RESILIENT INFRASTRUCTURE
June 1–4, 2016

THE USE OF ULTRASONIC WAVES AND ANALYTICAL MODELING TO ESTIMATE ELASTICITY MODULUS OF RUBBER CONCRETE SPECIMEN

Hadda Hadjab
University of Sciences & Technology Houari Boumediene, Algeria

Ahmed Arbia
University of Sciences & Technology Houari Boumediene, Algeria

Oussama Boulekfouf
University of Sciences & Technology Houari Boumediene, Algeria

ABSTRACT

This research work aims to evaluate the influence of the addition of rubber aggregates on elasticity modulus experimentally by using ultrasonic waves and theoretically by analytical models. Based on ultrasonic waves in concrete specimen at different percentages of rubber granules, one can evaluate the static modulus (Estatic) from the dynamic elastic modulus (Edynamic), according to British code [7]. From the obtained experimental results, one can conclude that the use of rubber granules has the potential for vibration damping capacity. In other words, the rubber granules, reduces the kinetics of ultrasonic pulses in the material. This reduction is due to the decreasing density of rubber granules (RG), with respect to gravel. The concrete base of its aggregates can be used such as paving of vibrating tools. Analytical modeling (Hill and BHS models) is used. The analytically obtained results converge with those from experimental procedure and give a good agreement to other researcher's works.

Keywords: concrete, aggregates, rubber, modulus of elasticity, ultrasonic waves

1. INTRODUCTION

The elastic modulus is a very important property for all materials. In the field of civil engineering evaluation of the elastic modulus of concrete is to determine the scope of its use (rigid pavements, retaining walls, structural elements ...). In our work we use a concrete aggregate base rubber, which has characteristics very promising in terms of reducing the kinetic ability of ultrasonic pulses through its decreased elastic modulus. The determination of the elastic modulus can be achieved by experimental methods and by analytical methods. The analytical methods are based on the use of different models, such as selected in this work as the "terminals" of Hill [1] and the bounds of Hashin-Shtrikman (BHS) [2]. These models will be used for a comparative study with the results found by the experimental method which is based on the evaluation of the modulus of elasticity from the ultrasonic test. The experimental procedure is a non-destructive technique to measure the travel time of the ultrasonic wave through the material.

2. MATERIALS & METHODS

2.1 Experimental Aspect

2.1.1 Principles of ultrasonic testing

The principle of ultrasonic testing is that an electrical signal is converted into a strain wave by a piezoelectric transducer. That wave propagates through a concrete specimen and is captured by the receiving transducer. The propagation time and the speed of the ultrasonic waves are deducted (see Fig. 1).

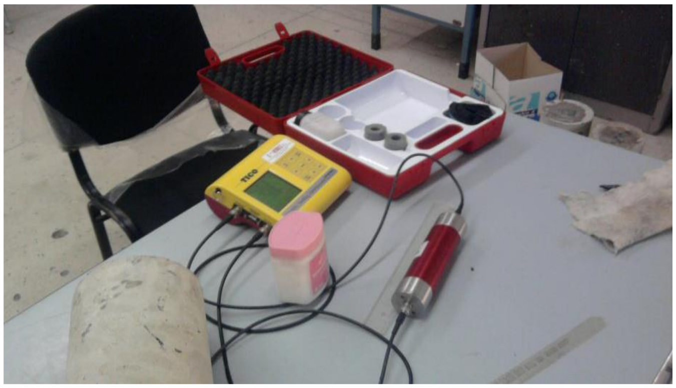

Fig.1. Ultrasonic test

2.2. Calculation Method

Due to the heterogeneity of the material, the interpretation of ultrasonic signals is not easy. Typically, tests are often based on the measurement of velocities of longitudinal ultrasonic waves (see Fig. 2). The speed of the wave through an elastic solid is given by the following expression [3]:

[1] $$V = \sqrt{\frac{E(1-\upsilon)}{\rho(1+\upsilon)(1-2\upsilon)}}$$

The Young's modulus (dynamic) E can be expressed as a function of V, and υ using the following formula:

[2] $$E = \frac{(1+\upsilon)(1-2\upsilon)}{1-\upsilon}\rho V^2$$

V: wave velocity measured in km/s.
E: modulus of elasticity in GPa.
υ: Poisson's ratio.

Fig.2. Ultrasonic measurements

2.3 Composition of different concretes

The rubber granules used have a size identical to that of gravel 3/8 so we replaced different percentages of Gravel 3/8 by rubber granules (25, 50 and 100%). We have four (04) concrete compositions:

* **Reference Concrete RC**: which has the following composition:

Table 1: Composition of the Reference Concrete

CONSTITUENTS	MASS (kg)
Sand	644
Gravel 3/8	206.7
Gravel 8/15	1004.6
Cement	400
Water	221

***Concrete including 25% of rubber granules C25RG**: with the following composition:

Table 2: Composition of C25RG

CONSTITUENTS	MASS (kg)
Sand	644
Gravel 3/8	155.02
Gravel 8/15	1004.6
Rubber Granules	51.67
Cement	400
Water	188
Adjuvant	4

***Concrete including 50% of rubber granules C50RG:** with the following composition:

Table 3: Composition of C50RG

CONSTITUENTS	MASS (kg)
Sand	644
Gravel 3/8	103.35
Gravel 8/15	1004.6
Rubber Granules	103.35
Cement	400
Water	188
Adjuvant	4

***Concrete including 100% of rubber granules C100RG:** with the following composition:

Table 4: Composition of C100RG

CONSTITUENTS	MASS (kg)
Sand	644
Gravel 3/8	0
Gravel 8/15	1004.6
Rubber Granules	206.7
Cement	400
Water	188
Adjuvant	4

3. ANALYTICAL MODELLING

3.1. Hill Model

Hill [1] considers a material with 2 phases: the inclusion and the matrix. To supervise the actual properties of these materials, it offers two models: parallel and series as shown in Fig. 3, which correspond to the "terminal" top and bottom respectively. Terminals Hill are best known for their simplicity and are also at the origin of models classified as parallel-series models with many applications for concrete.

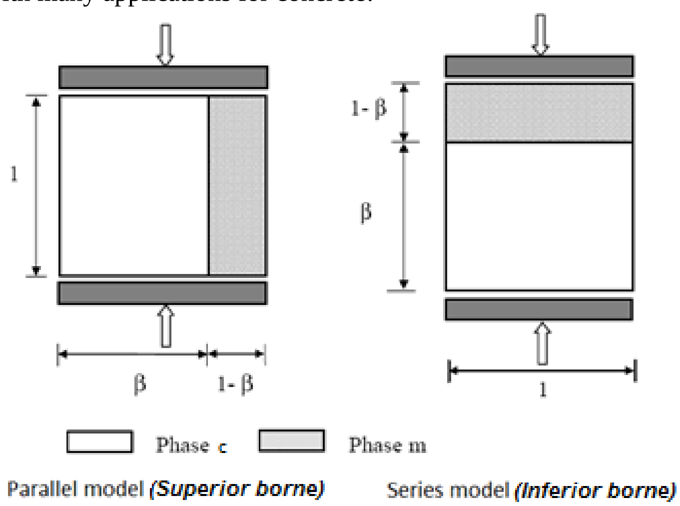

Fig.3. Schematic representation of Hill model [1]

We consider Ec and Em modulus of the two phases ''c'' (rubber) and ''m'' (matrix) respectively:

[3] Superior borne SB: $E_{sup}=\beta E_c+(1-\beta)E_m$

[4] Inferior borne IB: $\frac{1}{E_{inf}}=\frac{1}{E_C}+\frac{1-\beta}{E_m}$

These models can be adapted to predict the values wrapped in the elastic modulus of composite materials.
If we consider Vc, Ec and Vm, Em, as the volume fractions and elastic modulus of the two phases Rubber granules and Cement Matrix, we therefore proposed the following expressions by Hill [1]:

[5] SB: $E_{sup}=E_mV_m+E_cV_c$

[6] IB: $\frac{1}{E_{inf}}=\frac{V_m}{E_m}+\frac{V_c}{E_c}$

3. 2. Hashin-Shtrikman Model (BHS)

Hashin and Shtrikman [2] proposed limits established for a mixture of "n" isotropic elastic components, without any particular assumption about their form or their volume concentration. On concrete incorporating RG, if Kr, Km, Gr, Gm, Vr, Vm are the bulk modules compressibility, shear moduli and volume fractions of each phase and rubber matrix, K_{inf} and and K_{sup} (the lower and upper bounds of the modulus) and Gsup and Ginf (terminals of the shear modulus) of the composite can be put in the form of the following equation [4]:

[12] $$K_{inf} = K_r + \frac{Vm}{\frac{1}{Km - Kr} + \frac{3Vr}{3Kr + 4\,Gr}}$$

[13] $$K_{sup} = K_m + \frac{Vr}{\frac{1}{Kr - Km} + \frac{3Vm}{3Km + 4\,Gm}}$$

[14] $$G_{inf} = G_r + \frac{Vr}{\frac{1}{Gm - Gr} + \frac{6(Kr + 2Gr)Vr}{5Gr\,(3Kr + 4\,Gr)}}$$

[15] $$G_{sup} = G_m + \frac{Vm}{\frac{1}{Gr - Gm} + \frac{6(Km + 2Gm)Vm}{5Gm\,(3Km + 4\,Gm)}}$$

[16] We note that: $$K_{r,m} = \frac{E_{r,m}}{3(1 - 2\upsilon_{r,m})} \quad \text{et} \quad G_{r,m} = \frac{E_{r,m}}{2(1 + \upsilon_{r,m})}$$

Where $\nu_{r,m}$ and Er, m are the Poisson's ratios and modulus of elasticity of each phase, i.e. rubber and matrix respectively.

So we can express the modulus of elasticity as a function of the shear modulus G and the bulk modulus K according to the relation below:

[17] $$E = \frac{9KG}{3K + G}$$

And the relationship between the upper bound, lower elastic modulus and the experimental result is as follows:

[18] $$BHS_{inf} = \frac{9K_{inf}\,G_{inf}}{3K_{inf} + G_{inf}} \leq E_{exp} \leq \frac{9K_{sup}\,G_{sup}}{3K_{sup} + G_{sup}} = BHS_{sup}$$

4. RESULTS AND INTERPRETATIONS

4.1 Experimental Results:

After measurement of ultrasonic velocities, we use the formula (2) to evaluate the results of the dynamic modulus of elasticity of each concrete composition and which are illustrated in Table 5 and Fig. 4 given below:

Table 5: Variation of elasticity modulus versus rubber granules percentage

Concrete composition	RC	C25RG	C50RG	C100RG
Elasticity Modulus (GPa)	37.26	32.77	29.42	15.22

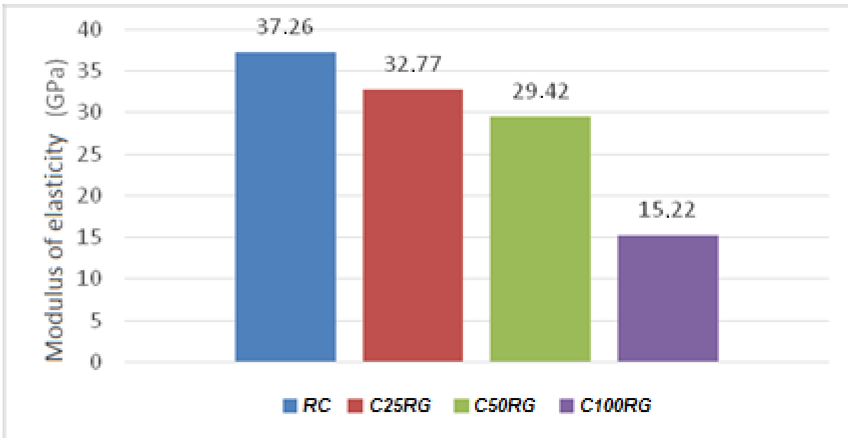

Fig.4. Influence of rubber aggregates on elasticity modulus

It can be seen from the graphic that the addition of rubber granules induced a significant drop in modulus of about 59% for concrete C100 RG, 21% for concrete C50RG and around 12% for concrete C25RG compared to the reference concrete. Many studies, such as those Güneyisi [5] Ganjian [6] and Cuong [4] confirmed that the incorporation of RG induced a significant drop in modulus of elasticity. Note that this drop is mainly due to the low stiffness of the RG, the poor quality of the transition zone between the rubber and the cement matrix and the porosity of the concrete. In order to proceed to the analytical modeling of the dynamic modulus of elasticity and the static modulus of elasticity and basing on the British Code [7] and the work of Lydon and Balendran [8], a linear correlation between the Young's modulus static and dynamic was performed. This correlation is limited to a compressive concrete resistance less than 40 MPa concrete. The following formula defines the relationship between Estatic and Edynamic:

[19] $E_{static} = 0.83\ E_{dynamic}$

The results are summarized and given in the following Table 6.

Table 6: Values of Estatic for different compositions of concrete

Concrete composition	RC	C25RG	C50RG	C100RG
$E_{dynamic}$ (GPa)	37.26	32.77	29.42	15.22
E_{static} (GPa)	30.92	27.19	24.41	12.63

4. 2. Results of Analytical Modeling

In this part we present only the modeling results for two models, i.e. the models according to HILL and the Hashin-Shtrikman terminals.

4.2.1. Hill Model

Using eqs. (5) and (6) we can calculate the lower and upper bounds and compare them with the experimental results. The modulus of elasticity of the vulcanized rubber is chosen to be 1.75 GPa.
The results are shown in Table 7 and Fig.5 below:

Table 7:.Modulus of elasticity: comparison of experimental results and the model of Hill

Type of concrete	V_c(%)	V_m(%)	Modulus of elasticity (GPa)		
			IB	SB	Experimental Results
RC	0	100	30.92	30.92	30.92
C25RG	9	91	12.36	28.29	27.17
C50RG	19.5	80.5	7.27	25.23	24.41
C100RG	39	61	4.13	19.53	12.63

The above figure shows that the experimental curve is between the two upper and lower bounds of the Hill model agrees best with the results given in relation to the upper bounds. The Hill model therefore allows describing the variation of the elastic modulus as a function of experimental substitution rate in RG. This statement is more accurate for a percentage of the aggregate less than 50% according to the graph in other words assays RG volume less than 20%.

4.2.2. The Terminals Hashin-Shtrikman (BHS)

Using eqs. (12), (13), (14), (15), (16), (17) and (18) for three concrete compositions the Reference concrete, concrete with a 50% RG and concrete with 100% RG, we find the results summarized in Table 8 below:

Table 8:.Bounds of Hashin-Shtrikman model results

Concrete	Vr	Vm	Kr	Km	Gr	Gm	Kinf	Ksup	Ginf	Gsup	BHSinf	BHSsup	Eexp
RC	0	100	14.583	18.405	0.591	12.672	396.726	18.405	1208.683	12.672	30.920	30.920	30.920
C50RG	19.5	80.5	14.583	18.405	0.591	12.672	67.189	28.064	6.504	19.032	18.903	46.568	24.410
C100RG	39	61	14.583	18.405	0.591	12.672	36.378	45.002	2.838	29.603	8.299	72.837	12.630

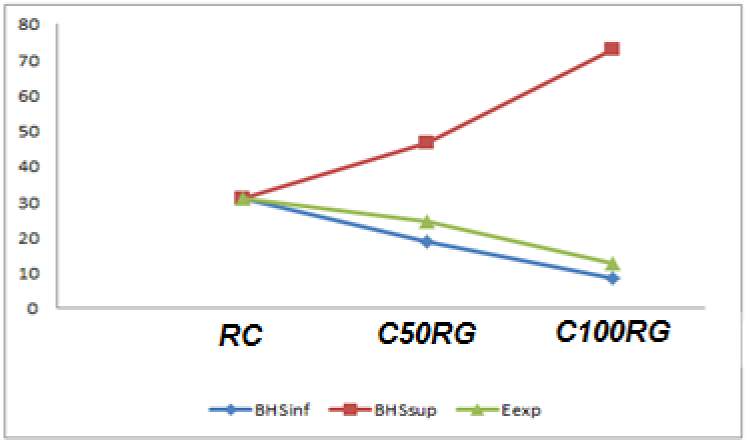

Fig.5. Modulus of elasticity, comparison of the experimental results and analytical model

It can be seen that the lower Hashin-Shtrikman agreement is better with the experimental results with a small difference in contrast to the upper bound. We can say therefore that the BHS yield very significant results since they integrate at the same time the bulk modulus, shear modulus and Poisson's ratio of two constituent phases of the composite.

5. CONCLUSIONS

In this work we discussed and compared the results of two analytical methods (model BHS HILL and terminals) to calculate the elastic modulus of concrete containing rubber granules from the experimental results in order to have a reliable prediction of this important feature. The HILL model which considers the material in two phases is one of the important tools to predict the modulus of elasticity of concretes and those incorporating rubber granules. This modeling approach is characterized by its simplicity and its appearance affordable based on physical characteristics and the volume fractions of each phase, but it is far from perfect. The lower Hashin-Shtrikman bound gives results much better and more realistic because this method incorporates bulk modulus, shear and Poisson's ratio of RG phases and cement matrix and therefore we can say that it can be used as a tool for prediction of the elastic modulus.

REFERENCES

[1] Hill R., (1963) Elastic proprieties of reinforced solids : some theorical priciples. J. Mech.Phys. Solids, 11, pp 357-372.

[2] Hashin Z. and Shtrikman S., (1963) A variational approache to the theory of the elastic behaviour of multiphase materials, J. of Mech. Phys. Solids, 11, pp.127-140

[3] Zdiri M (2007) Prediction and measurement of the elastic module of the RCC: Case of the low cement proportioning, International Journal of Physical Sciences.

[4] Cuong H A (2010) Doctoral thesis « Optimisation de la composition et caractérisation d’un béton incorporant des granulats issus du broyage de pneus usagés : Application aux éléments de grande surface », PhD Dissertation University of TOULOUSE, France.

[5] Güneyisi E., Gesoğlu M and Özturan T (2004) Properties of rubberized concretes containing silica fume.

[6] Ganjian E., Khorami M and Maghsoudi A. A (2009) Scrap-tyre-rubber replacement for aggregate and filler in concrete.

[7] British Standard Institution (1985). British code of concrete structure. London, UK.

[8] Lydon, F and Balendran, R (1986) Some observations on elastic properties of plain concrete. Cement and concrete research.