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# Count on Diversity: The Cognitive and Mathematical Profiles of Children in Early Elementary School

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by

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Abstract

The present study investigated the diverse cognitive profiles of children learning mathematics in early elementary school. Unlike other types of learning difficulties, mathematics impairments are not characterized by a single underlying cognitive deficit, instead multiple general and numeracy-specific cognitive skills have been proposed to underlie mathematics ability. Combining theory- and data-driven approaches, the study investigated cognitive mathematics profiles. Participants for this study were 97 children tracked longitudinally from senior kindergarten to Grade 2, as part of the Count Me In Study. Using numeracy, working memory, receptive language, and phonological awareness factors, a two-step cluster analysis revealed a three-cluster solution. The groups were characterized as (1) good-all-around (above average overall), (2) weak spatial system (average overall with weak visuospatial working memory), (3) multiple weaknesses (poor overall with strong visuospatial working memory). The good-all-around cluster demonstrated strengths in mathematics, reading, and nonverbal reasoning compared to the weak spatial system cluster and the multiple weaknesses cluster. Developmental trends and potential interventions are discussed.

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The Cognitive and Mathematical Profiles of Children in Early Elementary School

What makes some children better at math than others? At the far end of the ability spectrum, there is specific learning disability – mathematics, characterized by an individual having significantly lower math performance than their general performance predicts (Mash & Wolfe, 2013). Mathematics disability is characterized by core deficits, enumerating sets, comparing quantities, and other features (Butterworth & Reigosa, 2008). The reading comparison of mathematics disability, specific learning disability – reading, is connected to a core deficit in phonological processing. Interventions for reading disability can target this core deficit and improve reading ability (Mash & Wolfe, 2013). There is no agreed upon core deficit in mathematics disability. Instead, several underlying factors have been investigated, suggesting the existence of math disability subtypes (Szucs, Devine, Soltesz, Nobes, & Gabriel, 2013) and distinct pathways to mathematical success (Leferve et al., 2010; Sowinski et al., 2015). Theoretical approaches describe mathematics ability with domain-general and domain-specific explanations. Domain-general explanations involve individuals' skill in more general cognitive structures (e.g., visuospatial working memory, working memory). Domain-specific explanations emphasize underlying numeracy abilities (e.g., subitizing and magnitude representation; Butterworth & Reigosa, 2008).

These theoretical frameworks have led to the suggestion of mathematics difficulty subtypes (e.g., McCloskey et al., 1985; Temple, 1997) and the investigation of subtypes through data-driven approaches (Archibald, Cardy, Joanisse, & Ansari, 2013; Bartelet, Ansari, Vaessen, & Blomert 2014). The present study combined these approaches to explore the cognitive profiles of elementary school children, tracked longitudinally from senior kindergarten (SK) to Grade 2, learning mathematics.

Domain-specific explanations of mathematics ability and difficulty implement specific numeracy abilities proposed to underlie mathematics ability, including comparison tasks, subitizing, estimation, and neuroabnormalities in numeracy-dominated brain areas. In subitizing tasks, individuals enumerate sets of dots as quickly as possible without counting. Typically, children can subitize (enumerate without counting) three or four dots. Children with mathematics impairments typically show greater increases in response time (RT) as the size of the set increases. Children with typical mathematics skill usually display similar RTs for sets of 1-3 dots. Performance on subitizing tasks has been proposed as a key discriminator of math ability (Penner-Wilger et al., 2007). Among children with poor mathematics skill, subitizing slopes are much steeper (Landerl, 2013).

In contrast, domain-general explanations involve processes not specific to mathematics (e.g., executive control, language systems, the visuospatial system, Butterworth & Reigosa, 2008; Szucs, Devine, Soltesz, Nobes, & Gabriel, 2013). Examining children with “pure developmental dyscalculia” (children with a mathematics disability and without a comorbid disability), Szucs et al. (2013) contrasted five theories of developmental dyscalculia (magnitude representation, working memory, inhibition, attention and spatial processing). Using a variety of math-specific and general cognitive measures, the researchers supported deficits in working memory, inhibition, attention, and spatial processing, but did not find support for deficits in magnitude representation – which has been the dominate domain-specific explanation of mathematics difficulty (De Smedt & Gilmore, 2011; Piazza et al., 2010; Rouselle & Noel, 2007).

Neuroimaging data supports Szucs et al.'s (2013) assertion (Davis et al., 2009). Using functional magnetic resonance imaging (fMRI), Davis et al. (2009) illustrated neural activation differences in spatial working memory areas, not in magnitude representation areas, among children with mathematics difficulties, compared to matched children. Additionally, Davis et al. (2009) suggests children with mathematics learning difficulties use developmentally immature strategies to solve mathematics problems, compared to their non-impaired peers, as a result of their spatial memory deficits, which may lead to slower RTs in arithmetic fluency tasks. These results suggest the role of working memory in mathematic ability (Davis et al., 2009; Szucs et al., 2013).

Investigating children of all math performance levels, LeFevre et al. (2010) tested a model of associations between early cognitive precursors, numeracy skill, and math outcomes. This model identifies three pathways that precede math ability: quantitative (numeracy), linguistic (receptive vocabulary and phonological awareness), and spatial attention (visuospatial working memory). These pathways contribute independently to numeracy skills during the early years of formal education and are differently related to performance on many math outcome measures. Each of the pathways were related to performance on numeration and calculation ability, as well as symbolic number line estimation, but the spatial pathway was not involved in magnitude comparison. Least surprisingly, the linguistic pathway was the only pathway to account for variability in word reading, but more surprisingly, it was the only pathway to be involved in all mathematics outcomes. This research indicates the diversity present in math performance and suggests that an individual may compensate for weaknesses in one area of performance with strengths in other pathways. This research also highlights the importance of considering the role of linguistic skill in math performance, as indicated by the relation of the linguistic pathway to each of the math outcomes.

Recently, Sowinski et al. (2015) revised the Pathways Model (LeFevre et al., 2010) by considering more quantitative measures. In the refined model, only the quantitative and linguistic pathways, and not the working memory pathway, accounted for unique variance in calculation and number knowledge, suggesting the contribution of pathways depend on the cognitive task.

Data-driven approaches have used cluster analysis to investigate the cognitive profiles of children with mathematics difficulties in cross-sectional designs. Bartelet et al. (2014) used a variety of math-specific and general cognitive measures and identified six profiles of mathematics difficulty: (1) weak mental number line (poor number line task performance), (2) weak approximate number system (poor non-symbolic performance), (3) spatial difficulties (poor spatial working memory), (4) access deficit (poor symbolic knowledge and counting skills), (5) no numerical cognitive deficit (strong verbal working memory skills without concurrent deficits in numeracy measures), (6) garden variety (many numeracy and general cognitive deficits). These profiles are similar to many of the subtypes suggested by theory-driven research. Archibald, et al. (2013) investigated the cognitive profiles of children with language, reading, and math learning difficulties using a large epidemiological sample. Children were given a battery of standardized tests measuring language, reading fluency, phonological awareness, general intelligence, working memory, and arithmetic ability. Archibald et al., (2013)'s profiles were characterized by: (1) below average across most measures, (2) below average sentence recall (3) below average reading efficacy, (4) below average math and reading, (5) below average math fluency, (6) and above average overall. The math impairment group displayed high performances in general intelligence, despite arithmetic weaknesses. Since Archibald et al. (2013) did not include wider range of general (e.g., processing speed, nonverbal reasoning) and math-specific (e.g., subitizing, estimation) variables, these numeracy and cognitive skills within these profiles cannot be evaluated. However, Archibald et al. (2013) identified comorbidity between reading and mathematics difficulty, and the absence of comorbidity between mathematics difficulty and specific language impairments, suggesting the possible contribution of low reading skill to mathematics difficulty.

Mathematics ability cannot be explained by a singular factor. Rather, independent pathways to mathematics success exist along with distinct profiles of mathematics ability characterized by a range of deficits in numeracy and general cognitive abilities. Building from previous theoretical and data-driven approaches, the present study investigates the cognitive profiles of children by following clusters of children longitudinally from SK to Grade 2 and evaluating their performance on mathematical, reading and general cognitive measures. It is proposed that differences in mathematical ability between clusters will exist in mathematical, reading, and some general cognitive measures, but will not exist for processing speed or general working memory ability, based on the assertions of previous theoretical models and the results of data driven approaches.

# Methods

## The Count Me In Project

All data for the experimental studies reported in this study were collected as part of the Count Me In longitudinal project. The Count Me In project was a four-year research project investigating the precursors of mathematical skill in primary school children, and was funded by the Canadian Social Sciences and Humanities Research Council (SSHRC). Canadian children from eight schools in three cities: Ottawa, and Peterborough, Ontario and Winnipeg, Manitoba, participated in the Count Me In project annually from 2004 to 2007 ($N$ = 456). The project design was both longitudinal and cross-sectional, with two cohorts. Cohort 2, used for the present study, started in 2005 (students in Junior Kindergarten, Senior Kindergarten (SK), or Grade 1) and continued for three years. The present study tracked students beginning in SK and ending in Grade 2.

## Participants

Participants were 97 children (51 male) tracked longitudinally over three years from SK to Grade 2. In SK (2005), children were, on average, 5 years and 11 months old ($SD$ = 3.98 months, range = 18 months). In Grade 1 (2006), children were, on average, 6 years and 10 months old ($SD$ = 3.36 months, range = 13 months). In Grade 2 (2007), children were, on average, 7 years and 10 months old ($SD$ = 3.33 months, range = 13 months). Sixty-seven children were educated in Winnipeg, 27 in Ottawa, and three in Peterborough. Of the children involved in the present study, 95.9% spoke at least some English at home and 4.1% spoke only French at home. Sixty-eight percent of the children identified as Anglophone (English-first language), 22.7% as allophone (language other than French or English as first language), and 9.3% as bilingual (learned French and English at the same time). Thirty-six and one tenth of children involved in the study had at least one parent with a university degree, 23.7% had a parent with a college degree, 21.6% had a parent with a graduate or professional degree, 15.5% had a parent with only a high school diploma, and 2.1% had a parent with less than a high school education. Children who participated in the present study were largely right handed (91.8%), as identified by their preferred writing hand.

## Materials

**Screening variables (senior kindergarten [SK]).** Subitizing, visuospatial working memory, phonological awareness, and receptive vocabulary were used in the creation of clusters.

***Subitizing.*** Children's ability to enumerate sets without counting was measured using a subitizing task. In this task, 1-6 dots are displayed on a computer screen. Three trials were presented for each dot array, for a total of 18 trials. The dots for each trial were displayed in pseudo-random arrangements. Subitizing slopes were computed using the median (RTs) for 1-3 dots and the best fitting regression line was calculated for each child. This RT slope was used as the measure of subitizing. A higher slope suggests the child is counting the three dot display while a lower slope suggests that the child is subitizing the dot display.

***Visuospatial Working Memory.*** A computerized variant of the Corsi Block task was used to measure visual-spatial working memory. In this task, children viewed a frog jumping in sequence from one lily pad to another and included nine lily pads dispersed on the laptop screen. Once the frog finished its sequence, the child was asked to repeat the pattern by clicking on the appropriate lily pads in order (DeStefano & LeFevre, 2003; $\alpha_c = .699$, $N = 191$). Children completed one practice trial and 12 experimental trials, with the length of the span ranging from 2 to 7. Children's maximum span was used as the measure of visuospatial working memory.

***Phonological Awareness.*** Phonological awareness was measured using the Elision subtest of the Comprehensive Test of Phonological Processing (CTOPP; Wagner, Torgesen, Rashotte, 1999).

In this task, children must identify phonemes in words. Children are read a word and are asked to repeat it but without a certain phoneme (e.g. brat omitting the /r/). Cronbach's alpha was .90 for children in SK. Children's Elision grade standardized scores from SK ($M$ = 10, $SD$ = 3) were used as the measure of phonological awareness.

***Receptive Language.*** The Peabody Picture Vocabulary Test – Revised – Form B (PPVT; Dunn & Dunn, 1997) was used to measure receptive language. Children were shown a set of four pictures and chose the picture that corresponded with a verbally presented vocabulary word. The words increased in relative difficulty as the test progressed. The task was terminated after the child made six errors in eight consecutive questions. The split-half reliability of the PPVT is .95 (Dunn & Dunn, 1997). Due to the high performance level of the children participating in Count Me In, the starting set for the PPVT was raised to one set higher than suggested by Dunn and Dunn (1997). PPVT scores in SK, standardized by grade ($M$ = 100, $SD$ = 15), were used as the measure of receptive language.

**Evaluation variables (Grades 1 & 2).** Mathematical achievement, arithmetic fluency and ability, reading skill, nonverbal reasoning, phonological working memory, and processing speed were used to evaluate the created clusters.

***Mathematics Achievement (KeyMath Numeration).*** The Numeration subtest of the KeyMath Test-Revised (Connolly, 2000) covers concepts including quantity, order, and place value. Most of the items in the range for these children require knowledge of the symbolic number system. The reported alternate-form reliability coefficient for the grade-scaled Numeration subtest is .75 and the split-half reliability is .81 (Connolly, 2000). Raw Numeration scores from Grade 1 and 2 were used as measures of mathematical achievement.

***Mathematics Achievement (Woodcock-Johnson Calculation).*** The Calculation subtest of the Woodcock-Johnson Tests of Achievement (WJ-Math; Woodcock & Johnson, 1989) covers mathematical problems that increase in difficulty. This calculation measure involves all four operations (addition, subtraction, multiplication, and division), although most of the questions that were attempted by the children in the present study involved addition or subtraction only. The test was stopped once the child made six sequential errors or when they believed they could not answer any more questions. This test has a median reliability of .85 and a one-year test-retest correlation of .89 (Woodcock & Johnson, 1989). Children's Grade 1 and 2 raw scores were used as measures of mathematical achievement.

***Arithmetic Ability and Fluency.*** Children's arithmetic fluency and ability was measured in a single digit addition task. Children were instructed to sum single digit addends as quickly as possible without making many errors. All sums were less than ten. This task was stopped if the child made five sequential errors and timed out if the child did not respond within 20 seconds. Trials were initiated by the child pressing the "GO" button. Children provided their answers verbally. When the child spoke, the experimenter stopped the timer and recorded the child's response. Children's Grade 1 and 2 median RT (response time in milliseconds) and accuracy (percent correct) were used as measures of arithmetic fluency and ability, respectively.

***Reading Skill.*** The Word Identification subtest of the Woodcock Reading Mastery Test – Revised/ Normative Update, Form G (WJ-Reading; Woodcock, 1998) was used to assess reading skill. Children were shown a set of words (e.g., cat) and were asked to read each word. The words increased in relative difficulty as the test progressed. The test was terminated when the child made six consecutive errors, including errors of pronunciation. Children's Grade 1 Word

Identification Scores ($M$ = 100, $SD$ = 15) were used as the measure of reading skill. Reading measures were not collected in Grade 2.

***Nonverbal Reasoning.*** The analogy subtest of the Cognitive Intelligence Test (CIT; Gardner, 1990) was used to assess children's nonverbal reasoning. Children were presented with a pattern in blocks where one block was missing. Children were asked to select the correct response from a set of possible solutions arranged across the bottom of the page. The task was stopped after six consecutive errors. Using the KR-20, the total reliability of the CIT was determined to be .90. Children's standardized Grade 2 scores were used as the measure of nonverbal reasoning. Non-verbal reasoning measures were not collected in Grade 1.

***Phonological Working Memory***. Children completed a reverse digit-span task as a measure of working memory. Children were asked to recall a series of spoken digits presented by an on-screen dog character. Children repeated the numbers in the reverse order to which they were presented (14 trials; Orsini et al., 1987). Since children must hold and manipulate the numbers, rather than simply storing the numbers in this task, this task is considered to be a measure of phonological working memory. Children were given two trials for each set size. Digit lengths started at two and increased in length until the child was inaccurate for both trials of a certain length. Participants' Grade 1 maximum reverse span was used as the measure of phonological working memory. Phonological working memory measures were not collected in Grade 2.

***Processing Speed.*** Children completed a simple choice reaction time task as a measure of processing speed. Children were presented with one of two types of stimuli (X or O). The stimuli were displayed for one second, preceded by a half-second fixation point. Children then had to press the corresponding key as quickly as possible without making an error. The child positioned the index finger of their dominant hand on the keyboard key with an 'X' sticker (the 'X' key) and their middle finger on the key above it labeled 'O' (the 'D' key).  Left-handed children used similar stickers on the right side of the keyboard. Children completed 24 trials in one minute. Children’s mean RT from SK to Grade 2 was used as the measure of processing speed.

## Procedure

Parental consent was attained for all children who participated in the study. Ethical approval was gained from the Carleton, Trent, and Winnipeg University Ethics Review Boards and permission was granted at each school by the respective principals and school boards in Peterborough and Ottawa, Ontario, and in Winnipeg, Manitoba. In May of each year (Kindergarten – Grade 2), children were tested by trained research assistants. Computer tasks and pencil-and-paper tasks were given in two separate sessions lasting between 15 and 30 minutes, with sessions extended as necessary. The order of tasks was consistent each year. In the first session, children completed the subitizing, arithmetic fluency, Corsi, processing speed, and digit span measures. During the second session, children completed the CIT, KeyMath, WJ-Math, PPVT, CTOPP, and WJ-Reading measures. Each year, children were engaged in approximately one hour of testing time.

# Results

## Cluster Analysis

Four screening variables (subitizing, visuospatial working memory, phonological awareness, and receptive language) were entered into a two-step cluster analysis, with the log-likelihood as the distancing measure and Schwarz’s Bayesian Criterion (BIC) as the clustering criterion. The two-step cluster analysis revealed a three-factor solution with an average silhouette statistic of 0.4. Descriptive statistics for the clusters are reported in Table 1. The clusters were characterized by (1) good-all-around (above average overall), (2) weak spatial system (average overall with weak visuospatial working memory), and (3) multiple weaknesses (poor overall with relatively strong visuospatial working memory). More specifically, the good-all-around displayed average subitizing (approximately at median), very strong visuospatial working memory (cluster mean at 75th percentile of sample), very strong phonological awareness (cluster mean at 75th percentile of sample), and strong receptive language. The weak spatial system cluster showed very weak visuospatial working memory (cluster mean at 25th percentile of sample), average phonological awareness (cluster mean at sample median), slightly below average receptive language, and average subitizing. The multiple weaknesses cluster was characterized by very strong visuospatial working memory (cluster mean at 75th percentile of sample), very weak phonological processing (cluster mean below the 25th percentile of sample), very weak receptive language (cluster mean below the 25th percentile of sample), and slightly below average subitizing. Figure 1 illustrates the z-score distributions screening variables within each cluster.

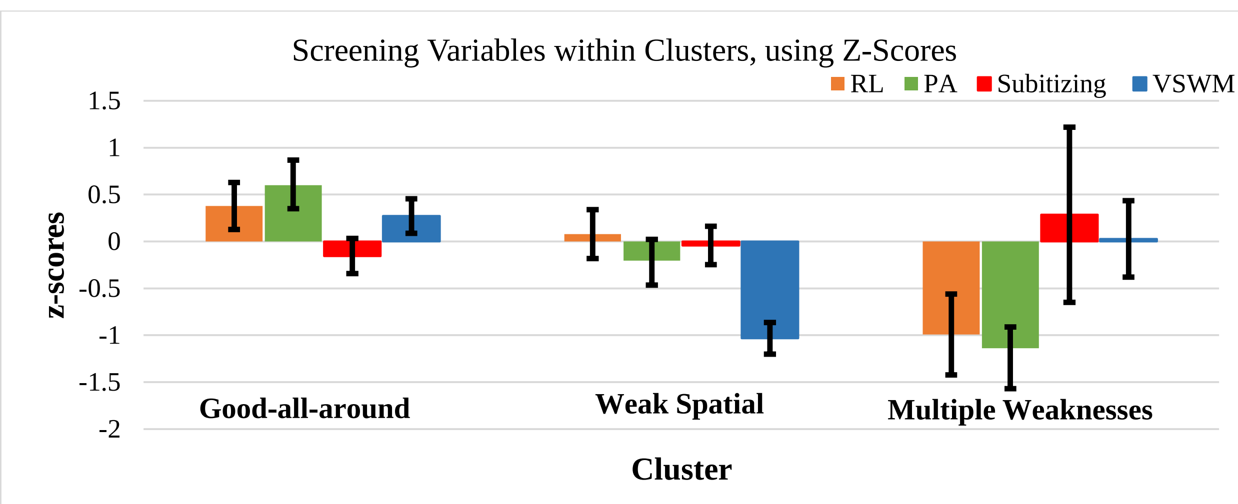


*Figure 1*. Z-score distributions of screening variables.

Table 1

*Descriptive Statistics of Screening Variables for Clusters*

| Cluster | N (male) | Age (SD) | Subitizing Slope (SD) | VSWM (SD) | PA (SD) | RL (SD) |
|---|---|---|---|---|---|---|
| **1 Good-all-around** | 45 (25) | 5 years, 11 months (3.88 months) | 162.0 (167.00ms) | 4.24 (.61) | 12.5 (2.36) | 114.7 (9.4) |
| **2 Weak Spatial System** | 35 (18) | 5 years, 11 months (3.91 months) | 194.5 (160.11ms) | 2.63 (.49) | 10.4 (1.77) | 111.0 (8.63) |
| **3 Multiple Weaknesses** | 17 (8) | 6 years, 0 months (4.56 months) | 290.7 (509.53) | 3.94 (.83) | 8.0 (1.23) | 97.88 (9.93) |

*VSWM is visuospatial working memory, PA is phonological awareness, RL is receptive language*

**Analysis of Demographic Control Variables**

Potential differences in demographic control variables between the clusters (gender, age, parent education, language spoken at home) were investigated. Differences in gender proportions (male, female) in the clusters were assessed using the chi square test of independence. The clusters were not found to differ in gender proportions, $X^2$ (2, n = 97) = .386, *ns*. Age differences (in months) among the clusters were assessed using a one-way between subjects ANOVA. The clusters were not found to differ in age, $F(2, 94) = .082$, *ns*. Differences in parent education (less than high school, high school, university, college/ vocational education, multiple degree/ post-graduate education) among the clusters was assessed using the chi square test of independence. The clusters were not found to differ in parent education, $X^2$ (2, n = 96) = 6.41, *ns*. Lastly, the chi square test of independence was used to investigate whether clusters differed in their proportions of languages spoken at home (allophone or Anglophone). Clusters did not differ in their proportions of languages spoken at home, $X^2$ (2, n = 97) = 2.061, *ns*.

**Analysis of Mathematical Outcomes**

To assess the longitudinal mathematical outcomes of the different clusters, a 2(Grade: one, two) x 3(cluster membership: good-all-around, weak spatial system, multiple weaknesses) mixed factorial ANOVA was performed with KeyMath, WJ-Math, addition ability and addition fluency as dependent variables.

For KeyMath Numeration, there was a main effect of grade; children correctly answered more questions in Grade 2 ($M$ = 13.9, $SD$ = .33) than they did in Grade 1 ($M$ = 10.8, $SD$ = .32), regardless of cluster membership, $F(1,85) = 99.25$, $p < .001$, $\eta^2 = .54$, power = 1.00. Additionally, there was a main effect of cluster membership, $F(2,85) = 9.09$, $p < .001$, $\eta^2 = .18$, power = .97. Using Tukey's HSD, post-hoc analyses revealed that children in the good-all-around cluster performed better on the KeyMath ($M$ = 13.9, $SD$ = .38) than children in weak spatial system cluster ($M$ = 12.2, $SD$ = .43, $t(74) = 2.38$, $p = .008$) and those in the multiple weaknesses cluster ($M$ = 11.1, $SD$ = .65, $t(56) = 3.24$, $p = .001$), regardless of grade. Children in the weak spatial system cluster did not differ from children in the multiple weaknesses cluster in terms of KeyMath scores, regardless of grade, $t(44) = .37$, *ns*. There was no interaction observed, $F(2, 85) = .59$, *ns*, $\eta^2 = .01$, power = .15.

For Woodcock-Johnson Calculation, there was a main effect of grade; children correctly answered more questions in Grade 2 ($M$ = 11.7, $SD$ = .30) than they did in Grade 1 ($M$ = 8.0, $SD$ = .28), regardless of cluster membership, $F(1,85) = 128.09$, $p < .001$, $\eta^2 = .60$, power = 1.00. There was also a main effect of cluster membership, $F(2,85) = 13.63$, $p < .001$, $\eta^2 = .24$, power = 1.00. Tukey's HSD post-hoc test revealed that children in the good-all-around cluster correctly solved more questions ($M$ = 11.4, $SD$ = .31) than those in weak spatial system cluster ($M$ = 9.1, *SD* = .36, $t(74) = 3.21$, $p < .001$) or those in the multiple weaknesses cluster ($M = 9.1$, $SD = .54$, $t(56) = 2.92$, $p = .002$), regardless of grade. Children in the weak spatial system cluster did not differ from children in the multiple weaknesses cluster in terms of Woodcock-Johnson calculation scores, regardless of grade, $t(44) < .01$, *ns* There was no interaction observed, $F(2, 85) = 1.82$, *ns*, $\eta^2 = .04$, power = .37.

Addition fluency and percent of addition problems solved correctly were also evaluated longitudinally. For addition fluency, a main effect of grade was observed. When in Grade 2, children were faster to solve small addition problems ($M = 2347$ ms , $SD = 75.6$ ms) than when in Grade 1 ($M = 4138$ ms , $SD = 182.4$ ms), regardless of cluster membership, $F(1,85) = 125.96$, $p < .001$, $\eta^2 = .60$, power = 1.00. Additionally, there was a main effect of cluster membership, $F(2,85) = 8.73$, $p < .001$, $\eta^2 = .17$, power = .97. Tukey's HSD post-hoc test revealed children in the good-all-around were faster ($M = 2646$ ms , $SD = 148.6$ ms) than children in the weak spatial system cluster ($M = 3364$ ms, $SD = 170.2$ ms, $t(74) = 3.21$, $p < .001$) or those in the multiple weaknesses cluster, regardless of grade ($M = 3719$, $SD = 257.4$, $t(56) = 3.25$, $p < .001$). Children in the weak spatial system cluster did not differ from children in the multiple weaknesses cluster in terms of addition fluency, regardless of grade, $t(44) = .04$, *ns.* Third, a quantitative interaction between cluster membership and grade was observed, $F(2,85) = 7.50$, $p = .001$, $\eta^2 = .15$, power = .94. As illustrated in Figure 2, although slower in Grade 1 than their weak spatial system peers, children in the multiple weaknesses cluster reached the same level of addition fluency in grade 2 as there their weak spatial system cluster peers. However, children in these clusters did not reach the addition fluency levels of their good-all-around cluster peers in Grade 2.

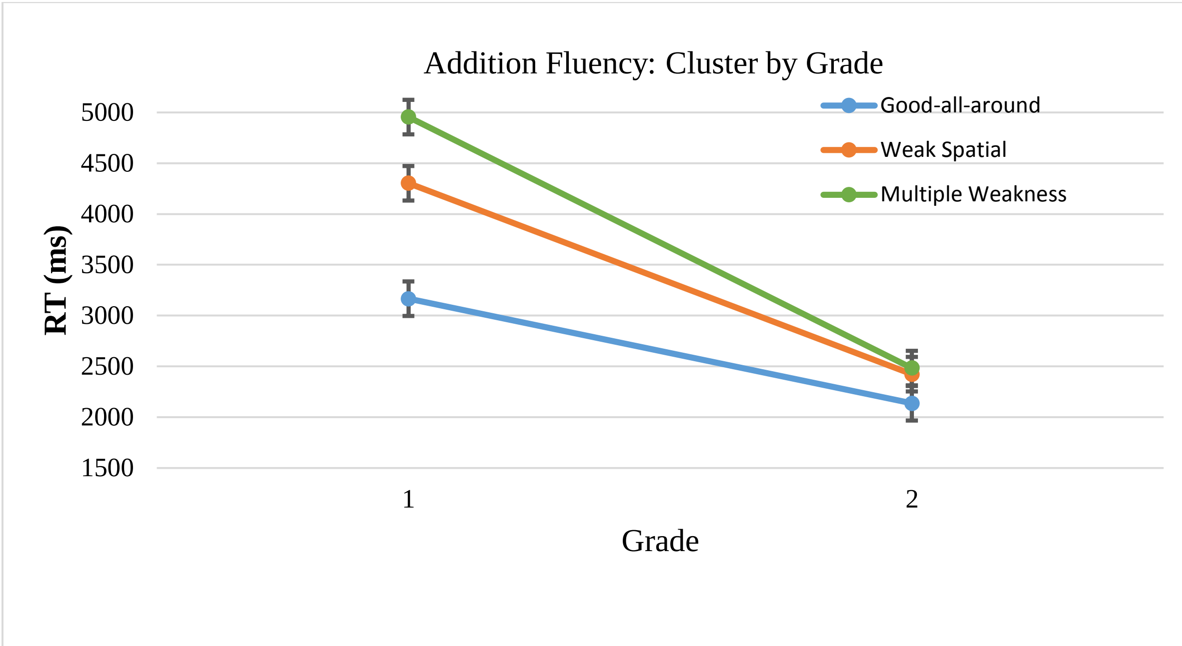


*Figure 2*. Interaction between cluster and grade for addition fluency.

In terms of percent of addition problems solved correctly, a main effect of grade was observed, $F(1, 85) = 56.19$, $p < .001$, $\eta^2 = .40$, power = 1.00. Regardless of cluster membership, children solved more small addition problems correctly in Grade 2 than they did in Grade 1. No main effect of cluster membership was observed, $F(2, 85) = 1.14$, *ns*, $\eta^2 = .03$, power = .24. Across clusters, children answered a statistically equivalent percentage of the small addition problems correctly. Additionally, no interaction was observed, $F(2,85) = 7.50$, $p = .001$, $\eta^2 = .15$, power = .94.

Given these results, a composite measure (composed of addition fluency and percent of addition problems solved correctly) was constructed and evaluated longitudinally. There was a main effect of grade observed, $F(1, 85) = 124.45$, $p < .001$, $\eta^2 = .59$, power = 1.00. Regardless of cluster membership, children were quicker and more accurate to solve small addition problems in

Grade 2 than in Grade 1. Children also differed with respect to their cluster membership, $F(2, 85) = 8.48$, $p < .001$, $\eta^2 = .86$, power = 1.00. Tukey's HSD post-hoc test revealed children in the good-all-around were faster and more accurate ($M = 3017$, $SD = 223.9$) than children in the weak spatial system cluster ($M = 4061$, $SD = 256.5$, $t(74) = 2.73$ , $p < .008$) or those in the multiple weaknesses cluster, regardless of grade ($M = 4631$, $SD = 387.8$, $t(56) = 3.24$ , $p < .002$). Children in the weak spatial system cluster did not differ from children in the multiple weaknesses cluster in terms of addition fluency, regardless of grade, $t(44) = .77$, *ns*. Third, a quantitative interaction between cluster membership and grade was observed, $F(2, 85) = 8.08$, $p = .001$, $\eta^2 = .16$, power = .95. Similar to the trend observed for addition fluency and as illustrated in figure 3, although slower in Grade 1 than their weak spatial system peers, children in the multiple weaknesses cluster reached the same level of addition fluency in Grade 2 as there their weak spatial system cluster peers. However, children in these clusters did not reach the addition fluency levels of their good-all-around cluster peers in Grade 2.

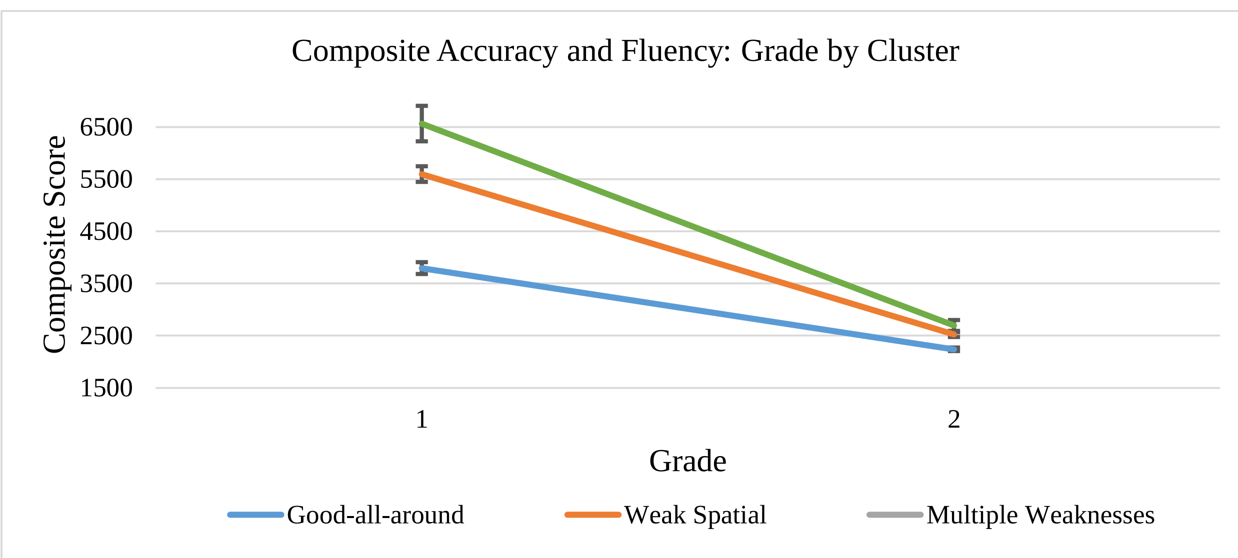


*Figure 3*. Interaction between cluster and grade for composite measure.

**Analysis of Reading Outcomes**

To assess the reading outcomes of the different clusters, a one-way between subjects ANOVA was performed with Grade 1 standardized Woodcock-Johnson Word Identification scores as the dependent variable and cluster membership as the independent variable. Reading scores were not collected in Grade 2. Participants scores differed based on their cluster membership, $F(2,86) = 7.90$, $p = .001$. Using Tukey's HSD, post-hoc analyses revealed that children in the good-all-around cluster had stronger reading skills ($M = 124.2$, $SD = 12.02$) than children in the weak spatial system cluster ($M = 117.6$, $SD = 12.01$, $t(74) = 1.67$, $p = .048$), and children in the multiple weaknesses cluster ($M = 110.4$, $SD = 12.70$), $t(56) = 3.25$, $p = .001$). Children's scores in the multiple weaknesses cluster and weak spatial systems cluster did not differ in reading skill, $t(45) = 1.49$, *ns*.

**Analysis of General Cognitive Outcomes**

To determine if the clusters differed on general cognitive outcomes, two separate one-way between subjects ANOVAs were conducted with nonverbal reasoning and phonological working memory as dependent variables and cluster membership as the independent variable. Additionally, a 3(grade: SK, 1, 2) x 3(cluster membership: good-all-around, weak spatial system, and multiple weaknesses) mixed factorial ANOVA was used to assess potential differences in processing speed.

The clusters were found to differ in their nonverbal reasoning ability, $F(2,94) = 4.62$, $p = .012$. In post-hoc analysis, Tukey's HSD revealed that children in the good-all-around cluster had stronger nonverbal reasoning skills ($M = 106.8$, $SD = 12.52$) than children in the weak spatial system cluster ($M = 97.9$, $SD = 14.24$, $t(80) = 1.99$, $p = .013$), but not stronger than children in the multiple weaknesses cluster (*M* = 99.4, *SD* = 14.77, $t(62) = 1.67$, *ns*). Children in the multiple weaknesses and weak spatial system cluster did not differ in nonverbal reasoning, $t(45) = 1.48$, *ns***.** The clusters were not found to differ in their phonological working memory, $F(75) = .45$, *ns*.

In the mixed design factorial ANOVA, a main effect of grade was observed for processing speed, $F(2,85) = 26.20$, $p < .001$, $\eta^2 = .24$, power = 1.0. Tests of within-subjects contrasts revealed a linear trend, indicating that children's processing speed increased (e.g., was faster) as they moved into older grades, $F(1,84) = 56.45$, $p < .001$, $\eta^2 = .40$, power = 1.0. There was no main effect of cluster membership, indicating that processing speed did not differ by cluster, $F(2,84) = 1.12$, *ns*, $\eta^2 = .03$, power = .24. Nor was there an observed interaction, $F(4,83) = .79$, *ns*, $\eta^2 = .02$, power = .25.

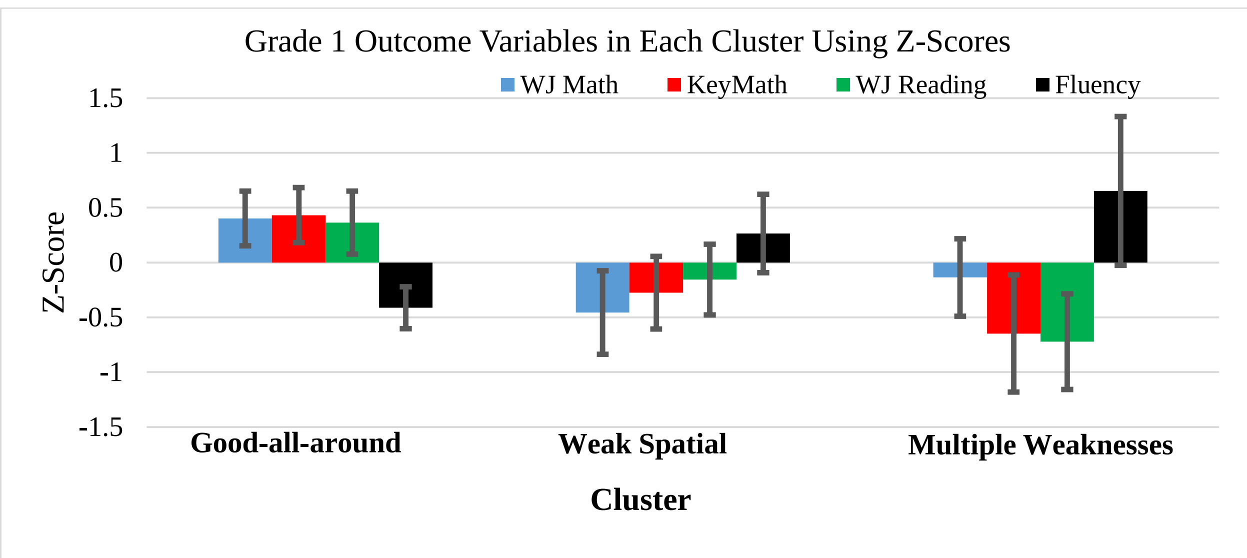


*Figure 4*. Z-score distributions of key outcome variables. Graph contains z-scores for math ability, math fluency, and reading outcome variables. WJ Math is Woodcock-Johnson math measure, KeyMath is the KeyMath Numeration subtest, WJ Reading is the Woodcock-Johnson standardized reading measure, fluency is the single digit addition fluency task.

## Discussion

The present study investigated cognitive profiles of children in kindergarten, formed using cluster analysis based on subitizing, visuospatial working memory, phonological awareness and receptive language, and the associated learning outcomes for the different cluster groups in Grade 1 and 2. The clusters were characterized as (1) good-all-around (strong receptive language, strong phonological awareness, strong subitizing ability, and strong visuospatial working memory), (2) weak spatial system (average receptive language, phonological awareness, and subitizing, very weak visuospatial working memory), and (3) multiple weaknesses (very weak receptive language and phonological awareness, weak subitizing, and strong visuospatial working memory). Across a variety of outcome measures, children in the good-all-around cluster outperformed their peers, longitudinally in both Grade 1 and 2. These children displayed strong math scores (arithmetic ability, arithmetic fluency, numeration, and calculation skill), and stronger reading skills than their peers in the weak spatial system and multiple weaknesses clusters. Encouragingly, these children represented the largest portion of the sample (47%). However, over half the sample, underperformed in comparison to the good-all-around cluster. Children in these clusters present with distinct cognitive profiles with unique strengths and weakness. Successful interventions may draw on the strengths of these children, to compensate for their weaknesses and to improve their achievement outcomes.

Longitudinal trends for arithmetic ability and fluency indicate that children in the good-all-around cluster maintained their advantage over the next two years. Though all children attained similarly high accuracy rates, children in the good-all-around cluster were consistently faster than their peers in the weaker clusters. In the present study, children in the weak spatial system and multiple weaknesses clusters did not compensate for their early performance deficits as formal education continued. In arithmetic fluency, children in the multiple weaknesses cluster, who were slower to accurately complete addition problems in Grade 1 than their peers, reached the performance level of their weak spatial system cluster peers by Grade 2. Despite these rapid performance increases, neither children in these clusters reached the performance level of children in the good-all-around cluster. In this study and in other work (Aunola et al., 2004), we see evidence that children who start school with poor numeracy skills do not catch up to their peers as formal education progresses. Therefore, there is a need for early identification of at-risk children and interventions targeted to children's pattern of strengths and weaknesses to increase performance outcomes.

The single digit arithmetic fluency task provides the only evidence of children with weaker cognitive profiles reaching the mathematical ability of their peers with stronger profiles. In Grade 1, children in the good-all-around cluster were fastest to complete single digit addition trials, followed by children in the weak spatial system cluster, and then children in the multiple weaknesses cluster. In Grade 2, children in the good-all-around cluster stayed faster than their peers, however, children in the multiple weaknesses cluster had reached the performance level of children in the weak spatial system cluster. Though these children were able to improve their performance at a faster rate than children with other profiles, these results are still concerning. The task asks children to add two single digit addends to a sum that is less than ten. Although accuracy rates were similar across clusters, the weaker clusters' slower response times suggest the possible use of developmentally immature strategies and a lack of automaticity in solving these problems. Discrepancies in response times between the profiles suggest that children in the weaker clusters have not yet mastered this skill at the level of children in the good-all-around cluster.

Not surprisingly, children in the good-all-around cluster outperformed children in the latter clusters in standardized reading. Children in the good-all-around cluster displayed early strengths in linguistic ability (as indexed by strong phonological awareness and receptive language scores), as compared to their peers in the weak spatial system cluster (average phonological awareness and receptive language) and children in the multiple weaknesses cluster (very weak receptive language and phonological awareness). However, children in these clusters had equivalent reading outcomes. The sizable weaknesses in early linguistic skill within the multiple weaknesses cluster would suggest later weaknesses in reading ability, however this result was not observed. This outcome was likely due to a small group size for this cluster.

Children in the weak spatial system showed weakness in their visuospatial working memory, but average phonological processing, receptive language, and subitizing. This spatial difficulties group has immerged in previous work including in the data-driven approach used by Bartelet et al. (2014). Leferve et al. (2010)'s Pathways Model would suggest that the strengths in the linguistic and quantitative pathways can be targeted to maximize the success of these students. Conversely, children in the multiple weaknesses cluster, who displayed weaknesses in in phonological awareness, receptive language, and subitizing, but strong visuospatial working memory systems might benefit from interventions targeted to utilize their visuospatial competence. Interventions targeted toward children's strength may offer greater likelihoods of success (Gary et al., 2012).

It is possible that the differences in cognitive profiles observed in the present study and in previous work (e.g. Archibald et al., 2013, Bartelet et al., 2014) are due to underlying differences in processing speed or general working memory capacity. However, no differences in processing speed or phonological working memory were seen between the clusters, suggesting it is visuospatial working memory, specifically, and subitizing abilities that underlie math ability, rather than broader processes involving pure speed or general working memory resources.

Some researchers suggest that differences in mathematical ability stem from core differences in magnitude comparison (De Smedt & Gilmore, 2011l Piazza et al., 2010; Rouselle & Noel, 2007). Others suggest differences in visuospatial working memory better account for differences in math ability (Davis et al., 2009; Szucs et al., 2013). Unfortunately, our study did not include a magnitude comparison task during SK. However our study does indicate the importance of the visuospatial system, along with quantitative and linguistic skills in math performance.

The present study is not without limitation. The data used in the present study is archival – data collection was concluded in 2007. As such, the study was not initially designed with our research question in mind. The sample size used is relatively small to use clustering techniques. Additionally, numerical cognition research has progressed greatly since 2007. The field is now aware of several additional predictors that are proposed to underlie later mathematic ability including magnitude comparison (De Smedt & Gilmore, 2011l Piazza et al., 2010; Rouselle & Noel, 2007), symbolic and non-symbolic versions of tasks (Bartelet et al., 2014), and ordinality tasks (Lyons, Ansari, & Beilock, 2012). Future studies would benefit from incorporating these more recent numerical abilities. In fact, more recent research into the cognitive profiles of children with mathematic learning difficulties have used a variety of numeracy tasks and have identified more diversity in math learning and math learning difficulties (e.g., Bartelet et al., 2014; Szucs et al., 2013).

Finally, due to the initial study design, there is a discrepancy between the phonological working memory and visuospatial working memory operational definitions used. Working memory involves the manipulation of information – not simply the storage of information. As such, when a participant is asked to repeat information (e.g., digits or pattern) in the reverse order to which is was presented, working memory is said to be involved. Reverse spans were used for phonological working memory tasks, but not for visuospatial working memory tasks. Future research should incorporate both forward and reverse spans for working and short term memory tasks. The inclusion of robust and well-defined working memory tasks is essential to contribute to the debate surrounding the role of working memory versus magnitude comparison and other numeracy tasks in math learning and later mathematics ability. Given a comprehensive spatial-task battery (including visuospatial working and short term memory, and trail-making), Szucs et al. (2013) identified deficits in spatial ability as the key feature of pure mathematics impairments. Although the results of the present study identify visuospatial working memory as a key profiling factor, without a more comprehensive numeracy and spatial system battery, it is difficult to determine the roles of these predictive factors across the spectrum of arithmetic ability.

Cluster analysis revealed one early cognitive profile that led to favourable mathematic, reading, and general cognitive outcomes and two cognitive profiles leading to similar unfavourable outcomes, two years into their formal education. Future research employing cluster analysis should incorporate additional numeracy skills suggested to underlie math ability in addition to the numeracy, linguistic, and general cognitive predictors used in the present study. Additionally, future research should track participants further into elementary schools to examine longitudinal trends. In the present study, without intervention, poorly performing children are not likely to reach the mathematical ability of their peers. Will this trend continue as education progresses? Will some children find strategies to compensate for their underlying weaknesses in numeracy, linguistics, or in spatial skills? These questions can be addressed through longer term longitudinal projects.

Children can show different early cognitive profiles with similar outcomes in later elementary school, their individual strengths and weaknesses need to be considered to develop interventions that will help them achieve their potential. The results of the current study suggest (1) early identification measures to determine which students are at risk for math difficulties and (2) cluster-based interventions that target children's strengths as a means to improve their math outcomes. Our lab is currently designing such interventions.

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