Western University Scholarship@Western

Economic Policy Research Institute. EPRI Working **Papers**

Economics Working Papers Archive

2003

2003-4 Intergenerational Mobility under Private vs. Public Education

Jim Davies

Jie Zhang

Jinli Zeng

Follow this and additional works at: https://ir.lib.uwo.ca/economicsepri_wp



Part of the Economics Commons

Citation of this paper:

Davies, Jim, Jie Zhang, Jinli Zeng. "2003-4 Intergenerational Mobility under Private vs. Public Education." Economic Policy Research Institute. EPRI Working Papers, 2003-4. London, ON: Department of Economics, University of Western Ontario (2003).

Intergenerational Mobility under Private vs. Public Education

by

James B. Davies, Jie Zhang and Jinli Zeng

Working Paper # 2003-4

January 2003



RBC Financial Group Economic Policy Research Institute EPRI Working Paper Series

Department of Economics
Department of Political Science
Social Science Centre
The University of Western Ontario
London, Ontario, N6A 5C2
Canada

This working paper is available as a downloadable pdf file on our website http://www.ssc.uwo.ca/economics/centres/epri/

Intergenerational Mobility under Private vs. Public Education

James B. Davies University of Western Ontario

Jie Zhang University of Queensland

Jinli Zeng National University of Singapore

January 2003

Abstract

This paper analyzes intergenerational earnings mobility in a model where human capital is produced using schooling and parental time. In steady-states more mobile societies have less inequality, but in the short-run higher mobility may result from an increase in inequality. Starting from the same inequality, mobility is higher under public than under private education. A rise in income shocks, for example due to increased returns to ability, or a switch from public to private schooling both increase inequality. However, increased shocks raise mobility in the short-run and do not affect it in the long-run, whereas an increased role for private schooling reduces mobility in both the short- and long-run. That these differences may help to identify the source of changes in inequality, and other real-world implications, are illustrated in a brief discussion of time trends and cross-country differences.

JEL classification: J62; D30; I20 Keywords: Mobility; Inequality; Education regimes

Correspondence:
James Davies
Department of Economics
Faculty of Social Science
The University of Western Ontario
London, Ontario
CANADA, N6A 5C2

Email: jdavies@uwo.ca; Fax: (519) 661-3666

I. Introduction

There has been much recent interest in modeling the implications of public vs. private education for economic growth and inequality. At the same time there has been increasing attention to the empirical study and theoretical modeling of intergenerational mobility. This paper brings these two themes together, showing how mobility can be analyzed in relation to inequality in a unified setting in which the impact of varying schooling regimes can be conveniently analyzed.

Our work builds on the modeling of parental and public investments in children's human capital initiated by Glomm and Ravikumar (1992) and carried forward by Saint-Paul and Verdier (1993), Eckstein and Zilcha (1994), Benabou (1996), and Zhang (1996). An important feature incorporated by Benabou was a source of persistent inequality, in the form of fresh shocks to income in each generation. Without such shocks it is found that (i) inequality disappears in the long-run under public education and under private education as well under a suitable concavity condition, (ii) inequality falls more quickly under public than under private education, and (iii) provided initial inequality is low the long-run growth rate is higher under private than under public education. With fresh shocks to income in each generation, Benabou showed that inequality has a lower steady-state value under public than under private education. Further, public education produces faster rather than slower long-run growth. While the effects of public investment in education on inequality and growth are at the heart in this literature, intergenerational mobility has not been explicitly analyzed in most of the work.

A popular approach to analyze intergenerational mobility focuses on the persistence coefficient in a log-linear model relating children's earnings to parents' (see Solon, 1999). However, this persistence coefficient only tells us about long-run mobility. Current mobility may be quite different. In the model presented here current mobility is captured by the inverse of the correlation coefficient between earnings of parents and children in two succeeding generations. This correlation interacts with changes in inequality and varies in transition. Its behavior therefore requires careful study.

¹Another popular approach focuses on the quantile transition matrix of income distribution across generations; see e.g. Galor and Tsiddon (1997), Owen and Weil (1998), Maoz and Moav (1999), Iyigun (1999), and Hassler and Mora (2000).

This paper analyzes the full path of intergenerational mobility in a model where human capital is produced using schooling and parental time. We show that, in the comparison of steady-states, more mobile societies also have less inequality. In addition, inequality is lower and mobility is higher in the long run under public than under private education. The same contrast between the schooling regimes is also found in transition, for societies beginning with the same level of inequality.

An important contribution of our analysis is to explore the distinction between long-run and current mobility. The former is the mobility observed in steady-state. It corresponds to the degree of intergenerational regression to the mean that empirical researchers have often estimated using earnings data. In the short-run, current mobility can differ from long-run by an amount that we refer to as transitional mobility. The latter is positive when inequality is rising toward its steady-state level, and negative when inequality is falling. Thus the two components of intergenerational mobility have a sharply contrasting character. Higher long-run mobility leads to lower long-run inequality, but positive transitional mobility is a reflection of increasing inequality. How current mobility is regarded may therefore depend on the relative contribution of these two components.

Inequality can rise due to either of two factors in our model. One is an increase in the variance of idiosyncratic shocks to earnings which may reflect a rise in the returns to unobserved skill or "ability". The other is a rise in the rewards to private education inputs, either due to a change in the human capital production function or a switch from public to private schooling. Either factor will lead to a period of rising inequality. However, increased shocks raise mobility in the short-run and do not affect it in the long-run, whereas an increased role for private schooling reduces mobility in both the short- and long-run. This contrast implies differing time paths of mobility in the two scenarios, which in principle can allow one to distinguish empirically between increases in inequality caused by rising returns to ability vs. increased importance of private education inputs. Thus the study of mobility is not only fruitful in its own right, but promises to enhance our understanding of changes in inequality as well.

The paper is organized as follows. The next section introduces the model. Section III provides

the analysis of mobility. The comparative dynamic impacts of various parameter changes are investigated in Section IV. Possible implications for the interpretation of observed differences in inequality and mobility both across countries and over time are considered in Section V. Section VI concludes.

II. The Model

The model is similar to that of Benabou (1996). There is a continuum of overlapping-generation families $i \in \Omega$, of unit measure. Parents work, consume, and spend a fixed amount of time in child care. At time zero with given h_0^i , the parent in dynasty i faces the problem:

$$\operatorname{Max} \ U_0^i = E_0 \left(\sum_{t=0}^{\infty} \rho^t \ln c_t^i \right)$$

subject to

$$c_t^i = (1 - \tau_t^i)y_t^i, \tag{1}$$

$$y_t^i = \nu_t^i h_t^i, \tag{2}$$

$$h_{t+1}^{i} = \kappa \xi_{t}^{i} [(1 - \nu_{t}^{i}) h_{t}^{i}]^{\alpha} (L_{t}^{i})^{\beta} (H_{t})^{\gamma}, \tag{3}$$

where c_t^i is consumption, y_t^i income, τ_t^i the proportion of the parent's income invested in human capital formation under private education and the tax rate under public education, $^2\nu_t^i$ the fraction of time spent working (the remaining $1-\nu_t^i$ being used in the child's education), h_t^i human capital, L_t^i the input of goods in education, and H_t the average human capital stock. The unpredictable ability of the child ξ_t^i follows a lognormal distribution $\ln \xi_t^i \sim \mathcal{N}(-s^2/2, s^2)$. And the distribution of initial human capital $\mu^i(h_0)$ is also lognormal: $\ln h_0^i \sim \mathcal{N}(m_0, \Delta_0^2)$.

While simple, the earnings process set out in (3) captures a range of important influences on inequality and mobility. As in Loury (1981) the stochastic component is an ability or endowment

²Here we treat the tax rate as a flat rate and abstract from redistribution through transfers or social insurance. A possible objection is that higher inequality will lead to greater redistribution. Benabou (2000) challenges this view by showing that multiple steady state equilibria can emerge in a voting model whereby high inequality is associated with low redistribution. Benabou and Ok (2001) also show that the poor may not support high levels of redistribution when there is the prospect of upward mobility. Introducing redistribution and progressive taxation may enrich the analysis of intergenerational earnings mobility, but needs to be done with great care and is beyond the scope of our analysis.

shock.³ The family has two kinds of influence. First, there is an exogenous impact of parents' human capital on children's, which allows a genetic influence. But, in addition to this, there is a discretionary parental effect via time spent with the child in the home, $1 - \nu_t^i$, and under private education purchased inputs, L_t^i . Under private education, the goods input in human capital formation, L_t^i , equals $\tau_t^i y_t^i = \tau_t^i \nu_t^i h_t^i$, where parents all choose $\nu_t^i = \nu = 1 - \rho \alpha$ and $\tau_t^i = \tau = \rho \beta/(1 - \rho \alpha)$. Under public education (with state funding), $L_t^i = \int_0^\infty \tau_t^i \nu_t^i h_t^i d\mu_t(h) = \tau_t \nu_t H_t$, where $\nu_t^i = \nu = 1 - \rho \alpha$ as in the previous case and $\tau_t^i = \tau_t$ is set exogenously by the government.⁴ The assumption of either parental or public financing of education is rather standard, reflecting the absence of education loans, as in Glomm and Ravikumar (1992) and Benabou (1996).⁵ A worker's income is given by $y_t^i = \nu h_t^i$. Therefore, income and human capital stock exhibit the same pattern of behavior.

Letting $R = \alpha + \beta + \gamma$, human capital accumulates according to

$$h_{t+1}^{i} = \Theta \xi_{t}^{i}(h_{t}^{i})^{\phi_{j}} H_{t}^{R-\phi_{j}}, \quad j = 1, 2, \quad \phi_{1} = \alpha + \beta, \quad \phi_{2} = \alpha; \quad 0 < \alpha, \beta, \gamma < 1; \quad \alpha + \beta < 1,$$

$$(4)$$

where ϕ_1 and ϕ_2 reflect the degree of *persistence* in human capital from generation to generation under private and public education respectively, and $\Theta = \kappa (1 - \nu)^{\alpha} (\nu \tau)^{1-\alpha}$. The persistence (i.e. parental characteristics influencing children's education) and the community factors are all standard assumptions, albeit some models use only one of them (e.g. parental factors are not allowed in Fernández and Rogerson, 1998). Taking logs in (4) with $\theta = \ln \Theta$ we obtain:

$$\ln h_{t+1}^{i} = \theta + \phi_{i} \ln h_{t}^{i} + (R - \phi_{i}) \ln H_{t} + \ln \xi_{t}^{i}, \tag{5}$$

which is of a form that has been used frequently in theoretical and empirical analyses of intergenerational mobility. This is a Galtonian process, in which earnings regress to the mean at the rate

³Becker and Tomes (1979) also incorporated "market luck", which would vary from year to year over the individual's life cycle, but argued that it was likely much less important than endowment luck.

⁴As we shall see the level of τ has no impact on inequality or mobility as long as $\tau > 0$. We therefore refrain from modeling how τ is determined in the political process.

⁵Owen and Weil (1998) also rule out education loans and own wage income as possible resources to purchase education, which has a fixed cost in their model and is only purchased through transfers from parents. Iyigun (1999) only considers free public education with competitive school admission. The fixed cost or the competitive admission serves to induce a binary outcome in regard to whether or not a child is educated in the models that follow the quantile transition matrix approach.

 $(1 - \phi_j)^{6}$ On our assumption that both initial human capital, h_0^i , and the ability shock, ξ_t^i , are lognormal, so is the distribution of h_t^i in each generation.

The distribution of human capital evolves according to:

$$m_{t+1} = \theta - s^2/2 + Rm_t + (R - \phi_i)\Delta_t^2/2,$$
 (6)

$$\Delta_{t+1}^2 = \phi_j^2 \Delta_t^2 + s^2, \quad \Delta_{\infty}^2 = s^2/(1 - \phi_j^2).$$
 (7)

Given that human capital has a lognormal distribution, the variance of logarithms, Δ_t^2 , is here an ideal measure of inequality. From (7) we have immediately that inequality is lower in all generations $t \geq 1$ under public than under private education, since Δ_0^2 is the same in the two regimes and $\phi_2 < \phi_1$. Also, of course long-run inequality, Δ_{∞}^2 , is lower under public education. Such a difference in how inequality behaves between the education regimes has been studied in Glomm and Ravikumar (1992) and Benabou (1996).

From (7) we have:

$$\Delta_{t+1}^2 = \Delta_{\infty}^2 + \phi_j^2 (\Delta_t^2 - \Delta_{\infty}^2), \quad j = 1, 2,$$
(8)

which indicates that a fraction $(1 - \phi_j^2)$ of the gap between current and steady-state inequality disappears in each generation. This means that inequality converges to its long-run value as shown in the left-hand panels of Figure 1. Clearly, the rate of convergence is slower under private than under public education because $\phi_1 > \phi_2$.

Finally, we note that from time to time the intergenerational earnings process may be shocked by changes in parameters. These can be thought of as resetting the clock to t = 0, and possibly resulting in a change in Δ^2_{∞} which will initiate a new path of convergence to long-run inequality of the form shown in Figure 1. But note that the *only* changes which can have such an effect are those in s^2 , α , or β . (Thus technological progress, reflected in κ , or levels of expenditure on education have no effect.) Long-run inequality is proportional to s^2 , but speed of convergence is unaffected

⁶Galton identified processes of this type in his study of the intergenerational correlation of such natural characteristics as height. See, e.g., Galton (1886). Recent studies using panel data and employing instrumental variables indicate that ϕ_j is in the range of 0.4-0.6 in the U.S. See Solon (1999).

by this variance of shocks. From (7), a rise in α will increase long-run inequality and slow down convergence in both education regimes, as will a rise in β under private education.

III. Analysis of Mobility

Under the earnings process set out above for both the private and public education regimes, the joint distribution of the log of human capital in any two generations is bivariate normal. In this situation a natural, and sufficient, indicator of intergenerational mobility is the inverse of the correlation coefficient for $(\ln h_t^i, \ln h_{t+1}^i)$. From (4) and (5) the correlation is:

$$r_{t+1} = \phi_j \Delta_t / \Delta_{t+1}, \quad j = 1, 2.$$
 (9)

Since $\phi > 0$ and $\Delta \ge 0$, (7) implies that the correlation r lies in [0, 1], i.e. a subset of its standard range [-1, 1]. Intergenerational mobility is then given by $\psi_{t+1} = 1 - r_{t+1}$ with $\psi \in [0, 1]$:

$$\psi_{t+1} = 1 - \phi_i \Delta_t / \Delta_{t+1}, \quad \psi_{\infty} = 1 - \phi_i.$$
 (10)

Note that there is an important distinction between immobility, r_{t+1} , and persistence, ϕ_j . The two are only the same in the long run, when $\Delta_t = \Delta_{t+1}$. Current mobility is therefore not identical to the rate of regression to the mean, $1 - \phi_j$. This distinction has significant consequences for the analysis of changing patterns of inequality and mobility, as we discuss below.

It is also worth noting from (10) that long-run mobility only depends on persistence. In particular it is independent of the level of inequality, or of the variance of earnings shocks, s^2 . This has seemingly paradoxical implications. For example, in a society with very low s^2 and low ϕ_j , long-run mobility will be high despite the fact that people all have very similar incomes. In contrast, a society with great inequality, but high ϕ_j , will have low mobility although absolute changes in income from parent to child will generally be much larger than in the former society. The reason is that mobility, as defined here, is a relative concept. If a large fraction of the variation in children's earnings is explained by parental differences, then we have low mobility, irrespective of how small the differences in earnings are within a generation.

⁷The mobility measure we define here has been christened the Hart index by Shorrocks (1993), who points out that the use of this index cannot be seriously challenged when earnings follow a Galtonian process.

From (10) it is immediate that when inequality is rising ($\Delta_t < \Delta_{t+1} < \Delta_{\infty}$) mobility will be above its long-run value (and that the converse holds for falling inequality). The reason is that past values of h_t^i were relatively concentrated, and differences in the parental contribution to h_{t+1}^i are small compared to those of the random shocks, ξ_{t+1}^i . Over the period of a generation, say 30 years, the levels of income inequality can change substantially. From (9) the result is that intergenerational earnings correlation (r_{t+1}) may differ significantly from the level of persistence, ϕ_j .

To study the time path of mobility more closely, note from (7) that:

$$\frac{\Delta_{t+1}}{\Delta_t} = \left(\phi_j^2 + \frac{s^2}{\Delta_t^2}\right)^{1/2}, \quad j = 1, 2. \tag{11}$$

Thus, when Δ_t is rising over time Δ_{t+1}/Δ_t is falling, and the reverse is true when Δ_t is falling. Hence, from (10) we have:

Proposition 1: As inequality, Δ_t^2 , converges to its steady-state level, Δ_∞^2 , mobility, ψ_t , (i) rises monotonically if Δ_t^2 is falling, and (ii) falls monotonically if Δ_t^2 is rising.

This proposition is illustrated in Figure 1. We see that mobility may be temporarily either above or below its long-run, or permanent level, $1 - \phi_j$. The latter reflects the rate of regression to the mean. We refer to the difference between current and long-run mobility as transitional mobility (see Figure 1).

It is important to note that long-run and transitional mobility have radically different implications. Long-run mobility reduces inequality both in the short-run and in steady-state. Transitional mobility is, in contrast, a byproduct of rising inequality.

It is also worth noting that mobility in our model does not depend directly on the levels or growth rates of income per person. Some studies have found contrasting results. For example, under private education in Owen and Weil (1998), mobility is higher in equilibrium with higher output; and under public education in Iyigun (1999) mobility may depend positively on the income level if the share of resources devoted to education is large enough to offset the relative advantage of having educated parents. This relationship between mobility and income levels is likely caused

by the assumption of fixed education cost per pupil in these models, since more children can be educated when education becomes cheaper. Moreover, Hassler and Mora (2000) found that fast growth promotes mobility by increasing the market return to innate ability that is essential for adoption of newer technology, while Galor and Tsiddon (1997) showed that major inventions enhance mobility. (In our model, by contrast, technological advance is absent.)

How does mobility compare under private vs. public education? From (10) and (11):

$$\psi_{t+1} = 1 - \left(1 + \frac{s^2}{\phi_j^2 \Delta_t^2}\right)^{-1/2},\tag{12}$$

which indicates that mobility is higher under public education, since ϕ_j and Δ_t are both lower than under private education. We thus have:

Proposition 2: Intergenerational mobility, ψ_t , is greater under public than under private education in all generations $t \geq 1$ and as $t \to \infty$.

The reason mobility is higher under public education is of course that long-run mobility, $1 - \phi_j$, is greater due to the absence of differences among children in the quality of education received. This result agrees with the numerical simulation of intergenerational mobility in the steady-state equilibrium of Fernández and Rogerson (1998). They found that a state-financing education regime has higher intergenerational mobility than a local-financing regime; in fact, absent parental influence on children' education in their model, there is perfect intergenerational mobility under state-financing.⁸

IV. Comparative Dynamics

We noted in Section II that changes in the parameters α , β and s^2 affect short-run and long-run inequality. Here we look at how both inequality and mobility are affected. One result is the emergence of patterns which could, in principle, help to identify the source of secular trends in inequality. In all the analysis of this section we assume that society is in a steady-state at t=0 with $\Delta_0^2 = \frac{s^2}{1-\phi_s^2}$.

⁸While these results agree with our Proposition 2, note that our result applies in transition as well as in steady state, and that mobility is less than perfect in both education regimes in our model.

It is interesting to see the effects of changes in the variance of the shocks on mobility and inequality in both the short run and the long run. First, as we have seen, the level of long-run mobility is not affected by the variance of the shocks. However, will a permanent rise in s^2 have any impacts on mobility in the transition to a new steady-state equilibrium? We answer this question by analyzing the effects of a rise in s^2 to \tilde{s}^2 which happens unexpectedly at t=0. The appendix shows that the results extend to the case where the economy is not initially in steady state.

Proposition 3: Starting from a steady state path at t=0 with $\Delta_0^2=\Delta_\infty^2$ and $\psi_0=\psi_\infty$, the result of a permanent rise in s^2 to \tilde{s}^2 is: (i) in transition, $\tilde{\Delta}_t^2$ rises monotonically, while $\tilde{\psi}_t$ initially rises and then falls monotonically; and (ii) along the new steady state path, $\tilde{\Delta}_\infty^2>\Delta_\infty^2$ and $\tilde{\psi}_\infty=\psi_\infty$.

Proof. By (7), $\tilde{\Delta}_{t+1}^2 = \phi_j^2 \tilde{\Delta}_t^2 + \tilde{s}^2$ for $t \geq 0$ with $\tilde{\Delta}_0^2 = \Delta_0^2$. This equation has the following solution:

$$\begin{split} \tilde{\Delta}_t^2 &= (\phi_j^2)^t \Delta_0^2 + \left[\frac{1 - (\phi_j^2)^t}{1 - \phi_j^2} \right] \tilde{s}^2 \\ &= \frac{(\phi_j^2)^t s^2 + [1 - (\phi_j^2)^t] \tilde{s}^2}{1 - \phi_j^2}, \text{ since } \Delta_0^2 = s^2 / (1 - \phi_j^2) \end{split}$$

So $\tilde{\Delta}_{t+1}^2 - \tilde{\Delta}_t^2 = (\tilde{s}^2 - s^2)(\phi_j^2)^t > 0$ (due to $\tilde{s}^2 > s^2$), i.e. monotonically increasing. Also, $\tilde{\Delta}_{\infty}^2 = \tilde{s}^2/(1-\phi_j^2) > \Delta_{\infty}^2 = s^2/(1-\phi_j^2)$.

On the other hand, $\tilde{\psi}_{t+1} = 1 - [1 + \tilde{s}^2/(\phi_j^2 \tilde{\Delta}_t^2)]^{-1/2}$ for $t \ge 0$ according to (12). For t = 0, it follows that $\tilde{\psi}_1 = 1 - [1 + \tilde{s}^2/(\phi_j^2 \Delta_0^2)]^{-1/2}$. Thus, we have

$$\tilde{\psi}_1 - \psi_0 = [1 + s^2/(\phi_i^2 \Delta_0^2)]^{-1/2} - [1 + \tilde{s}^2/(\phi_i^2 \Delta_0^2)]^{-1/2}$$

which is increasing in \tilde{s}^2 . Also, the right-hand side of this equation would equal zero if \tilde{s}^2 were equal to s^2 . Thus, $\tilde{\psi}_1 > \psi_0$ must hold under $\tilde{s}^2 > s^2$. Moreover, note that $\tilde{\psi}_{t+1}$ is decreasing in $\tilde{\Delta}_t^2$. So as $\tilde{\Delta}_t^2$ rises in transition, $\tilde{\psi}_{t+1}$ must fall after the immediate rise. Furthermore, it is obvious that $\tilde{\psi}_{\infty} = 1 - \phi_j = \psi_{\infty}$.

The transition to the new equilibrium is illustrated in Figure 2. The pattern shown forms an interesting contrast with Figure 3 which shows the effect of a rise in persistence, ϕ_j . The latter may occur due to a rise in the effectiveness of parental time inputs, α , or as a result of an increase

in the effectiveness, β , of schooling under private education. (A rise in β has no effect on ϕ under public education.) The effect of a rise in ϕ_j is stated in:

Proposition 4: Starting from a steady state path at t = 0 with $\Delta_0^2 = \Delta_\infty^2$ and $\psi_0 = \psi_\infty$, the result of a permanent rise in ϕ_j to $\tilde{\phi}_j$ is: (i) in transition, $\tilde{\Delta}_t^2$ rises monotonically, while $\tilde{\psi}_t$ falls monotonically; and (ii) along the new steady state path, $\tilde{\Delta}_\infty^2 > \Delta_\infty^2$ and $\tilde{\psi}_\infty < \psi_\infty$.

Proof. By (7), $\tilde{\Delta}_{t+1}^2 = \tilde{\phi}_j^2 \tilde{\Delta}_t^2 + s^2$ for $t \geq 0$ with $\tilde{\Delta}_0^2 = \Delta_0^2$. Its solution is

$$\tilde{\Delta}_{t}^{2} = (\tilde{\phi}_{j}^{2})^{t} \Delta_{0}^{2} + \left[\frac{1 - (\tilde{\phi}_{j}^{2})^{t}}{1 - \tilde{\phi}_{j}^{2}} \right] s^{2}$$

$$= \frac{(\tilde{\phi}_{j}^{2})^{t} s^{2}}{1 - \phi_{j}^{2}} + \left[\frac{1 - (\tilde{\phi}_{j}^{2})^{t}}{1 - \tilde{\phi}_{j}^{2}} \right] s^{2}, \text{ since } \Delta_{0}^{2} = s^{2}/(1 - \phi_{j}^{2})$$

The new steady state inequality is then $\tilde{\Delta}_{\infty}^2 = s^2/(1-\tilde{\phi}_j^2)$. We thus have:

$$\tilde{\Delta}_{t+1}^{2} - \tilde{\Delta}_{t}^{2} = \frac{s^{2}[(\tilde{\phi}_{j}^{2})^{t+1} - (\tilde{\phi}_{j}^{2})^{t}](\phi_{t}^{2} - \tilde{\phi}_{j}^{2})}{(1 - \phi_{j}^{2})(1 - \tilde{\phi}_{j}^{2})}$$

$$= \frac{s^{2}(\tilde{\phi}_{j}^{2})^{t}(\tilde{\phi}_{j}^{2} - \phi_{j}^{2})}{1 - \phi_{j}^{2}} > 0, \quad \text{under } \tilde{\phi}_{j} > \phi_{j}$$

$$\tilde{\Delta}_{\infty}^{2} - \Delta_{\infty}^{2} = \frac{s^{2}(\tilde{\phi}_{j}^{2} - \phi_{j}^{2})}{(1 - \tilde{\phi}_{j}^{2})(1 - \phi_{j}^{2})} > 0, \quad \text{under } \tilde{\phi}_{j} > \phi_{j}$$

With respect to mobility, by (12), $\tilde{\psi}_{t+1} = 1 - [1 + s^2/(\tilde{\phi}_j^2 \tilde{\Delta}_t^2)]^{-1/2}$, which converges to $\tilde{\psi}_{\infty} = 1 - \tilde{\phi}_j < \psi_{\infty} = 1 - \phi_j$ under $\tilde{\phi}_j > \phi_j$. For t = 0, $\tilde{\psi}_1 = 1 - [1 + s^2/(\tilde{\phi}_j^2 \Delta_0^2)]^{-1/2}$. Thus, we have

$$\tilde{\psi}_1 - \psi_0 = [1 + s^2/(\phi_i^2 \Delta_0^2)]^{-1/2} - [1 + s^2/(\tilde{\phi}_i^2 \Delta_0^2)]^{-1/2}$$

which is decreasing in $\tilde{\phi}_j$. Also, the right-hand side of this equation would be zero if $\tilde{\phi}_j$ were equal to ϕ_j . So $\tilde{\psi}_1 < \psi_0$ must hold under $\tilde{\phi}_j > \phi_j$. Moreover, $\tilde{\psi}_{t+1}$ is decreasing in $\tilde{\Delta}_t^2$. Hence, as $\tilde{\Delta}_t^2$ rises in transition, $\tilde{\psi}_{t+1}$ must fall. \blacksquare

The reason that the comparison between a rise in s^2 vs. one in ϕ_j is interesting is that while the impact on inequality is qualitatively similar, the effect on mobility is very different. A change in s^2 has no effect on long-run mobility while a rise in ϕ_j reduces it. Further, the immediate effect of a rise

in s^2 is to create a sharp increase in (transitional) mobility. In contrast, a rise in α lowers mobility sharply. This contrast creates the possibility of distinguishing empirically between secular increases in inequality which may be created by these contrasting forces. If intergenerational mobility takes a large temporary increase, then idiosyncratic shocks may be driving the rise in inequality. On the other hand if there is a decrease in mobility then the current model says that it is more likely that inequality is rising due to greater importance of parental inputs.

Note also that a permanent regime switch from a steady-state equilibrium under public education to private education should have the same pattern of effects on inequality and mobility as those of a permanent rise in α or β . Such a regime switch increases inequality and reduces mobility in the short run and the long run by replacing $\phi_1 = \alpha$ by $\phi_2 = \alpha + \beta$.

Finally, note that comparing the original and new paths of inequality and mobility associated respectively with the old and new values of s^2 or ϕ_j is analogous to comparing inequality and mobility across economies that start with the same initial inequality but have different values of s^2 or ϕ_j . Given the same initial inequality the economy with a higher s^2 will always have higher inequality, and have higher mobility in all finite future periods but share the same steady-state mobility with others, as shown in Proposition 7 in the Appendix. On the other hand, the economy with a higher ϕ_j will have higher inequality and lower mobility both for all finite periods, and in the steady-state.

V. Applications

It is interesting to ask what implications our model may have for trends over time and cross-country comparisons. In order to do so it is useful to think in terms of smaller differences, or smoother changes, in ϕ_1 and ϕ_2 than considered above.

In the model we have set out, a pure private system has $\phi_1 = \alpha + \beta$, and a public system has $\phi_2 = \alpha$. We have studied the consequences of moving from ϕ_1 to ϕ_2 or vice versa. A simple generalization of our model would have a single parameter ϕ lying between α and $\alpha + \beta$ depending on the degree to which the system was "private" or "public". Other things equal, one could then

 $^{^9}$ This result can be obtained in a model where the goods component, L_i , in the human capital production function

think of ϕ as having fallen in most western countries in the 19th century with the spread of public education, and as having decreased further or stayed low throughout the 20th century.¹⁰ In the U.S., the U.K., and perhaps some other countries it may have increased in recent decades, with greater differences in public school quality, more private schooling, and increased participation in the (relatively heterogeneous) post-secondary system. Absent other factors, these trends should have produced a decline in inequality combined with rising intergenerational mobility across most countries up to around the middle of the 20th century. In the U.S. and U.K., if we are right in thinking that ϕ has been rising recently, inequality would have initially declined, but then would have bottomed out and started to increase in recent decades.

The predicted trends in earnings inequality roughly resemble what has been observed. In Europe, while inequality rose during the industrial revolution, at some point in the late 19th century (varying from 1870 to 1900 in different countries) pre-tax income inequality began to decrease, and continued to decline through to the 1970's. (See Morrison, 2000, and Lindert, 2000.) In the U.S., the decline in income inequality set in later, but was dramatic from the immediate pre-WWII to post-war periods (Goldin, 1992). Subsequently there was little trend until the early 1970's, when inequality began to increase. (See e.g. Gottschalk, 1997). Aside from the fact that the initial decline of inequality was delayed in the U.S. until WWII, these trends are as predicted by the changes in education systems over time mentioned above.

The delayed fall in inequality in the U.S. has an intriguing possible implication. Throughout the 19th century and for much of the 20th, the U.S. was regarded as a land of equal opportunity and high mobility – especially in comparison with Europe. Our model indicates that, in part, this contrast may have been due not to differences in equality of opportunity, but to positive transitional mobility, due to rising inequality, in the U.S. vs. the opposite in Europe. If so, U.S. social history in the late 19th and early 20th centuries may need to be seen in a new light.

With the help of our model, observations on trends in intergenerational earnings mobility would

is a composite of a range of inputs each of which can be provided publicly or privately.

¹⁰See e.g. Boyd and King (1980) on the timing of the spread of public education.

make possible some guesses about the sources of rising or falling inequality. Unfortunately, it is only with the advent of panel data that estimates of this mobility have become possible, and we do not yet have an indication of trends over time. On the other hand, sociologists have long been interested in trends in intergenerational occupational mobility, and their results are suggestive. Blau and Duncan (1967), Featherman and Hauser (1978), Hout (1988), and Hauser et al. (1996) together document a rise in intergenerational occupational mobility that continued from the earlier years of the century through the 1980's. ¹¹ The rate of increase slowed in the 1980's, however, which is perhaps consistent with the finding by economists that *intra*generational mobility did not rise after the 1970's (Gottschalk, 1997).

Suppose trends in intergenerational earnings mobility were roughly similar to those in occupational mobility in the U.S. in the $20^{\rm th}$ century. Then there would be an important contrast between trends from the 1930's to the 1970's vs. those observed more recently. The earlier period would be one of declining inequality and rising mobility, while the latter period would feature rising inequality and mildly increasing or constant mobility. In terms of our model, falling ϕ (consistent with schooling trends) must have dominated in the earlier period, since it reduces inequality and raises mobility. Falling s^2 would have reduced mobility. On the other hand, since the 1970's both rising s^2 and increasing ϕ may have been at work. (Both would raise inequality, but their effects on current mobility would offset.) This story is consistent with the finding of increased returns to unobserved ability (see Juhn et al., 1993) and the apparently increasing "privateness" of the U.S. education system in recent decades.

Solon (1999) has surveyed the comparative cross-country evidence on intergenerational mobility. He finds (p. 1787) that "...the United Kingdom and United States do appear to be less mobile societies than Canada, Finland, and Sweden". While intuition might suggest that countries with less regulated markets would have greater mobility, the evidence is that it is the more social democratic countries, with their relatively homogeneous systems of public education (lower ϕ), where mobility

¹¹These sociologists make a distinction between structural mobility, which would occur due to changes in the shape of the earnings distribution alone, and "circulation mobility", which represents changes in relative rank. Their empirical result is that circulation mobility rose throughout.

is higher. This is strikingly consistent with our model.

An apparent challenge to our model is provided by the comparison between Italy and the U.S. Checchi et al. (1999) and others (e.g. Benabou and Ok, 2000) have found that Italy has both lower inequality and lower intergenerational mobility than the U.S. In terms of our model, this could only occur in steady state if Italy has higher ϕ and lower s^2 .¹² But Italy is said to have a "centralised and egalitarian school system" (Checchi et al.). So why does it not have more mobility than the U.S., along with Canada, Finland and Sweden? The answer may lie in the fact that although the Italian education system is financed through taxation and is relatively standardized, educational attainment is found to be more closely related to parents' than it is in the U.S. In other words, in Italy public schooling does not play the role of reducing differences in goods inputs in human capital production that it is assumed to play in our model. This observation sounds an important cautionary note. The mobility-enhancing effects of public schooling can only be felt if public schooling is also more equal schooling.

VI. Conclusion

We have shown how intergenerational mobility can readily be analyzed in relation to inequality in a simple two period overlapping generations model. We have found that societies with more long-run mobility, or regression toward the mean, also have less long-run inequality *ceteris paribus*. Since in our model a public education regime produces more regression to the mean, public education leads to lower long-run inequality than private education. For societies beginning with the same level of inequality, public education also gives more mobility and less inequality in transition.

In the short-run, current mobility can differ from long-run mobility. We have labeled the difference transitional mobility. Transitional mobility is above zero when inequality must rise over time to reach the steady state, and it is negative if inequality is falling from generation to generation. Thus, while long-run mobility is a force for greater equality, transitional mobility has no such significance. It is merely a reflection of rising inequality. This means that it is important

¹²Higher ϕ is needed to explain the lower mobility in Italy, but would, in itself, create higher inequality. So s^2 must be sufficiently smaller to produce lower inequality.

to distinguish between these two forms of mobility both theoretically and empirically. It may be, for example, that current mobility has not declined in the recent years of rising inequality but that long-run mobility and equality of opportunity have done so.

The two determining factors in the earnings process we study are the rate of regression to the mean (long-run mobility) and the variance of shocks in each generation. The latter can be interpreted in our model as the returns to unobserved skill or "ability". Inequality can rise either because of increased returns to ability, or because of a greater impact of private schooling. Either will lead to higher inequality, but the immediate impact of increased shocks is to raise mobility, while that of more private schooling is to reduce mobility. Moreover, long-run mobility does not change in response to increased shocks, whereas it falls with a larger role for private schooling. These differences imply that the two sources of increased inequality generate quite different transitional profiles of mobility. In principle, this provides a method of identifying better the dominant influences during periods of secularly changing inequality.

We have illustrated the possible implications of our model by discussing historical trends and national differences. We noted, for example, that in the late 19th and early 20th centuries the U.S. showed rising inequality, whereas Europe largely showed the opposite. According to our model this means that there would have been positive transitional mobility in the U.S. at the time, but negative transitional mobility in Europe. This difference may have been partly responsible for the contrast between high mobility in the New World and lower mobility in the Old, which was evident in this period.

While observations on trends over time in intergenerational earnings mobility are not yet available, the evidence is that intergenerational occupational mobility rose throughout most of the 20th century in the U.S., although at a falling rate after the 1970's. Suppose the trend in earnings mobility was roughly similar. In terms of our model, rising current mobility would imply that the reduction of inequality from the 1930's to the early 1970's must have been due mainly to increasing long-run mobility. In contrast, in the most recent period it appears likely that both rising ability shocks and reduced long-run mobility were at work.

Turning to international differences, the evidence is that a number of countries with fairly standardized and near-universal public schooling – Canada, Finland and Sweden – have both lower inequality and higher intergenerational earnings mobility than the U.S. and U.K., whose education systems are arguably more towards the private pole. This is consistent with our model. On the other hand, Italy has a fairly uniform tax-financed education system combined with both lower inequality and lower mobility than the U.S. We have argued that this is explicable in terms of our model since schooling attainment in Italy is more closely linked to parental attainment than in the U.S.

Finally, we must acknowledge the limitations of our model. It is built on simple functional forms and has ignored technological innovations, progressive taxation, and income redistribution through means other than public education. Extending the model in these dimensions would enrich our understanding of intergenerational mobility. For example, incorporating technological innovation may increase the rate of return on human capital investment and can thus increase intergenerational mobility as in Hassler and Mora (2000). The effects of a variety of public policies that may affect the rate of innovation, such as R&D subsidies, can be studied in such a framework. Bringing in progressivity and redistribution would allow us to analyse e.g. the effects of reducing education expenditures and increasing transfers, or to allow for the fact that more redistribution often goes along with more standardized and egalitarian public education.

Appendix

Letting $\Lambda(\tilde{s}^2) = [\tilde{s}^2(1-\phi_j^2)/(\tilde{s}^2-\phi_j^2s^2)]^{1/2}$ where $0 < \Lambda < 1$, we provide more general initial conditions for Propositions 3 and 4 below.

Proposition 5: Given (Δ_t, ϕ_j, s^2) for $t \leq 0$, and $(\tilde{\Delta}_t, \phi_j, \tilde{s}^2)$ for $t \geq 1$ such that $\tilde{s}^2 > s^2$ and $\tilde{\Delta}_{t+1}^2 = \phi_j^2 \tilde{\Delta}_t^2 + \tilde{s}^2$ for $t \geq 0$, we have (i) $\tilde{\Delta}_1 > \Delta_0$ if $\Delta_0 < \tilde{\Delta}_\infty$, (ii) $\tilde{\psi}_1 > \psi_0$ if $\Delta_0 > \Delta_\infty \Lambda$, (iii) $\tilde{\Delta}_\infty > \Delta_\infty$, (iv) $\tilde{\psi}_\infty = \psi_\infty$, (v) $\tilde{\Delta}_t$ rises monotonically from t = 0 to $t = \infty$ if $\Delta_0 < \tilde{\Delta}_\infty$, and (vi) $\tilde{\psi}$ falls monotonically from t = 1 to $t = \infty$ if $\Delta_0 < \tilde{\Delta}_\infty$.

Proof. From (7) $\tilde{\Delta}_1^2 - \Delta_0^2 = (1 - \phi_j^2)(\tilde{\Delta}_\infty^2 - \Delta_0^2)$. Thus, $\tilde{\Delta}_1 > \Delta_0$ if $\Delta_0 < \tilde{\Delta}_\infty$ (part (i)). From (12), $\tilde{\psi}_1 - \psi_0 = [1 + s^2/(\phi_j^2 \Delta_{-1}^2)]^{-1/2} - [1 + \tilde{s}^2/(\phi_j^2 \Delta_0^2)]^{-1/2}$ where both terms in brackets are positive. Then sign $(\tilde{\psi}_1 - \psi_0) = \text{sign } \{[1 + s^2/(\phi_j^2 \Delta_{-1}^2)]^{-1} - [1 + \tilde{s}^2/(\phi_j^2 \Delta_0^2)]^{-1}\} = \text{sign } (\tilde{s}^2 \Delta_{-1}^2 - s^2 \Delta_0^2)$. In addition, from (7) $\Delta_{-1}^2 = (\Delta_0^2 - s^2)/\phi_j^2$. So sign $(\tilde{\psi}_1 - \psi_0) = \text{sign } [\tilde{s}^2(\Delta_0^2 - s^2) - \phi_j^2 s^2 \Delta_0^2] = \text{sign } (\Delta_0^2 - \Delta_\infty^2 \Lambda^2) = \text{sign } (\Delta_0 - \Delta_\infty \Lambda)$ where $\Delta_\infty^2 = s^2/(1 - \phi_j^2)$. Part (ii) then follows. The proof of other parts is similar to that in Propositions 1 and 3.

Proposition 6: Given (Δ_t, α, s^2) for $t \leq 0$, and $(\tilde{\Delta}_t, \tilde{\alpha}, s^2)$ for $t \geq 1$ such that $\tilde{\alpha} > \alpha$, $\tilde{\phi}_j > \phi_j$; and $\tilde{\Delta}_{t+1}^2 = \tilde{\phi}_j^2 \tilde{\Delta}_t^2 + s^2$ for $t \geq 0$, we have (i) $\tilde{\Delta}_1 > \Delta_0$ if $\Delta_0 < \tilde{\Delta}_\infty$, (ii) $\tilde{\psi}_1 < \psi_0$ if $\Delta_0 < \tilde{\Delta}_\infty$, (iii) $\tilde{\Delta}_\infty > \Delta_\infty$, (iv) $\tilde{\psi}_\infty < \psi_\infty$, (v) $\tilde{\Delta}_t$ rises monotonically from t = 0 to $t = \infty$ if $\Delta_0 < \tilde{\Delta}_\infty$, and (vi) $\tilde{\psi}_j$ falls monotonically from t = 0 to $t = \infty$ if $\Delta_0 < \tilde{\Delta}_\infty$.

Proof. Similar to that of Proposition 5.

According to Proposition 5, the results in Proposition 3 hold so long as initial inequality is below the new higher steady-state inequality and above a level which is lower than the initial steady-state inequality (since $0 < \Lambda < 1$). According to Proposition 6, the results in Proposition 4 hold provided initial inequality is below the new higher steady-state inequality.

Proposition 7: For two economies with (Δ_0, ϕ_j, s^2) and $(\Delta_0, \phi_j, \tilde{s}^2)$ such that $\tilde{s}^2 > s^2$, we have: $(i) \ \tilde{\Delta}_t > \Delta_t \ from \ t = 1 \ to \ t = \infty, \ and \ (ii) \ \tilde{\psi}_t > \psi_t \ from \ t = 1 \ to \ t < \infty \ and \ \tilde{\psi}_{\infty} = \psi_{\infty}.$ Proof. Part (i) is obvious from (7). For part (ii), note that (7) implies $\Delta_t^2 = \phi_j^{2t} \Delta_0^2 + (1 - \phi_j^{2t}) \Delta_\infty^2$ and $\tilde{\Delta}_t^2 = \phi_j^{2t} \Delta_0^2 + (1 - \phi_j^{2t}) \tilde{\Delta}_\infty^2$. Combining these equations with (9) and (10), we have

$$\begin{split} \tilde{\psi}_{t} - \psi_{t} &= r_{t} - \tilde{r}_{t} = \frac{r_{t}^{2} - \tilde{r}_{t}^{2}}{r_{t} + \tilde{r}_{t}} \\ &= \frac{\phi_{j}^{2} (\Delta_{t-1}^{2} \tilde{\Delta}_{t}^{2} - \Delta_{t}^{2} \tilde{\Delta}_{t-1}^{2})}{(r_{t} + \tilde{r}_{t}) \Delta_{t}^{2} \tilde{\Delta}_{t}^{2}} \\ &= \frac{\Delta_{0}^{2} \phi_{j}^{2t} (\tilde{s}^{2} - s^{2})}{(r_{t} + \tilde{r}_{t}) \Delta_{t}^{2} \tilde{\Delta}_{t}^{2}}, \quad t \geq 1. \end{split}$$

Therefore, $\tilde{\psi}_t > \psi_t$ for $1 \le t < \infty$ under $\tilde{s}^2 > s^2$. As $t \to \infty$, it is evident that $\psi_\infty = \tilde{\psi}_\infty$ since $\lim_{t \to \infty} \phi_j^t = 0$.

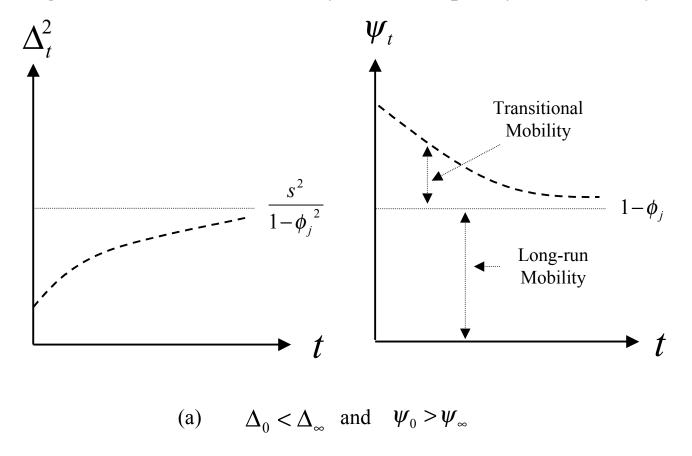
References

- Becker, Gary S. and Tomes, Nigel. "An Equilibrium Theory of the Distribution of Income and Intergenerational Mobility." *Journal of Political Economy*, December 1979, 87(6), pp. 1153-89.
- Benabou, Roland. "Heterogeneity, Stratification, and Growth: Macroeconomic Implications of Community Structure and School Finance." *American Economic Review*, June 1996, 86(3), pp. 584-609.
- Benabou, Roland. "Unequal Societies: Income Distribution and the Social Contract." American Economic Review, March 2000, 90 (1), pp. 96-129.
- Benabou, Roland and Ok, Efe A. "Social Mobility and the Demand for Redistribution: The POUM Hypothesis." Quarterly Journal of Economics, May 2001, 116(2), pp. 447-87.
- Benabou, Roland and Ok, Efe A. "Mobility as Progressivity: Ranking Income Processes According to Equality of Opportunity", NBER Working Paper No. 8431, August 2001.
- Blau, Peter M. and Duncan, Otis Dudley. *The American Occupational Structure*. New York: John Wiley and Sons, 1967.
- Boyd, William and King, Edmund J. *The History of Western Education*. (11th Ed.) Totowa, N.J.: Barnes and Noble, 1980.
- Checchi, Daniele; Ichino, Andrea and Rustichini, Aldo. "More Equal But Less Mobile? Education Financing and Intergenerational Mobility in Italy and in the US", *Journal of Public Economics*, 74 (3), 1999, pp. 351-394.
- Eckstein, Zvi and Zilcha, Itzhak. "The Effect of Compulsory Schooling on Growth, Income Distribution and Welfare." *Journal of Public Economics*, July 1994, 54(3), pp. 339-59.
- Featherman, David L. and Hauser, Robert M. Opportunity and Change. New York: Academic Press, 1978.
- Fernández, Raquel and Rogerson, Richard. "Public Education and Income Distribution: A Dynamic Quantitative Evaluation of Education-Finance Reform." *American Economic Review*, September 1998, 88(4), pp. 813-33.
- Galor, Oded and Tsiddon, Daniel. "Technological Progress, Mobility, and Economic Growth." *American Economic Review*, June 1997, 87(3), pp. 363-82.
- Galton, F. "Regression towards Mediocrity in Hereditary Stature." Journal of the Anthropological Institute of Great Britain and Ireland, 1886, 15, pp. 246-63.
- Glomm, Gerhard and Ravikumar, B. "Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality." *Journal of Political Economy*, August 1992, 100(4), pp. 818-34.

- Goldin, Claudia. "The Great Compression: The Wage Structure in the United States at Mid-Century." Quarterly Journal of Economics, February 1992, 107, pp. 1-34.
- Gottschalk, Peter. "Inequality, Income Growth, and Mobility: The Basic Facts." *Journal of Economic Perspectives*, Spring 1997, 11, pp. 21-40.
- Hassler, John and Mora, José V. Rodríguez. "Intelligence, Social Mobility, and Growth." *American Economic Review*, September 2000, 90(4), pp. 888-908.
- Hauser, Robert M.; Warren, John R.; Huang, Min-Hsiung and Carter, Wendy Y. "Occupational Status, Education, and Social Mobility in the Meritocracy." Center for Demography and Ecology Working Paper No. 96-18, University of Wisconsin, 1996.
- Hout, Michael, "More Universalism, Less Structural Mobility: The American Occupational Structure in the 1980s." *American Journal of Sociology*, May 1988, 93(6), pp. 1358-1400.
- Iyigun, Murat F. "Public Education and Intergenerational Economic Mobility." *International Economic Review*, August 1999, 40(3), pp. 697-710.
- Juhn, Chinhui; Murphy Kevin M. and Pierce, Brooks. "Wage Inequality and the Rise in the Returns to Skill", *Journal of Political Economy*, 101, 1993, pp. 410-442.
- Lindert, Peter H. "Three Centuries of Inequality in Britain and America", in A.B. Atkinson and F. Bourguiguon (eds.), *Handbook of Income Distribution*. Amsterdam: North Holland, 2000, pp. 167-216.
- Loury, Glenn. "Intergenerational Transfers and the Distribution of Earnings." *Econometrica*, July 1981, 49(4), pp. 843-67.
- Maoz, Yishay D. and Moav, Omer. "Intergenerational Mobility and the Process of Development." *Economic Journal*, October 1999, 109(458), pp. 677-697
- Morrison, Christian. "Historical Perspectives on Income Distribution: The Case of Europe", in A.B. Atkinson and F. Bourguiguon (eds.), *Handbook of Income Distribution*. Amsterdam: North Holland, 2000, pp. 217-260.
- Owen, Ann L. and Weil, David N. "Intergenerational Earnings Mobility, Inequality and Growth." Journal of Monetary Economics, February 1998, 41, pp. 71-104.
- Saint-Paul, Gilles and Verdier, Thierry. "Education, Democracy and Growth." *Journal of Development Economics*, December 1993, 42(2), pp. 399-408.
- Shorrocks, Anthony F. "On the Hart Measure of Income Mobility", in Casson, Mark and John Creedy (eds.), *Industrial Concentration and Economic Inequality: Essays in Honour of Peter Hart*, Aldershot, U.K.: Elgar; distributed in the U.S. by Ashgate, Brookfield, Vt., 1993, pp. 3-21.

- Solon, Gary. "Intergenerational Mobility in the Labor Market", in Ashenfelter, Orley and David Card (eds.), *Handbook of Labor Economics* Volume 3A, Amsterdam: Elsevier, 1999, pp. 1761 1800.
- Zhang, Jie. "Optimal Public Investments in Education, and Endogenous Growth." Scandinavian Journal of Economics, September 1996, 98(3), pp. 387-404.

Figure 1: Transition to Steady-State Inequality and Mobility



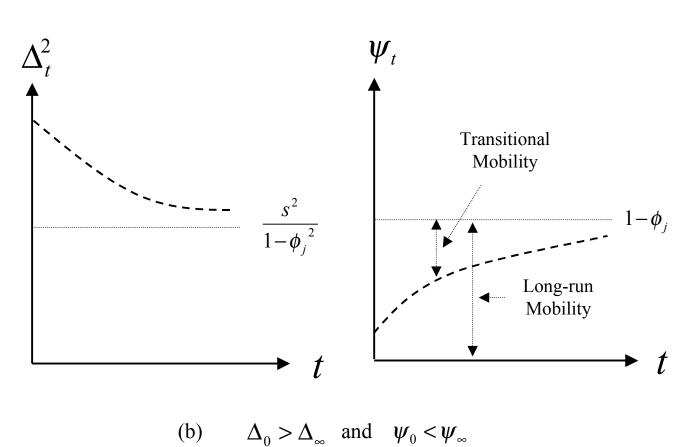


Figure 2: Effects of a Rise in S^2 on Inequality and Mobility Starting From Initial Steady State

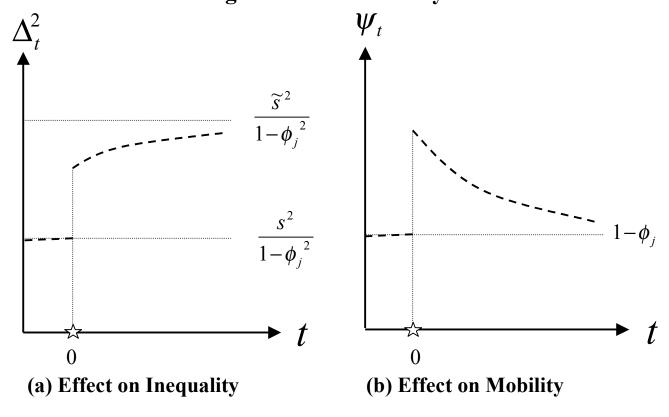


Figure 3: Effects of a Rise in ϕ_j on Inequality and Mobility Starting From Initial Steady State

