are the price and inverse demand elasticity of a representative differentiated good, respectively.  $n^i$  and  $X^i$  are the number of varieties produced at equilibrium  $i$  and the amount of a representative good produced at that equilibrium. The right-hand term in (20) can be rearranged as follows:

$$(21) \quad (n^f X^f - n^a X^a) = (X^f/X^a - n^a/n^f)(n^f X^a).$$

The expansion condition is therefore satisfied if  $X^f/X^a \geq n^a/n^f$ , an inequality which does hold in Krugman's model. In Krugman's case, trade rationalizes production, with each country producing larger amounts of each of fewer varieties in the post-trade equilibrium (the number of varieties consumed, of course, increases). The fact that  $X^f/X^a > n^a/n^f > 1$  follows from two factors: (A) Since there are only differentiated products, total resources allocated to these goods is unaffected by trade; (B) with IRS, the amount of the representative good produced must expand more than in proportion to the number of varieties lost by virtue of the lower average cost of larger production (total resources held constant). Note finally that in this model



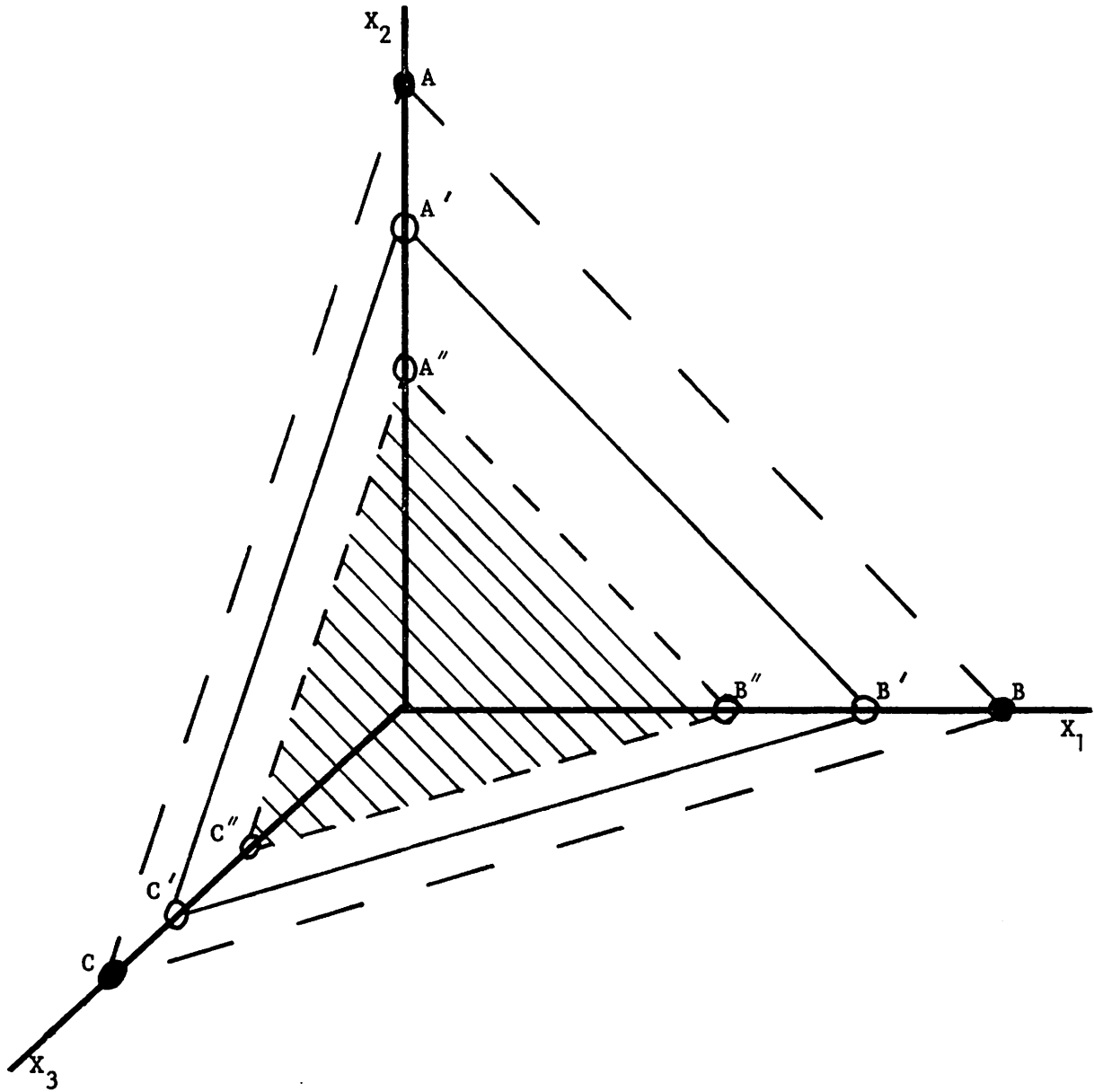
the IRS expansion condition is sufficient for GFT. With the  $w_i$ 's equal and only IRS goods produced, the common  $w^f$  can be factored out of (20) and (20) is reduced to equation (1).

Now consider the convexity condition, again using Krugman's (1979) model. Krugman uses a simple one-factor model in which each good requires an initial lump-sum of labour and can thereafter be produced at constant marginal cost. The resulting production set is illustrated in Figure 10 for a three-dimensional sub-space of the infinite-dimensional space of possible varieties.

If the economy produces only one good, it can be at any of points A, B, or C in Figure 10. If two goods are produced, a lump-sum of resources is lost in fixed costs, so that the economy "drops down" toward the origin and produces somewhere on the boundary of the triangle  $A'B'C'$ , vertices excluded. If three goods are produced, another lump of resources is used up and the economy produces in the interior of the planar surface bounded by the triangle  $A''B''C''$ . This diagram thus hopefully helps to illustrate the tradeoff between scale economies and product diversity.

The production set in Figure 10 is quite obviously non-convex. How then are GFT guaranteed in Krugman's analysis? We maintain that GFT probably rely on a number of restrictive assumptions. The first is that there are infinitely many perfectly symmetric goods that can be produced. The second is the usual monopolistic competition assumption about entry. This implies that firms will enter whenever prices of existing products exceed average costs, thereby guaranteeing average cost pricing. A final assumption is that only differentiated goods are produced which ensures the expansion effects mentioned in (20) and (21) above. The first two assumptions guarantee that all goods produced trade for the same price. More to the point, the assumptions collectively guarantee that the price plane will lie on the

FIGURE 10



convex hull of the production set (if this is still appropriate terminology) in the sub-space of goods produced and consumed.

Suppose for example that in autarky the economy produces  $X_1$  and  $X_2$  in Figure 10. Production and consumption thus take place on the line segment A'B'. Suppose with trade the economy reduces the number of varieties produced to one and increases the number consumed to three as per the above results. The Krugman assumptions imply that whichever variety is produced, the price plane coincides with the plane defined by ABC in Figure 10. The price plane thus forms a separating hyperplane and GFT are assured.

Several restrictive features of the assumptions used here have already been discussed by Dixit and Norman (1980) and Eaton and Kierzkowski (1982). They show that under somewhat different assumptions trade may not increase product variety and free entry may not be sufficient to induce average-cost pricing. We think in particular that the Eaton/Kierzkowski result is important in emphasizing that average-cost pricing should not be automatically assumed, but rather that the equilibrium pricing configuration should be derived from underlying behavioral assumptions.

For our part, we would like to briefly comment on the role of the symmetry assumptions used above. Consider briefly the production side and retain for the moment the symmetry assumptions on the demand side. If there are different technologies for different goods then price ratios will not in general equal ratios of marginal costs. For goods  $X_i$  and  $X_j$ , the equilibrium conditions give us

$$(22) \quad \frac{p_i (1 - 1/\eta_i)}{p_j (1 - 1/\eta_j)} = \frac{MC_i}{MC_j} .$$

Let us retain all of Krugman's assumptions except let the constant marginal costs be related by  $MC_i > MC_j$ . Figure 10 continues to be perfectly applicable except that the cords such as  $A'B'$  and  $B'C'$  will now have different slopes (equal to  $MC_i/MC_j$ ). If the fixed costs are in a similar ratio across goods, then these marginal costs define a plane coincident with  $A'B'C'$  as well.

If  $(p_i/p_j) = (MC_i/MC_j)$  in (22), it must be the case that  $\eta_i = \eta_j$ , but this can only occur if  $X_i = X_j$  and  $p_i = p_j$  due to the demand assumptions. Thus the equilibrium in (22) cannot be characterized by  $(p_i/p_j) = (MC_i/MC_j)$  and the price plane will not lie on the convex hull of the production subspace in Figure 10.

A similar problem occurs with respect to consumption. Suppose we now retain all of Krugman's production assumptions but make goods non-symmetric in demand ( $MC_i/MC_j = 1$ ). For  $(p_i/p_j) = (MC_i/MC_j)$  in (22) we must have  $\eta_i = \eta_j$  and  $p_i = p_j$ . But if  $i$  is the preferred good,  $p_i = p_j$  must imply  $X_i > X_j$ . Given the structure of technology, however,  $X_i > X_j$  and  $p_i = p_j$  must imply that profits in  $i$  exceed profits in  $j$ . Depending on the behavioral assumptions adopted, this will generally not be an equilibrium (e.g., average-cost pricing).

Referring back to Figure 9, let the plane  $ABC$  correspond to  $ABC$  in Figure 10. The above analysis implies that with asymmetries in production and/or consumption, the price plane will generally not coincide with  $ABC$ . The equilibrium prices could look something like the dotted triangle in Figure 9. Identical countries will enjoy different welfare levels and some countries may suffer losses depending upon the pattern of specialization.

VI. An Application to the Multinational Enterprise

The purpose of this section is to show briefly how the tangency condition can be expressed in terms of profits when returns to scale are internal to the firm and then show how this result is helpful in analyzing GFT in the presence of multinational enterprises (MNE). Suppose for simplicity that the production set is convex despite IRS in sector  $X_1$ . The only other good,  $X_0$ , is produced with CRS by a competitive industry. We know then that

$$p_0^f = MC_0^f \text{ and } p_1^f(1-w_1^f) = MC_1^f, \quad w_1^f = 1/\pi_1^f.$$

Equation (10) above then becomes

$$\begin{aligned} (23) \quad \Sigma p_i^f C_i^f &\geq \Sigma p_i^f C_i^a + p_1^f w_1^f (X_1^f - X_1^a) \\ &= \Sigma p_i^f C_i^a + (p_1^f - MC_1^f) (X_1^f - X_1^a). \end{aligned}$$

But with IRS in  $X_1$ ,  $MC_1 \leq AC_1$ . Thus if  $X_1$  expands with trade, (23) can be rewritten as

$$\begin{aligned} (24) \quad \Sigma p_i^f C_i^f &\geq \Sigma p_i^f C_i^a + (p_1^f - AC_1^f) (X_1^f - X_1^a) \\ &= \Sigma p_i^f C_i^a + (\pi_1^f - \pi_1^{af}), \end{aligned}$$

where  $\pi_1^f$  denotes profits at the free-trade equilibrium and  $\pi_1^{af}$  denotes profits at the autarky output at free-trade prices. Equation (24) states that there will be GFT if free-trade profits at  $p^f$  exceed autarky profits at  $p^f$  (subject also of course to  $X_1^f > X_1^a$ ).

The geometry of the situation is illustrated in Figure 11. Using  $X_0$  as numeraire,  $I^f$  gives total income while  $I_r^f$  gives income of the

FIGURE 11

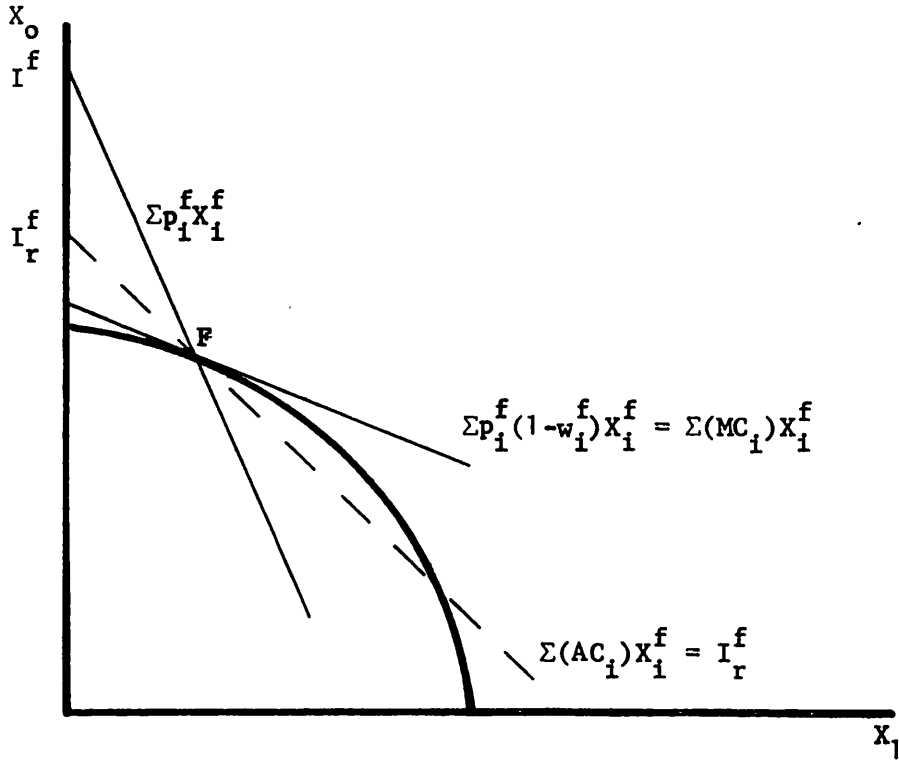
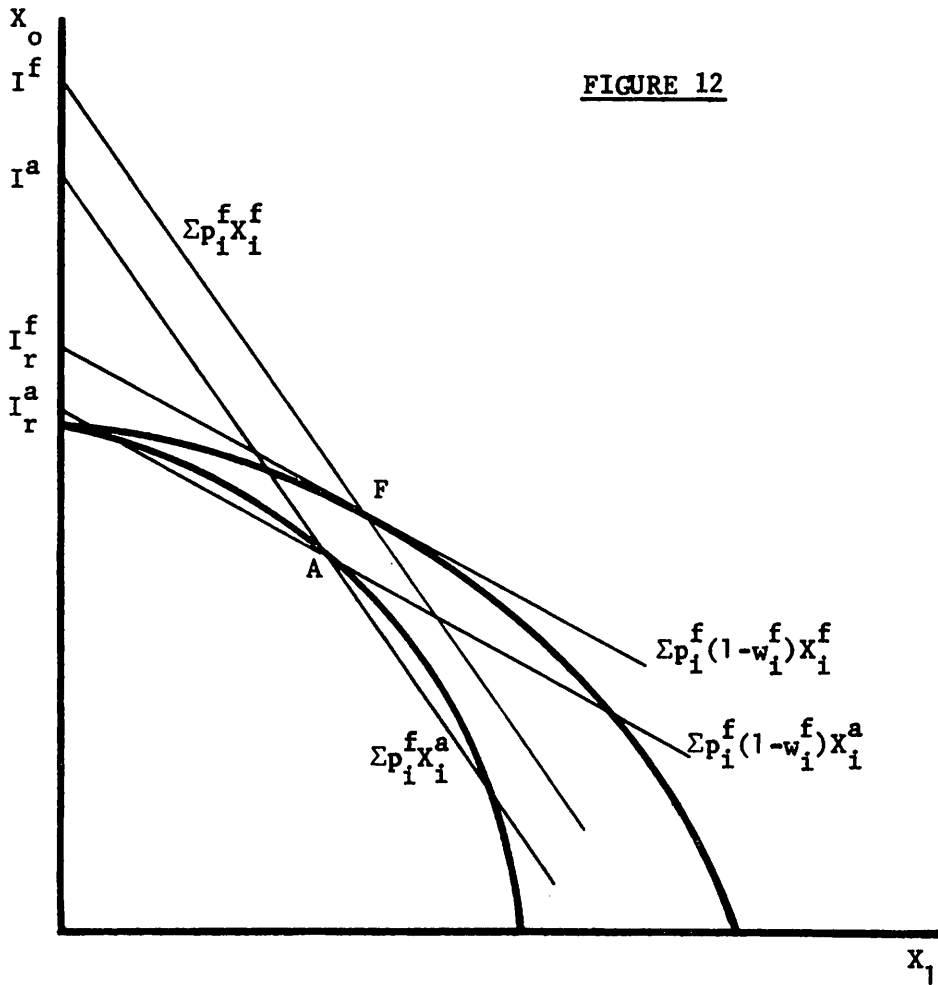


FIGURE 12



domestic resource owners. The difference between  $I^f$  and  $I_r^f$  thus constitutes monopoly profits.  $I_r^f$  will lie somewhere between  $I^f$  and the tangent at F depending on the strength of the IRS and the possibilities of entry.

By itself, equation (24) adds little since it is only true when  $X_1^f > X_1^a$  and we know that is sufficient for GFT in any case. But there are situations in which this formulation is very useful. Figure 12 applies this analysis to a simple MNE problem (Markusen (1982)). Suppose that the economy initially produces at A realizing an income of  $I^a$  at  $p^f$ . Suppose instead that a MNE controls the  $X_1$  sector by virtue of superior technology. This technology expands production possibilities for  $X_1$  and MNE production is assumed to occur at point F (satisfying our restriction that  $X_1^f > X_1^a$ ). Income is now higher at  $I^f$ . But if the MNE is entirely foreign-owned and profits are repatriated, domestic citizens could end up with an income as low as  $I_r^f$ , corresponding to the lower bound case in Figure 11 where  $MC_1 = AC_1$ . Domestic factor owners do gain in this case ( $I_r^f > I_r^a$ ) but total domestic income is lower due to the fact that profits which would have accrued to domestic entrepreneurs now go to foreigners.

Let  $\pi^*$  denote profits repatriated by the MNE. The balance of payments constraint (2) above now becomes  $\sum p_i^f X_i^f - \pi^* = \sum p_i^f C_i^f$ . Substitution will give us a new equivalent to (24).

$$(25) \quad \sum p_i^f C_i^f \geq \sum p_i^f C_i^a + (\pi^f - \pi^{af} - \pi^*)$$

so that GFT are assured if  $(\pi^f - \pi^{af} - \pi^*) \geq 0$ . A simple rule falls right out of this equation: if the host country can retain a share of the MNE profits  $(\pi^f - \pi^*)$  that is at least equal to the profits that would have been earned by a domestic monopolist in the absence of MNE activity  $(\pi^{af})$  and the MNE expands production, then GFT are assured.

### VII. An Algebraic Example of Non-Convexities

The shape of the production surface is determined by the interaction of factor intensity effects (which tend to make the surface concave to the origin) with IRS effects (which tend to make the surface convex to the origin). The purpose of this section is to present an example of how these effects interact to determine the shape of the frontier.

Assume that there are two goods (X,Y), produced from two factors (K,L) in inelastic supply. X is characterized by multiplicatively separable external economies (Herberg and Kemp (1969), Markusen and Melvin (1981), Helpman (1982)) while Y is characterized by CRS. Production functions are C.E.S. and are given by

$$(26) \quad X = (X^T)(aL_x^{-\beta} + bK_x^{-\beta})^{-1/\beta} \quad Y = (cL_y^{-\gamma} + dK_y^{-\gamma})^{-1/\gamma} \quad -1 < \beta, \gamma < \infty ; 0 < T < 1.$$

Let  $p = p_x/p_y$  and  $\omega = w/r$  (i.e.,  $\omega$  equals the wage/rental ratio). Herberg and Kemp's results together with Markusen and Melvin's results give the following relationship:

$$(27) \quad \frac{dp}{p} / \frac{d\omega}{\omega} + T \frac{dX}{X} / \frac{d\omega}{\omega} = \frac{1}{1+k_x/\omega} - \frac{1}{1+k_y/\omega} \quad k_x = K_x/L_x, \quad k_y = K_y/L_y.$$

Assume throughout that X is labour intensive so that  $k_x < k_y$ . If there were CRS in X ( $T=0$ ), then  $(dp/p)/(d\omega/\omega) > 0$  and the relative price of X and the wage/rental ratio rise together. Since  $dX/d\omega > 0$  this in turn implies that  $dX/dp > 0$  or that the supply price of X rises with output. Since the supply price is related to the MRT by  $p(1-T) = MRT$ , this in turn implies that the MRT rises with X and the production frontier is concave. But when  $T > 0$ ,  $(dp/p)/(d\omega/\omega)$  could be negative and the production frontier convex.

The outcome turns out to depend very much on the elasticities of substitution in production, given by  $\sigma_x$  and  $\sigma_y$ .



$$(28) \quad \sigma_x \frac{d\omega}{\omega} = \frac{dk_x}{k_x} = \frac{dK_x}{K_x} - \frac{dL_x}{L_x}; \quad \sigma_x = \frac{1}{1+\beta}; \quad \omega = \frac{a}{b} \left(\frac{K_x}{L_x}\right)^{\beta+1} = \frac{c}{d} \left(\frac{K_y}{L_y}\right)^{\gamma+1}.$$

Differentiating the production function for X, dividing through by X, and substituting for  $dK_x/K_x$  from (28), we have

$$(29) \quad (1-T) \frac{dX}{X} = \frac{aL_x^{-\beta}}{aL_x^{-\beta} + bK_x^{-\beta}} \frac{dL_x}{L_x} + \frac{bK_x^{-\beta}}{aL_x^{-\beta} + bK_x^{-\beta}} \frac{dK_x}{K_x}; \quad (1-T) \frac{dX}{X} / \frac{d\omega}{\omega} =$$

$$\frac{dL_x}{L_x} / \frac{d\omega}{\omega} + \frac{bK_x^{-\beta}}{aL_x^{-\beta} + bK_x^{-\beta}} \sigma_x = \frac{dL_x}{L_x} / \frac{d\omega}{\omega} + \sigma_x \left[ \frac{a}{b} \left(\frac{L_x}{K_x}\right)^{-\beta} + 1 \right]^{-1}.$$

Since  $L_x/K_x$  falls with an increase in X, the second term on the right-hand side of the second line of (29) increases with X if  $\beta < 0$  ( $\sigma_x > 1$ ), falls with X if  $\beta > 0$  ( $\sigma_x < 1$ ), and remains constant with X in the special case of unitary elasticity of substitution (the Cobb-Douglas case). Turning to the first term on the right-hand side of (29), we note from (28) that this can be written as

$$(30) \quad \frac{dL_x}{L_x} / \frac{d\omega}{\omega} = \sigma_x \left[ \frac{dK_x}{K_x} / \frac{dL_x}{L_x} - 1 \right]^{-1}.$$

Total differentiation of  $\omega$  in (28) will show that

$$(31) \quad \frac{dK_x}{K_x} / \frac{dL_x}{L_x} = \frac{s+r(k_y/k_x)}{s+r} > 1 \quad \begin{aligned} s &= (\beta+1)(a/b) \\ r &= (\gamma+1)(c/d)(L_x/L_y)(k_y/k_x)^{\gamma} \end{aligned}$$

It can be shown that the value of (31) will fall with increases in X provided that  $k_y/k_x$  increases with X. It follows from (28) and a corresponding equation for Y that this will occur if and only if  $\sigma_x < \sigma_y$ . Given this restriction, (31) falls with X implying in turn that (30) falls with X. Given the discussion following equation (29), it then follows that  $\sigma_x \leq 1$  and  $\sigma_x < \sigma_y$  is sufficient for  $(dX/X)/(d\omega/\omega)$  to fall monotonically with X.

Finally, note with reference to the right-hand side of (27) that  $k_x/\omega$  decreases with  $X$  if and only if  $\sigma_x < 1$ . Similar comments apply to  $k_y/\omega$ . Thus sufficient conditions for the right-hand side of (27) to be non-decreasing in  $X$  are that  $\sigma_x \leq 1$  and  $\sigma_y \geq 1$ . Note that the right-hand side of (27) is constant when  $\sigma_x = \sigma_y = 1$ . In total, we have

$$(32) \quad \frac{dp}{p} / \frac{d\omega}{\omega} \text{ rises monotonically with } X \text{ if } \sigma_x \leq 1 \text{ and } \sigma_y \geq 1 .$$

Since Kemp has shown that (27) must be negative in the neighborhood of  $X=0$  (i.e., the production possibility curve must be convex in the neighborhood of  $X=0$ ), this implies that the production possibility curve has at most one inflection point and that it cannot be everywhere concave. More specifically, the Kemp result allows us to state unambiguously where the concave and convex segments of the production-possibility curve occur. When it does exist under the restrictions noted on  $\sigma_x$  and  $\sigma_y$ , the concave section necessarily occurs at high levels of  $X$  production.

What this amounts to is the fact that the elasticities of substitution play an important role in determining the relative strengths of the factor intensity and IRS effects. With  $\sigma_x$  small and  $\sigma_y$  large, the factor intensity effect becomes relatively stronger as the output of  $X$  increases. With  $\sigma_x$  large and/or  $\sigma_y$  small more complex outcomes can occur. We have for example generated by computer simulation the case in which there are convex segments at each end of the frontier with a concave segment in the middle.

The functional forms in (26) can be used to show that the IRS expansion condition of equation (10) is not sufficient for GFT in the presence of non-convexities as noted in Section V. Suppose that two goods,  $X_0$  and  $X_1$ , are

both characterized by IRS so that  $T_0, T_1 > 0$ . Assume that the production functions are given by

$$(33) \quad X_0 = (X_0^T)F(L_0, K_0), \quad X_1 = (X_1^T)F(L_1, K_1)$$

where the F's are identical so that there are no factor intensity effects and the contract curve is the diagonal of the factor box. The production frontier is clearly convex to the origin throughout. Let  $\alpha = F(\bar{L}, \bar{K}) = (\bar{X}_1 / \bar{X}_1^T)$  where  $\bar{L}$  and  $\bar{K}$  are the total endowments of L and K and  $\bar{X}_1$  is the maximum production of  $X_1$ . Finally, assume that the economy specializes in  $X_1$ . The results of Markusen and Melvin will show that the supply price ratio is given by

$(p_1/p_0 = X_0^T / X_1^T)$ . Thus, strictly speaking, the minimum value of  $p_1/p_0$  consistent with specialization in  $X_1$  approaches zero which would guarantee losses from trade. But lest we be accused of creating a pathological case by focussing on a local maximum, let  $X_0^T = 1$  so that  $(p_1/p_0) = (1/X_1^T)$  at  $X_1 = \bar{X}_1$ . For the parameter values used below, this supply price ratio will imply that  $p_0 < AC_0$  over most of the production frontier even if prices remained constant after moving away from  $\bar{X}_1$ . If demand is relatively inelastic,  $(X_1 = \bar{X}_1, X_0 = 0)$  will indeed be a global profit maximum at these prices. In any case, we have

$$(34) \quad p_1/p_0 = (1/\bar{X}_1^T) = (\alpha/\bar{X}_1) = (\alpha/\alpha^{T_1^*}) = \alpha^{1-T_1^*}$$

where  $T_1^* = 1/(1-T_1) > 1$ . The slope of the cord connecting the ends of the production frontier is on the other hand given by

$$(35) \quad (\bar{X}_0/\bar{X}_1) = \alpha^{T_0^* - T_1^*}.$$

Thus for  $\alpha > 1$ , we have

$$(36) \quad p_1/p_0 < (\bar{X}_0/\bar{X}_1) \quad \text{or} \quad (p_1/p_0)(\alpha^{T_0^*-1}) = (\bar{X}_0/\bar{X}_1).$$

The price ratio through  $\bar{X}_1$  lies below the cord  $\bar{X}_0/\bar{X}_1$  and must cut the production frontier, implying that GFT are not assured. (Note that with CRS in  $X_0$  ( $T_0^* = 1$ ), the price ratio equals the slope of the cord, a special result due to the absence of factor intensity effects. Positive factor intensity effects ensure that  $p^f$  exceeds the slope of the cord, as demonstrated in Section V for CRS in  $X_0$ .)

Let the relevant parameters take on the following values:  $\alpha = 1.5$ ;  $T_0 = .5$  (or  $T_0^* = 2$ );  $T_1 = .9$ . From the second equation in (36), we then have  $(p_1/p_0)(1.5) = (\bar{X}_0/\bar{X}_1)$ . Since  $p_i(1-T_i) \leq MC_i$  as noted above, the IRS expansion condition in (10) could be expressed as

$$(37) \quad (p_1/p_0)(T_1/T_0) \geq (\bar{X}_0/\bar{X}_1) \quad \text{since} \quad (\bar{X}_0/\bar{X}_1) > (X_0^a/(X_1 - X_1^a)).$$

Since  $T_1/T_0 = 1.8$ , we have

$$(38) \quad (p_1/p_0)(T_1/T_0) = (p_1/p_0)(1.8) > (p_1/p_0)(1.5) = (\bar{X}_0/\bar{X}_1).$$

Thus the IRS expansion condition holds, but the production inequality  $\sum p_i^f X_i^f \geq \sum p_i^f X_i^a$  does not follow. For at least some bundles  $X^a$  near  $X_1 = 0$  we must have  $\sum p_i^f X_i^f < \sum p_i^f X_i^a$ .

### VIII. Summary and Conclusions

The primary purpose of this paper was to look for a unified approach to the gains from trade in the presence of increasing returns that could be applicable across a wide range of models. These models include analyses based on (A) homogeneous goods with external economies, (B) homogeneous goods with internal economies, and (C) differentiated products produced in a setting of monopolistic competition. We argued that in all three cases a failure to realize GFT could occur for the same two reasons. First, in all cases, the IRS goods are priced above marginal cost and thus the price plane may cut the production surface at the free-trade production point. We referred to this as a failure of the "tangency" condition. Second, IRS may imply that the production set is non-convex and thus even if the tangency condition is satisfied, the price plane may not form a separating hyperplane to the production set. This was referred to as a failure of the convexity condition. The tangency condition relates to economic pricing behavior while the convexity condition relates to the structure of technology.

With respect to the tangency condition, results (generally well known) show that losses from trade may occur if trade contracts the IRS industries. The intuition is fairly straightforward. With prices greater than marginal costs in autarky, the economy is already under-producing the IRS goods. If trade reduces production further, the economy may be moving away rather than towards its optimal production mix. We suggested that a sufficient condition for GFT is that trade have a certain rationalizing effect on production. This is a rather crude notion to the effect that surviving industries expand output more than in proportion to the number of IRS industries lost due to the opening of trade. We argued that this is in fact a reasonable outcome

(although hard to pin down rigorously) provided that trade does not decrease the total resources devoted to the IRS industries.

Non-convexities present a more difficult problem. On the one hand, we are able to show that the same expansion of all IRS industries that is sufficient for GFT in the convex case continues to be sufficient in the presence of non-convexities. Further, this result does not rely on restrictive functional forms, specialization in production, or on average-cost pricing in the IRS industries. On the other hand, the weighted increase in the outputs of the IRS industries that is sufficient in the convex case is no longer sufficient with non-convexities. We are thus still without a sufficient condition for GFT in the realistic case in which trade expands some IRS industries and contracts others. Further work which exploits profit restrictions, stability conditions, and restrictions on oligopolistic behavior may help produce such conditions.

Section V emphasized the applicability of these results to recent monopolistic competition models and noted how some of the very restrictive assumptions used in that literature imply that both the tangency condition and the convexity condition are satisfied. These assumptions include (A) symmetry assumptions which ensure that price ratios equal ratios of marginal costs, (B) free entry that results in average-cost pricing. Together these assumptions imply that the price plane will not cut the convex hull of the production set (i.e., the price plane is a separating hyperplane).

Section VI showed how the tangency condition can be expressed in terms of profits in addition to the more usual formulation in terms of output levels. This turns out to be quite useful in analyzing the gains from trade in the presence of multinational enterprises and allows the GFT condition to be expressed in terms of restrictions on profit repatriation by the MNE.

Footnotes

<sup>1</sup>For treatments of external economies, see for example Jones (1968), Herberg and Kemp (1969), Melvin (1969), Kemp (1969), Kemp and Negishi (1970), Eaton and Panagariya (1979), Markusen and Melvin (1981), and Panagariya (1981). Markusen (1981) treats internal economies with homogeneous products. As noted, see Helpman (1982) for an excellent survey of these contributions. One type of external economy which will not be treated here is "international external economies" (Ethier (1979)) in which returns to scale depend on the total world output of a good.

<sup>2</sup>Recent monopolistic competition/product differentiation models include Helpman (1981), Krugman (1979, 1980), Lancaster (1980), and Ethier (1982).

<sup>3</sup>These assumptions as discussed below include specific functional forms for production and utility functions, symmetry assumptions, free entry, average-cost pricing and so forth.

<sup>4</sup>Herberg and Kemp (1969) show that for a special type of technology, the production frontier must be convex in the neighborhood of zero production of an IRS good. This technology relies on a separable externality effect and "Heckscher-Ohlin" technology (all factors used in the production of each good). This convex segment need not occur for example with a specific factors technology.

<sup>5</sup>This criterion makes more sense in the formulations of Dixit and Stiglitz (1977) and Krugman (1979) in which consumers are identical than in the formulation of Lancaster (1979, 1980) in which consumers are heterogeneous (also used by Helpman (1981)). More work needs to be done on the welfare properties of this latter approach.

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