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Consumer Location and Local Public Goods

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1. Introduction

For years economists have recognized that consumer migration interacts with the quantities of private and local public goods exchanged in different communities.[1] Charles M. Tiebout [13] observed that a rational consumer will move to a political jurisdiction offering a vector of public goods which best satisfies his preferences. James Buchanan [2] and Mark Pauly [12] have recognized migration can change the preferences, endowments and technology in a jurisdiction, and consequently induce a new allocation of public and private goods. Martin McGuire [9] has extended Buchanan's and Pauly's work. He develops an economy with one private good and one local public good produced under conditions of crowding.[2] McGuire derives necessary conditions for consumers to be partitioned into a Pareto optimal jurisdiction structure when they have identical preferences and endowments. He then supposes there are two types of consumers and argues geometrically that this heterogeneous population must be partitioned into segregated jurisdictions to achieve optimality. Jurisdictions integrated by preferences or incomes are Pareto inferior.

McGuire's work advances the theory of local public goods but leaves several problems unsolved. His model contains no explicit relative prices or market behavior, so he cannot analyze whether his optimality conditions will be generated or sustained as the outcome of some market process. He claims his conclusion that integrated jurisdictions are suboptimal does not depend on his assumption that all residents pay equal taxes. In section 4 of this paper I will illustrate that a variable tax-price scheme makes both segregated and integrated consumer partitions Pareto optimal. I also prove that if crowding is sufficiently strong, an integrated consumer partition can be sustained as an equilibrium with no migration. Furthermore, a segregated partition will never be an equilibrium because a low income consumer living in a low income community can always increase his utility by moving into a rich community and sharing its public goods. Finally, McGuire ignores the indivisibility problems inherent in consumer migration. In deriving his optimality results he looks for the intersection of two curves which are assumed to be continuous functions of the number of residents. But since the number of residents can only be a positive integer, the possibility arises that these curves are either not well defined or do not intersect. Consumer indivisibilities cast doubt on the existence of optima in McGuire's model.

Indivisibilities also create doubts about the existence of a market equilibrium. When a consumer migrates, he exits from public good markets in one community and enters markets in another. His exit and entry cause discrete jumps of public good demand curves in both communities. Crowding in the production of local public goods creates additional problems by causing discrete supply curve shifts when someone moves. Migration causes demand and supply jumps which can induce more migration leading to further supply and demand adjustments, additional migration, and so on. The system may never settle down to an equilibrium.

Bryan Ellickson [4] explicitly addresses the problems of consumer migration and indivisibility in an economy where local public goods are allocated using Lindahl prices. The Lindahl scheme requires that each consumer pay a tax price for each public good equal to his marginal rate of substitution between the public good and a numeraire private good. In equilibrium the sum of all residents' tax prices equals the marginal cost of producing the public good. Ellickson develops several examples to illustrate problems caused by consumer indivisibilities. In one case where consumers have transferable utility,[3] he constructs an economy which has no feasible Lindahl equilibrium. He can compute an equilibrium but it is not feasible because it requires an individual to simultaneously consume a public good in more than one jurisdiction. Since the person cannot live in two places at once, the Lindahl allocation is an infeasible convex combination of two feasible allocations, one with the consumer in each jurisdiction. Consumer indivisibility leads to a non-convex aggregate production set, which implies no Lindahl price system exists which can always clear private and public good markets and reduce voluntary consumer migration to zero.

But why are consumer indivisibilities a problem? Why do some situations require a consumer to live in two places at once? The basic underlying reason for non-existence of a Lindahl equilibrium is a conflict among the consumers over who will live together. If all consumers can agree a particular partition is best, they can always form that partition and find an equilibrium with no migration. But when consumers disagree about where each of them should live, no stable partition will form, and an equilibrium does not exist because people keep moving from place to place. When consumers cannot agree on a partition they cannot agree on a technology since with crowding the technology used depends on which partition is generated. Consumer indivisibility implies the consumers cannot resolve their conflict by employing a convex combination of all most-preferred technologies associated with people's most-preferred partitions, since this would require that some consumers live in more than one jurisdiction. But without disagreement over which partition is best, the desire to employ more than one technology would never arise.

The main purpose of this paper is to extend the theory of local public goods by generating restrictions on preferences, endowments or technology which eliminate consumer conflict and insure the existence of a Lindahl equilibrium with no consumer migration. No other author has developed such existence theorems. The paper also explores the optimality properties of these equilibria.[4] If the restrictions seem reasonable, and if the equilibria are optimal, then a Lindahl scheme for allocating resources would benefit society. If not, then society needs to devise some alternate scheme for assigning people to cities.

My model differs from Ellickson's in three major respects. It permits interjurisdictional trading of private goods so consumers can live in a suburb and work in a central city, go on shopping trips to adjacent communities, and in general pursue consumption activities common in urban areas. Ellickson's model has more of an interurban interpretation since no intercity trading of private goods is more likely to occur when there are greater distances between cities. The difference between these economies is significant. As proved in section 3, people in the interurban economy will always voluntarily form one political jurisdiction to jointly consume locally pure public goods, while consumers in the intraurban economy may not. Another major difference in our models occurs in the handling of technology. I work with production sets which vary with the number of consumers assigned to different jurisdictions. This makes easier the explicit modeling of crowding in public goods production. A final difference occurs in the treatment of prices. I explicitly account for endogenous price changes resulting from migration. Induced income changes and income effects resulting from these price changes have a considerable impact on migratory behavior.

In section 2 I define a local Lindahl equilibrium which involves utility and profit maximization, and cleared markets in every jurisdiction given any fixed partition of consumers into jurisdictions. One can conceptually compute a local equilibrium for every possible consumer partition. A consumer most prefers that partition which gives him the greatest local equilibrium utility, but he can choose only his own location while others determine where they want to live. Then the resulting partition is the outcome of all consumers' residential choices, and yields associated local equilibrium utilities to the consumers.

A partition and its associated local equilibrium is a global equilibrium if no consumer can increase his utility by changing his location given the locational choices of the other consumers. When a global equilibrium occurs, consumer migration is zero and all public and private markets clear. This definition of global equilibrium is equivalent to the definition of a pure strategy Nash equilibrium point in an n-person non-zero-sum game. Hence the search for restrictions on preferences, endowments and technology which insure existence of a global equilibrium can be seen in game theoretic terms as a search for restrictions on the players' payoff functions or admissable outcomes which guarantee existence of a pure strategy Nash equilibrium. But rather than arbitrarily imposing restrictions directly on outcomes in the game, I work with the underlying economic process which generates these outcomes. Hopefully this method will prove useful in analyzing other problems which have a similar hierarchical structure.

Section 3 of this paper explores existence when public goods are locally pure. In addition to proving existence in an interurban economy, I prove existence of an intraurban global equilibrium when producers use a generalized Leontief technology or when consumers are identical. I also show any global equilibrium with everyone living together when public goods are pure is globally Pareto optimal. Section 4 analyzes economies with crowded public goods. In addition to proving the previously mentioned results about integrated and segregated partitions, I show that an economy with identical consumers can generate a global equilibrium. Finally, I give an example of an economy which has a global equilibrium that is not globally Pareto optimal.

2. Definitions and Notation

A. Jurisdictions and Partitions

The i=1,...,I consumers in the economy are partitioned into j=1,...,J jurisdictions. A partition q specifies a location j for each i. For example, if J=2 and I=3, then q=(2,1,2) means consumers 1 and 3 live together in jurisdiction 2 and consumer 2 lives in 1. In general there are $Q=J^I$ possible partitions of I people into J communities. Let $i \underset{q}{\epsilon} j$ mean consumer i is assigned to j by partition q. Let j_i be i's location with q suppressed. Similarly, the h=1,...,H producers in the economy are partitioned into the jurisdictions. Assume that producer locations are fixed, so that the locational subscripts for producers may be deleted.

There are NJ private goods and MJ local public goods distinguished by their physical characteristics and the jurisdiction in which they are exchanged. Each consumer may purchase and sell private goods in any jurisdiction but can consume local public goods only where he lives.[5] Furthermore, all consumers living in a given jurisdiction must consume equal quantities of the local public goods provided there.

B. Production

A production for firm h given consumer partition q is $(v_h,z_h)^q$, an NJ vector of private inputs and outputs (v_h) and an MJ vector of public outputs (z_h). Let V_h^q be the production set for h given q. Then the aggregate production set V^q equals $\sum_{h=1}^{H} V_h^q$. A point in V^q is $(v,z)^q = \sum_{h=1}^{H} (v_h,z_h)^q$, an $(N+M)J$ vector of private and public goods provided to each jurisdiction by producers in all jurisdictions.

Note the aggregate set distinguishes commodities by their location; in effect there is a complete set of private and public markets in each jurisdiction. Also observe that firm and aggregate production sets are defined relative to a consumer partition. When production sets can vary with partition changes, crowding in the production of public goods can be analyzed.

Make the following assumptions about V^q and V_h^q :

(A.1) If $(v_h,z_h)^q \in V_h^q$ and $0 \leqq t$, then $t(v_h,z_h)^q \in V_h^q$. (Constant returns to scale)[6]

(A.2) V^q is closed.

(A.3) $0 \in V_h^q$. (Possibility of inaction)

(A.4) $V^q \cap R^{(M+N)J+} = 0$. (Impossibility of free production)

(A.5) $R^{(M+N)J-} \subset V^q$. (Free disposal)

(A.6) For any $(v_h,z_h)^q$, $(\bar{v}_h,\bar{z}_h)^q \in V_h^q$ and $0 \leqq t \leqq 1$,

$$t(v_h,z_h)^q + (1-t)(\bar{v}_h,\bar{z}_h)^q \in V_h^q . \text{ (Convexity)}$$

(A.7) There exists $(v,z)^q \in V^q$ such that $z > 0$.

(All public goods can be simultaneously produced)

(A.8) If $(v,z)^q \in V^q$ and $\bar{z}_{mj} = z_{mj}$ when $z_{mj} \geqq 0$ but $\bar{z}_{mj} = 0$ when $z_{mj} < 0$, then $(v,\bar{z})^q \in V^q$. (No public good is required as a production input)

(A.9) For any $\bar{z}$ there is a $\bar{v}$ such that $(\bar{v},\bar{z})^q \in V^q$.

C. Consumption

Given $i \underset{q}{\varepsilon} I$, a consumption for i is (x_i, y_i), an NJ + M vector of NJ private goods exchanged in all jurisdictions and M public goods provided in the jurisdiction to which i is assigned by q. By allowing individuals to buy and sell private goods outside of their resident jurisdictions, situations where consumers live in one jurisdiction and work in another can be analyzed. All suburban commuters are clearly in this position.

Of course transportation costs will generally keep a consumer from choosing a residence hundreds of miles from his workplace. In this paper transportation costs are implicitly zero, which gives the model an intra-urban interpretation since short distances make transport costs much less significant. In an interurban version of the model with very large inter-jurisdictional transport costs, private consumptions are restricted to commodities found in jurisdictions of residence.

Let X_i be the set of all possible consumptions for i given some q. Each i has a complete preference pre-ordering $\underset{i}{\succsim}$ defined on X_i and an initial endowment of private goods $\omega_i \in R^{NJ}$. Let $\omega = \sum_{i=1}^{I} \omega_i$ and $\omega_j^q = \sum_{i \underset{q}{\varepsilon} j} \omega_i$.

Make the following assumptions about X_i, $\underset{i}{\gtrsim}$ and ω_i for every i:

(A.10) $X_i = R^{(M+NJ)+}$.

(A.11) for every $(\bar{x}_i,\bar{y}_i) \in X_i$, the sets $\{(x_i,y_i) \in X_i | (x_i,y_i) \underset{i}{\gtrsim} (\bar{x}_i,\bar{y}_i)\}$ and $\{(x_i,y_i) \in X_i | (x_i,y_i) \underset{i}{\lesssim} (\bar{x}_i,\bar{y}_i)\}$ are closed in X_i.

(Continuity of preferences)

(A.12) If $(x_i,y_i) \underset{i}{\succ} (\bar{x}_i,\bar{y}_i)$ and if $a,b > 0$ with $a+b = 1$, then $a(x_i,y_i) + b(\bar{x}_i,\bar{y}_i) \underset{i}{\succ} (\bar{x}_i,\bar{y}_i)$. (Convexity of preferences)

(A.13) If $(x_i,y_i) \geq (\bar{x}_i,\bar{y}_i)$ then $(x_i,y_i) \underset{i}{\succ} (\bar{x}_i,\bar{y}_i)$. (Monotonicity)

(A.14) $\omega_i \geq 0$.

D. Prices, Profits and Feasible Allocations

Let $p^q = (p^q_{11},\ldots,p^q_{NJ})$ be an NJ vector of private good prices given partition q. Let $s^q_{ij} = (s^q_{ij1},\ldots,s^q_{ijM})$ be an M vector of personalized public good prices for $i \underset{q}{\varepsilon} j$. Let $s^q_j = \sum\limits_{i \underset{q}{\varepsilon} j} s^q_{ij}$ be the total prices paid for public goods in j by its residents. Let $s^q = (s^q_1,\ldots,s^q_J)$.

Assume that the economy is a private ownership economy so that profits of firm h are distributed to consumers according to fixed shares in each firm $\theta_{ih} \geq 0$, $\sum\limits_{i=1}^{I} \theta_{ih} = 1$ for each $h = 1,\ldots,H$. Profits to h depend on all prices and can be written $\pi_h(p^q,s^q)$. Then i's wealth constraint is

$$w^q_i = p^q\,\omega_i + \sum_{h=1}^{H} \theta_{ih}\,\pi_h\,(p^q,s^q).$$

For each q there is a local private ownership economy

$E^q = \{(X_i, \succsim_i, \omega_i, \theta_{ih}), V_h^q\}$ for $i = 1,\ldots,I$ and $h = 1,\ldots,H$. Given any q, the set of locally feasible allocations A^q is the set of all consumptions and productions such that consumers and producers are in their consumption and production sets, each consumer in any jurisdiction consumes the entire vector of public goods produced for that jurisdiction, and the sum of all private good consumptions minus the sum of all private good productions equals the sum of consumers' initial endowments. Formally, a feasible allocation $a^q \varepsilon A^q$ is an $[(NJ + M)I + (N + M)JH]$ vector $[(x_1, y_1) \varepsilon X_1, \ldots, (x_I, y_I) \varepsilon X_i, (v_1, z_1)^q \varepsilon V^q, \ldots, (v_H, z_H)^q \varepsilon V_H^q]$ such that:

(1) $$y_i = \sum_{h=1}^{H} z_{jh}^q \text{ for all } i \underset{q}{\varepsilon} j \text{ for every } j = 1,\ldots,J.$$

(where z_{jh}^q is the M vector of public goods produced for j by h given q)

(2) $$\sum_{i=1}^{I} x_i - \sum_{h=1}^{H} v_h^q = \omega .$$

E. Equilibria and Optima

A local Lindahl equilibrium L^q given partition q is a locally feasible allocation $a^q = [(x_1, y_1), \ldots, (v_H, z_H)^q]$ and an $NJ + MI$ vector of prices $(p^q, s_1^q, \ldots, s_I^q) \geq 0$ [7] such that:

(3) $$(p^q, s^q) \cdot (v_h, z_h)^q \geqq (p^q, s^q) \cdot (\bar{v}_h, \bar{z}_h)^q \text{ for any } (\bar{v}_h, \bar{z}_h)^q \varepsilon V_h^q$$

for every $h = 1,\ldots,H$. (Producers maximize profits)

(4) For any $(\bar{x}_i,\bar{y}_i) \in X_i$, if $(\bar{x}_i,\bar{y}_i) \underset{i}{>} (x_i,y_i)$ then $(p^q,s_i^q) \cdot (\bar{x}_i,\bar{y}_i) > (p^q,s_i^q) \cdot (x_i,y_i) = w_i^q$ for every $i = 1,\ldots,I$. (No consumer can afford a vector of goods preferred to his equilibrium vector)

At least one local Lindahl equilibrium is defined for each partition $q = 1,\ldots,Q$.

Now that a local Lindahl equilibrium has been defined consumer migration can be formally modelled. It can be shown that assumptions (A.10 - A.14) insure the existence for each i of a continuous utility function u_i defined over X_i. Given any partition $\bar{q}$ consumer i can evaluate his utility function at his local Lindahl equilibrium consumption $(\bar{x}_i,\bar{y}_i)$. Let $U_i(\bar{q}) = u_i(\bar{x}_i,\bar{y}_i)$ for any $\bar{q}$ and call it i's equilibrium utility when $\bar{q}$ obtains.

Consider all possible equilibrium utilities available to i under all possible partitions. Since these $U_i(q)$'s represent i's constrained preferences, $U_i(q) > U_i(q')$ means i prefers partition q to partition q'. If i could choose any partition, he would pick $\bar{q} = \max_q [U_i(1),\ldots,U_i(Q)]$. But then i would be dictating a partition to all other consumers in the economy; he would be dictating everyone's residential location. Rather than give anyone this power, I assume that each individual is free to choose his own location from among the $j = 1,\ldots,J$ jurisdictions. Then the resulting partition and equilibrium utilities to all I consumers depend on all the individual choices made.

The intuitive notion underlying a global equilibrium L is that people arrive at some partition such that no one wants to move given everyone else's location. Before L is formally defined, notice that the I consumers' problem of choosing a partition can be represented as an I-person non-zero-sum game. Let $J_i = \{1,\ldots,J\}$ be i's choice or strategy set, and let j_i be the jurisdiction chosen by i. Then the resulting partition is $q = (j_1,\ldots,j_I)$. The utility or payoff to i is $U_i(q) = U_i(j_1,\ldots,j_I)$. Formally, a global Lindahl equilibrium L is a local Lindahl equilibrium L^q with $q = (j_1,\ldots,j_I)$ such that $U_i(j_1,\ldots,j_I) \geq U_i(j_1,\ldots,j_{i-1},\bar{j}_i,j_{i+1},\ldots,j_I)$ for all $\bar{j}_i \in J_i$ for every $i = 1,\ldots,I$. This equilibrium condition is equivalent to a pure strategy Nash equilibrium in the I-person game.[8,9]

If local equilibria are not unique, then $U_i(q)$ will not be unique, but this causes no problems. Given a local equilibrium L^q, each consumer knows his allocation and $U_i(q)$ is unique. If there are multiple local equilibria given some other partition $\bar{q}$, let $U_i(\bar{q})$ be the maximum utility i can get under $\bar{q}$. Then $U_i(\bar{q})$ is well defined, and the consumer will never choose to move and change q to $\bar{q}$ if $U_i(\bar{q}) \leqq U_i(q)$.

Finally the concepts of local and global Pareto optimality can be defined. A locally feasible allocation $a^q = [(x_1,y_1),\ldots,(v_H,z_H)^q]$ is locally Pareto optimal if, given q, there does not exist another locally feasible allocation $\bar{a}^q = [(\bar{x}_1,\bar{y}_1),\ldots,(\bar{v}_H,\bar{z}_H)^q]$ such that $(\bar{x}_i,\bar{y}_i) \underset{i}{\gtrsim} (x_i,y_i)$ for all i and $(\bar{x}_i,\bar{y}_i) \underset{i}{\succ} (x_i,y_i)$ for at least one i. A local Pareto optimum $a^q = [(x_1,y_1),\ldots,(v_H,z_H)^q]$ is globally Pareto optimal if there does not exist another partition $\bar{q}$ with an associated locally feasible allocation $\bar{a}^{\bar{q}} = [(\bar{x}_1,\bar{y}_1),\ldots,(\bar{v}_H,\bar{z}_H)^{\bar{q}}]$ such that $(\bar{x}_i,\bar{y}_i) \underset{i}{\gtrsim} (x_i,y_i)$ for all i and $(\bar{x}_i,\bar{y}_i) \underset{i}{\succ} (x_i,y_i)$ for at least one i.

F. Some Properties of the Local Economy

The following theorems about the economy with any arbitrary but fixed partition will be useful in analyzing the global economy. The proofs are a straightforward generalization of Foley's work [5].

Theorem 2.1: Given (A.2 - A.14) and any fixed partition, there exists a local Lindahl equilibrium.

Theorem 2.2: Given (A.2 - A.14) any local Lindahl equilibrium is locally Pareto optimal.

Suppose there are two or more identical consumers in the economy with identical preferences and equal endowments. Define a symmetric local Lindahl equilibrium as any local equilibrium which treats identical consumers equally whenever they live in the same jurisdiction. Equal treatment means all identical residents of a jurisdiction pay the same private and public prices, and receive the same quantities of private and public goods.

Theorem 2.3: Given (A.2 - A.14) and any partition, there exists a symmetric local Lindahl equilibrium.

3. Economies with Locally Pure Public Goods

Public goods are locally pure if changes in the consumer partition do not induce any change in the technology available to the economy; $V^q = V^{q'}$ for all $q, q' = 1, \ldots, Q$. When public goods satisfy this condition, consumers have a considerable incentive to live together. Adding consumers to a jurisdiction does not increase the resource cost of producing fixed public good quantities but allows sharing of this cost among more consumers. Hence when more people live in a jurisdiction each resident can have the same quantities as before at lower personal Lindahl prices. These notions can be formalized in the following:

<u>Theorem 3.1</u>: Given (A.1 - A.14) with locally pure public goods, any feasible allocation a^q associated with any partition q which has consumers living in two or more jurisdictions is weakly Pareto inferior to some allocation a^1 where partition 1 has all consumers living together in jurisdiction 1.

<u>Proof</u>: Let a^q be any feasible allocation given q. Then $\sum_{i=1}^{I} x_i^q - \sum_{h=1}^{H} v_h^q = \sum_{i=1}^{I} \omega_i$ and $y_i^q = \sum_{h=1}^{H} z_{jh}^q$ for all $i \underset{q}{\varepsilon} j$ for every $j = 1,\ldots,J$.

Define the allocation a^1 as follows:

Let $(x_i^1, y_i^1) = (x_i^q, \sum_{j=1}^{J} \sum_{h=1}^{H} z_{jh}^q)$ for every i, and let

$(v_h^1, z_h^1) = (v_h^q, z_h^q)$ for every h.

Then $\sum_{i=1}^{I} x_i^1 - \sum_{h=1}^{H} v_h^1 = \sum_{i=1}^{I} \omega_i$ and $y_i^1 = \sum_{j=1}^{J} \sum_{h=1}^{H} z_{jh}^q$ for every i.

a^1 is feasible because a^q is feasible; firms merely transport the public goods they produce under q to jurisdiction 1.

By monotonicity $(x_i^1, y_i^1) \underset{i}{\succsim} (x_i^q, y_i^q)$ for every i. Hence a^1 is weakly preferred to a^q. Proved.

<u>Remark</u>: If a^q happens to be a local Lindahl equilibrium allocation for partition q, Theorem 3.1 says this local equilibrium is not globally Pareto optimal whenever positive quantities of any public good are produced.

Since there are obvious benefits from living together when public goods are locally pure, the question arises as to whether prices will induce consumers to stay together and jointly consume public goods. Remarkably, the answer under a constant returns technology is no. In some cases a consumer can actually increase his utility by moving out of the jurisdiction where all others live. This situation can arise because the consumer causes a change in the aggregate demand for public goods when he moves which can induce a change in the technology used to produce all commodities. Under some circumstances this change in technology results in a large enough increase in the mover's income to more than compensate him for the loss of joint public goods consumption.

<u>Example 1</u>: Suppose one producer transforms quantities of four commodities (v_1, v_2, v_3, z) for two jurisdictions {A,B} according to the linear activities α: $-3.924\ v_2 \geqq v_1 + .8z$ and β: $-4v_3 \geqq v_1 + z$, both of which must be satisfied. Suppose there are two consumers $i = 1,2$ with identical utility functions $u_i = x_{1i}^{\frac{1}{2}} + .2\ x_{2i}^{\frac{1}{2}} + .2\ x_{3i}^{\frac{1}{2}} + 2\ y_i^{\frac{1}{2}}$ and unequal private endowments $\omega_1 = (0,1,9)$ and $\omega_2 = (0,9,1)$.

Consider partition $q = 1 = (A,A)$ which assigns both consumers to jurisdiction A. Then activity β is the binding production constraint, consumer 2's income is only $p^1\ \omega_2 = (1,.0447,4) \cdot (0,9,1) = 4.4023$, and $U_2(q=1) = 6.628$. But when 2 moves out and forms $q = 2 = (A,B)$, activity α is binding and consumer 2's income rises to $p^2\ \omega_2 = (1,3.924,.0447) \cdot (0,9,1) = 35.3607$ and his utility increases to $U_2(q=2) = 36.8365$. Consumer 2 prefers living alone with his larger income while consumer 1 prefers living together to jointly consume public goods, and there is no global equilibrium.

In an interurban world with large distances between jurisdictions consumers will always choose to live together. Large distances imply large transportation costs which can be implicitly modeled by prohibiting all inter-jurisdictional trading of private and public goods. With no trade between communities, any consumer disrupting a partition with everyone living together will have to produce with his own endowment his entire consumption of private and public goods after he moves. He will never prefer any feasible consumption he can produce by himself to his equilibrium consumption when he lives with everyone.

Theorem 3.2: Under (A.1 - A.14) when public goods are locally pure and private goods are not traded between jurisdictions, there exists a global Lindahl equilibrium.

Proof: Let $q = 1 = (1,\ldots,1)$ assign all consumers to jurisdiction one. Suppose a local Lindahl equilibrium (a^1,p^1,s^1) is not a global equilibrium. Then some consumer i can move to a jurisdiction j creating $q = 2 = (1,\ldots,1,j,1,\ldots,1)$ with the j in the i^{th} position such that $(x_i^2,y_i^2) \succ_i (x_i^1,y_i^1)$ where (x_i^2,y_i^2) is i's equilibrium allocation when $q = 2$. $(x_i^2,y_i^2) \succ_i (x_i^1,y_i^1)$ implies $p^1\, x_i^2 + s_i^1\, y_i^2 > p^1\, x_i^1 + s_i^1\, y_i^1 = p^1\, \omega_i$. Since everyone but i lives in jurisdiction 1 when $q = 2$, i cannot trade with anyone and must produce (x_i^2,y_i^2) himself. Hence $(x_i^2 - \omega_i, y_i^2) \in V^2$. Since public goods are locally pure $V^2 = V^1$, so $(x_i^2 - \omega_i, y_i^2) \in V^1$. Profit maximization and constant returns when $q = 1$ imply $0 = (p^1,s^1)\cdot(v^1,z^1) \geqq (p^1,s^1)\cdot(v,z)$ for all $(v,z) \in V^1$. In particular $0 \geqq (p^1,s^1)\cdot(x_i^2 - \omega_i, y_i^2)$. Since $s^1 = \sum_{i=1}^{I} s_i^1 \geqq s_i^1$ and $y_i^2 \geqq 0$, $0 \geqq (p^1,s_i^1)\cdot(x_i^2 - \omega_i, y_i^2)$. Hence i can afford (x_i^2,y_i^2) at (p^1,s_i^1), a contradiction.

In an intra-urban economy where interjurisdictional trades of private goods are permitted, Theorem 3.2 implies that a market boycott of the mover's initial endowment organized by non-movers would keep all consumers in the same jurisdiction. In some sense this market boycott is a threat strategy by the coalition of non-movers which works via the market mechanism; by merely refusing to trade with any mover the remaining consumers in an intra-urban economy can sustain a partition which has everyone living together.

Although constant returns with locally pure public goods does not insure existence of a global Lindahl equilibrium in an intra-urban economy, other restrictions on technology or preferences do generate zero consumer migration. Consider an economy, hereafter the Leontief economy,[10] where production satisfies the following assumptions as well as (A.1 - A.9).

(A.15) Each producer h produces only one commodity v_h or z_h.

(A.16) The first commodity (labour) is the only non-produced commodity.

(A.17) No output can be produced without some labour input.

(A.18) $\exists\ (\bar{v},\bar{z}) \in V$ such that $\bar{v}_h > \bar{v}_{1h} + \dots + \bar{v}_{(N+M)J,h}$ for all h, where v_{nh} is the quantity of h used in producing $\bar{v}_n$. (The economy is productive.)

Production in the Leontief economy can be characterized by technical coefficients:

$$a_{ij} = \begin{cases} -\dfrac{v_{ij}}{v_i} & \text{for } j = 2,\dots,NJ \text{ and } i = 2,\dots,NJ \\ -\dfrac{v_{ij}}{z_i} & \text{for } j = 2,\dots,NJ \text{ and } i = NJ+1,\dots,NJ+MJ \\ 0 & \text{otherwise.} \end{cases}$$

where a_{ij} denotes the quantity of input j required for production of one unit of output i. In this economy equilibrium profits to each firm are zero. Setting $p_1^q = 1$ as numeraire, zero profits in the production of commodity i can be expressed as:

$$a_{i1} + \sum_{j=2}^{NJ} p_j^q a_{ij} + \sum_{j=NJ+1}^{NJ+MJ} s_j^q \cdot a_{ij} = \begin{cases} p_i^q & \text{if i is a private good} \\ s_i^q & \text{if i is a public good} \end{cases}$$

Let A be the square $(N+M)J-1$ matrix of technical coefficients $[a_{ij}]$ with $i,j = 2,\ldots,(N+M)J$. Let I be the $(N+M)J-1$ identity matrix, and let $a_1' = (a_{21},\ldots,a_{(N+M)J})'$ be the transpose of labour input requirements for commodities $2,\ldots,(N+M)J$. With equilibrium production prices (p^q,s^q), zero profits in all industries can be written:

$$(5) \qquad (p^q,s^q)(I-A) = a_1$$

Since a_1' is positive by (A.17), equilibrium prices (p^q,s^q) solving (1) are unique and positive.[11] Of course if some commodity v_i is not produced then its equilibrium price p_i can be less than the resource cost of producing one unit. This situation will necessarily arise under some consumer partitions; if everyone lives together the outputs of local public goods in all empty jurisdictions will be zero. Nevertheless, the following result obtains:

<u>Theorem 3.3</u>: In the Leontief economy with locally pure public goods, there exists a global Lindahl equilibrium.

<u>Proof</u>: Consider a local Lindahl equilibrium (a^1,p^1,s^1) under $q = 1 = (1,\ldots,1)$ where everyone lives together. Suppose it is not a global equilibrium. Then some consumer i can move to an empty jurisdiction and create

$q = 2 = (1,\ldots,1,j,1,\ldots,1)$ with an associated local Lindahl equilibrium (a^2,p^2,s^2) such that $(x_i^2,y_i^2) \underset{i}{>} (x_i^1,y_i^1)$. This implies $p^1\, x_i^2 + s_i^1\, y_i^2 > p^1\, x_i^1 + s_i^1\, y_i^1 = p^1\, \omega_i$. But $p^1\, \omega_i = p_1^1\, \omega_{1i}$ since labour is the only non-produced good. By assumption $p_1^1 = p_1^2 = 1$, so $p^1\, \omega_i = p^2\, \omega_i$. Hence

(6) $$p^1\, x_i^2 + s_i^1\, y_i^2 > p^2\, \omega_i = p^2\, x_i^2 + s_i^2\, y_i^2 .$$

(6) can only be satisfied if there is some private good n such that:

(7) $$p_n^1\, x_{in}^2 > p_n^2\, x_{in}^2$$

or some public good m such that:

(8) $$s_{im}^1\, y_{im}^2 > s_{im}^2\, y_{im}^2 .$$

(7) requires $p_n^1 > p_n^2$ and $x_{in}^2 \neq 0$. But $p_n^1 > p_n^2$ implies $v_n^2 = 0$, and $v_n^2 = 0$ implies $x_{in}^2 = 0$. Therefore (7) cannot occur for any n.

(8) requires $s_{im}^1 > s_{im}^2$ and $y_{im}^2 \neq 0$. Since i lives alone when $q = 2$, $s_{im}^2 = s_m^2$. $y_{im}^2 \neq 0$ implies $s_m^2 = \sum_{j=2}^{(N+M)J} p_j^2\, a_{mj} + a_{m1}$, which is always greater than or equal to s_m^1. But $s_m^1 \geq s_{im}^1$, so $s_{im}^2 \geq s_{im}^1$ and (8) cannot occur for any m.

Since (7) and (8) cannot occur, (6) cannot be satisfied for any consumer i and no consumer will want to disrupt the partition $q = 1$. Proved.

Now suppose the economy operates under a rule which says equals must be treated equally in the following sense:

(A.19) Any consumers with the same preferences and endowments who live in the same jurisdiction must pay equal prices for each public good.

(A.19) insures all local Lindahl equilibria are symmetric.[12] When consumers are identical, (A.19) is satisfied, and public goods are locally pure, consumers will choose to live together.

Theorem 3.4: Under (A.1 - A.14) and (A.19), when consumers are identical there exists a global Lindahl equilibrium.

Proof: Let $q=1$ assign all consumers to jurisdiction 1, and let $L^1 = (a^1,p^1,s^1)$ be the symmetric local Lindahl equilibrium for $q=1$. If it is not a global equilibrium, some consumer i will move to some empty jurisdiction j creating $q=2=(1,\ldots,1,j,.,\ldots,1)$ with an associated symmetric local equilibrium $L^2 = (a^2,p^2,s^2)$ such that $(x_i^2,y_i^2) \underset{i}{>} (x_i^1,y_i^1)$. Since L^1 is symmetric, $(x_i^1,y_i^1) = (x_j^1,y_j^1)$ for every consumer j. By identical preferences, $(x_i^2,y_i^2) \underset{j}{>} (x_j^1,y_j^1)$ for all j.

At (p^2,s^2), $p^2\,\omega_j = p^2\,x_i^2 + s_i^2\,y_i^2 = p^2\,\omega_i$ for all j since endowments are equal. Since everyone else lives together, $\sum_{\substack{j=1 \\ j\neq i}}^{I} s_j^2 = s^2$, so $s_j^2 \leq s_i^2\ \forall_j$. Hence $p^2\,\omega_j = p^2\,x_i^2 + s_i^2\,y_i^2 = p^2\,x_i^2 + s_j^2\,y_i^2\ \forall_j$. Every j can afford to purchase (x_i^2,y_i^2) at (p^2,s_j^2) and by revealed preference, $(x_j^2,y_j^2) \underset{j}{\geq} (x_i^2,y_i^2)\ \forall_j$.

By transitivity of preferences $(x_j^2,y_j^2) \underset{j}{>} (x_j^1,y_j^1)\ \forall_j$. But by

Theorem 3.1, there exists a feasible allocation $\tilde{a}^1$ under $q=1$ such that $(\tilde{x}_j^1,\tilde{y}_j^1) \succ_j (x_j^2,y_j^2)\ \forall_j$. By transitivity $(\tilde{x}_j^1,\tilde{y}_j^1) \succ_j (x_j^1,y_j^1)\ \forall_j$ including consumer i. Since $\tilde{a}^1$ is feasible and unanimously preferred to a^1, a^1 is not locally Pareto optimal. Hence by Theorem 2.2, (a^1,p^1,s^1) cannot be a local Lindahl equilibrium, a contradiction.

Not surprisingly, any global equilibrium with all consumers living together when public goods are locally pure is globally Pareto optimal. Special cases include the economies with identical consumers, with a Leontief technology, and with no interjurisdictional trading of private goods (Theorems 3.2 - 3.4).

<u>Theorem 3.5</u>: Given (A.1 - A.14) with locally pure public goods, any global Lindahl equilibrium with everyone living together is globally Pareto optimal.

<u>Proof</u>: Let $q=1$ assign all consumers to the same jurisdiction with (a^1,p^1,s^1) a global equilibrium. Suppose a^1 is not globally Pareto optimal. Then $\exists$ some $q=2$ and an associated feasible allocation a^2 such that $(x_i^2,y_i^2) \succsim_i (x_i^1,y_i^1)\ \forall i$ and $(x_i^2,y_i^2) \succ_i (x_i^1,y_i^1)$ for at least one i. If $q=2$ has consumers living in two or more jurisdictions, by Theorem 3.1 there exists a feasible allocation $\tilde{a}^1$ such that $(\tilde{x}_i^1,\tilde{y}_i^1) \succsim_i (x_i^2,y_i^2)\ \forall i$. By transitivity of preferences $(\tilde{x}_i^1,\tilde{y}_i^1) \succsim_i (x_i^1,y_i^1)\ \forall i$ and $(\tilde{x}_i^1,\tilde{y}_i^1) \succ_i (x_i^1,y_i^1)$ for at least one i. Hence a^1 is not locally Pareto optimal and (a^1,p^1,s^1) cannot be a local Lindahl equilibrium by Theorem 2.2. Since it is not a local equilibrium, it cannot be a global equilibrium, a contradiction.

So far the discussion has focused on locally pure public goods. But local public goods are often assumed to be produced under conditions of crowding. Aside from problems of dispersing quantities of public goods geographically, the primary reason for providing public goods locally rather than nationally or internationally is the existence of congestion. Theorem 3.1 indicates consumers can always improve their situation by forming a single political jurisdiction whenever public goods are pure. This will not be true when public goods are produced under conditions of congestion.

4. Economies with Crowded Public Goods

When crowding exists in the production of public goods, an increase in the number of residents in a jurisdiction can increase the per-person cost of providing a fixed quantity of the good. Crowding can be introduced by specifying the relationship between the projections of V^q and $V^{q'}$ onto jurisdiction j's commodity space. Let $n_j(q)$ be the number of consumers assigned to jurisdiction j by partition q. Crowding is present in the production of z_{mj} if $n_j(q) > n_j(q')$ implies the cost per person of providing any fixed quantity of z_{mj} is higher under q. Clearly a necessary condition for the presence of crowding is $\text{proj}_{z_{mj}} V^q \subset \text{proj}_{z_{mj}} V^{q'}$ whenever $n_j(q) > n_j(q')$. It is often assumed crowding does not occur until a jurisdiction's population exceeds some number of consumers $\bar{n}$. If $n_j(q) < \bar{n}$ z_{mj} is pure, and if $n_j(q) > \bar{n}$ z_{mj} is crowded. Suppose $n_j(q') = \bar{n}$. Then $\text{proj}_{z_{mj}} V^q \subseteq \text{proj}_{z_{mj}} V^{q'}$ for all $q = 1,\ldots,Q$.

I will assume that crowding affects all public goods equally in the following sense: if $\bar{n}$ minimizes the cost of producing any quantity of some public good m in jurisdiciton j, then $\bar{n}$ minimizes the cost of producing every public good in every jurisdiction.[13]

If consumers have different preferences or endowments a global equilibrium may not exist when public goods are crowded. In the following example the advantages to a poor consumer from joint public good consumption with a rich consumer more than compensate the poor consumer for the increased costs of crowding that occur when he moves in with the wealthier consumer.

Example 2: Suppose there is only one producer who transforms quantities of three goods (v_1, v_2, z) for two jurisdictions (A,B) according to the technology $v_1 + b_A^q z_A + b_B^q z_B = v_2$. With $p_1^q = 1$ as numeraire, profit maximization yields prices $p_1^q = p_2^q = 1$ and $s_A^q = b_A^q$, $s_B^q = b_B^q$. Let (A,A) = 1, (A,B) = 2, (B,A) = 3 and (B,B) = 4 denote the four possible partitions of two consumers into A and B. Crowding requires the per-person costs increase as the number of residents increases, or $\frac{b_A^1}{n_A(1)} > \frac{b_A^2}{n_A(2)} = \frac{b_A^3}{n_A(3)}$ and $\frac{b_B^4}{n_B(4)} > \frac{b_B^2}{n_B(2)} = \frac{b_B^3}{n_B(3)}$. Let $b_A^1 = b_B^4 = 5$ and $b_A^2 = b_B^2 = b_A^3 = b_B^3 = 2.2$.

Suppose the consumers have utility functions $u_1 = x_{11}^{1/2} x_{21}^{1/12} y_{j1}^{5/12}$, $u_2 = x_{12}^{1/3} x_{22}^{1/12} y_{j2}^{7/12}$ and endowments $\omega_{21} = 12$, $\omega_{22} = 6$. Then solution of the system yields:

$$U_1(A,A) = U_1(B,B) = u_1(6,1,1.7) = 3.05 < U_1(A,B) = U_1(B,A) = u_1(6,1,2.27) = 8.43$$

and

$$U_2(A,A) = U_2(B,B) = u_2(2,\tfrac{1}{2},1.7) = 1.62 > U_2(A,B) = U_2(B,A) = u_2(2,\tfrac{1}{2},1.59) = 1.56$$

Consumer 2 prefers living with 1 under (A,A) or (B,B) while consumer 1 prefers living alone. No partition is an equilibrium because the poor consumer chases the rich consumer from place to place.

This example demonstrates that when public goods are crowded, a segregated partition cannot usually be sustained as an equilibrium. It also shows that both integrated and segregated partitions can be Pareto optimal, with integrated partitions favouring less wealthy consumers and those who have relatively weaker preferences for public goods.

As with locally pure public goods, restrictions on technology or preferences insure existence of a global Lindahl equilibrium. Suppose inter-jurisdictional trading of private goods is permitted. As before, let $\bar{n}$ be the number of consumers minimizing the cost of producing any given quantity of every public good in every jurisdiction. Now suppose there exists a partition q such that $n_j(q) = \bar{n}$ for every j (i.e., consumers divide evenly into minimum cost-sized jurisdictions). Then:

<u>Theorem 4.1</u>: Under (A.1 - A.19) when consumers are identical there exists a global Lindahl equilibrium.

<u>Proof</u>: Let $q = 1$ assign $\bar{n}$ consumers to each non-empty jurisdiction. If the associated local equilibrium (a^1, p^1, s^1) is not a global equilibrium, then some consumer wants to move. Suppose $(x_1^2, y_1^2) \succ_1 (x_1^1, y_1^1)$ for consumer 1 in jurisdiction j', where $q = 2$ assigns 1 to some jurisdiction j which is either empty or contains $\bar{n}$ consumers before 1 moves. $(x_1^2, y_1^2) \succ_1 (x_1^1, y_1^1)$ implies $p^1 x_1^2 + s_1^1 y_1^2 > p^1 x_1^1 + s_1^1 y_1^1 = p^1 \omega_1$.

Since the economy is a Leontief economy, $p^1 \omega_1 = p^2 \omega_1$.

$p^2 x_1^2 \geqq p_1 x_1^2$ since for every private good n, either $p_n^1 \leqq p_n^2$ or $x_{in}^2 = 0$ for all i [i.e., if $p_n^2 < p_n^1$, x_n is not produced when $q = 2$]. So $p^2 x_1^2 + s_1^1 y_1^2 \geqq p^1 x_1^2 + s_1^1 y_1^2 > p^2 \omega_1^2 = p^2 x_1^2 + s_1^2 y_1^2$ and $s_1^1 y_1^2 > s_1^2 y_1^2$. Therefore, for at least one public good m,

$$s_{m1}^1 > s_{m1}^2 .$$

By symmetry of the equilibrium with identical consumers, $s_{m1}^1 = \frac{s_{mj'}^1}{\bar{n}}$ and $s_{m1}^2 = \frac{s_{mj}^2}{\bar{n}+1}$ for <u>any</u> public good m. Since $\bar{n}$ is the cost-minimizing number of consumers per jurisdiction and $s_{mj'}^1 = s_{mj}^1$, the definition of crowding implies $\frac{s_{mj'}^1 \, y_m}{\bar{n}} < \frac{s_{mj}^2 \cdot y_m}{\bar{n}+1}$ for any fixed y_m, so $\frac{s_{mj'}^1}{\bar{n}} < \frac{s_{mj}^2}{\bar{n}+1}$; the per person average cost of m increases as a jurisdiction gets larger or smaller than $\bar{n}$. Hence

$$s_{m1}^1 = \frac{s_{mj'}^1}{\bar{n}} < s_{m1}^2 = \frac{s_{mj}^2}{\bar{n}+1}$$ for every m, a contradiction. Proved.

<u>Remark</u>: If $\bar{n}$ is the total population, then Theorem 4.1 is a special case of Theorems 3.3 and 3.4. If $\bar{n} = 1$ there are no advantages to living with others and all consumers will live alone.

Finally, consider an economy with $t = 1,\ldots,T$ types of consumers. If consumers can be appropriately divided into optimally-sized communities and have homothetically separable preferences,[14] a global equilibrium exists. This equilibrium involves identically "integrated" jurisdictions and is globally Pareto optimal. Let $n_{jt}(q)$ be the number of consumers of type t assigned to jurisdiction j by partition q. Suppose there exists a q such that:

(9) $n_{jt}(q) = n_{j't}(q)$ for all j,j' for every consumer class $t = 1,\ldots,T$ and

(10) $\sum_t n_{jt}(q) = n_j(q) = \bar{n}$ for every j, where $\bar{n}$ as before is the congestion-minimizing number of consumers per jursidiction.

Then q creates $I/\bar{n}$ identical jurisdictions of optimal size where I is the total number of consumers in the whole economy. Suppose:

(A.20) Consumers have homothetically separable preferences: for each i, $u_i(x_i,y_i) = f[g_{1i}(x_{1i}),\ldots,g_{NJ+M,i}(y_{Mi})]$ where the $g_{ni}(\cdot)$'s are homogeneous of degree one.

An example of (A.20) is any Cobb-Douglas utility function.

Several authors [9, 12, 13] have discussed the optimality of segregated partitions, but only McGuire [9] has analyzed integrated partitions. He concludes segregation is always Pareto superior to integration, but Theorem 4.3 below shows this is not true in my model; usually these two situations are Pareto noncomparable. Furthermore, Example 2 above demonstrates that segregated partitions cannot be sustained as equilibria, while Theorem 4.2 proves that integrated partitions do generate global equilibria. The policy implications are dramatic. If society wants consumers to form stable, Pareto optimal residential patterns, it should encourage integration of all communities and discourage segregation.

To insure existence of a global equilibrium with all jurisdictions integrated, a slightly different notion of crowding must be introduced. Until now crowding required that the average price per consumer of a public good with one additional resident exceed its current average consumer price.

For the following theorem, assume that crowding exists in the production of a public good if the maximum price paid by any consumer with one additional resident exceeds the maximum price paid currently by any consumer. This assumption substitutes maximum prices paid for average prices paid in the definition of crowding. Formally:

(A.21) $\max_i (s^2_{mji}) > \max_i (s^1_{mji})$ for all j such that

$n_j(2) - 1 = n_j\ (1) \geqq \bar{n}$ whenever $y^2_{mj} \neq 0$.

<u>Theorem 4.2</u>: Given (A.1 - A.21) and an integrated partition q = 1 satisfying (9) and (10), there exists a global Lindahl equilibrium (a^1, p^1, s^1).

<u>Proof</u>: Suppose not. Then there is a consumer, say $i \underset{q=1}{\varepsilon} 1$ for whom $(x^2_i, y^2_i) \underset{i}{>} (x^1_i, y^1_i)$ where q = 2 assigns consumer i to jurisdiction j and keeps everyone else where they are. $(x^2_i, y^2_i) \underset{i}{>} (x^1_i, y^1_i)$ implies $p^1\ x^2_i + s^1_i\ y^2_i > p^1\ x^1_i + s^1_i\ y^1_i = p^1\ \omega_i$. Since the technology is Leontief $p^1\ \omega_i = p^2\ \omega_i$, so $p^1\ x^2_i + s^1_i\ y^2_i > p^2\ x^2_i + s^2_i\ y^2_i$, and either

(11) $p^1_n\ x^2_{ni} > p^2_n\ x^2_{ni}$ for some private good n, or

(12) $s^1_{mi}\ y^2_{mi} > s^2_{mi}\ y^2_{mi}$ for some public good m.

(11) implies $p^1_n > p^2_n$ and $x^2_{ni} \neq 0$. But $x^2_{ni} \neq 0$ implies x_n produced when q = 2, which requires $p^2_n \geqq p^1_n$. So (11) cannot hold.

(12) implies $s^1_{mi} > s^2_{mi}$ and $y^2_{mi} \neq 0$. $y^2_{mi} \neq 0$ and (A.21) imply $s^2_{mi'} > s^1_{mi'}$ for some consumer $i' \underset{2}{\varepsilon} j$ who is paying the largest

personal price for y_{mj} given $q=2$. Homothetically separable preferences imply expenditures on each commodity are a constant fraction of income for each consumer. Since $p^1 \omega_i = p^2 \omega_i$, $s^1_{mi} > s^2_{mi}$ and homotheticity imply $y^1_{mi} < y^2_{mi}$. Since $p^1 \omega_{i'} = p^2 \omega_{i'}$, $s^2_{mi'} > s^1_{mi'}$ and homotheticity imply $y^1_{mi'} > y^2_{mi'}$.

Symmetry of equilibrium and identical jurisdictions and production prices when $q=1$ imply $y^1_{mi} = y^1_{mi'}$ even though i and i' live apart when $q=1$. Since i and i' live together when $q=2$, $y^2_{mi} = y^2_{mi'}$ by feasibility. Therefore $y^1_{mi} = y^1_{mi'} > y^2_{mi'} = y^2_{mi} > y^1_{mi}$, a contradiction. Hence (12) cannot hold. Since (11) and (12) cannot hold, consumer i cannot prefer $q=2$, and (a^1,p^1,s^1) is a global equilibrium.

This theorem says a society using different taxes for different consumers when it allocates public goods is more likely to achieve some stable equilibrium with no consumer migration if it encourages the formation of integrated rather than segregated communities. The incentives for any individual consumer to move in an integrated world are somewhat less. The next theorem demonstrates that a global equilibrium achieved in an integrated society is globally Pareto optimal. This provides some additional justification for encouraging integrated jurisdictions.

Theorem 4.3: The global Lindahl equilibrium in Theorem 4.2 is globally Pareto optimal.

Proof: Suppose not. Then there exists some partition $q=2$ with an associated locally feasible allocation a^2 such that $(x^2_i, y^2_i) \underset{i}{\succsim} (x^1_i, y^1_i)$

for all i and $(x_i^2, y_i^2) \underset{i}{>} (x_i^1, y_i^1)$ for at least one i. Construct an allocation $\bar{a}^1$ as follows: let $(\bar{x}_i^1, \bar{y}_i^1) = (x_i^2, y_i^2)$ for all i. Then $(x_i^2, y_i^2) \underset{i}{\sim} (\bar{x}_i^1, \bar{y}_i^1)$ for all i. So by transitivity of preferences $(\bar{x}_i^1, \bar{y}_i^1) \underset{i}{\geq} (x_i^1, y_i^1)$ for all i and $(\bar{x}_i^1, \bar{y}_i^1) \underset{i}{>} (x_i^1, y_i^1)$ for some i. But $\underset{j}{\text{proj}}\ V^2 \subseteq \underset{j}{\text{proj}}\ V^1$ for every j by the definition of crowding, since $n_j(1) = \bar{n}$ for all j. Hence $\bar{a}^1$ is feasible because a^2 is feasible. Since $\bar{a}^1$ is feasible and preferred to a^1, a^1 is not locally Pareto optimal, so (a^1, p^1, s^1) cannot be a local equilibrium by Theorem 2.2. Therefore (a^1, p^1, s^1) cannot be a global equilibrium, a contradiction.

So far in this paper, every global equilibrium has been shown to be globally Pareto optimal. This leads naturally to the question of whether all global equilibria are globally optimal. If one observes migration falling to zero and all markets clearing in the society, can one assume no relocation of consumers by a planning authority could increase the society's welfare? The following example shows one cannot. It is quite possible for the economy to achieve a global Lindahl equilibrium which is not globally Pareto optimal.

Example 3: Suppose there is one producer who transforms quantities of three goods (v_1, v_2, z) for four jurisdictions {A,B,C,D} according to the technology $v_1 + b_A^q z_A + b_B^q z_B + b_C^q z_C + b_D^q z_D = v_2$. With $p_1 = 1$ as numeraire, profit maximization yields prices $p_2 = 1$, $s_A^q = b_A^q$, $s_B^q = b_B^q$, $s_C^q = b_C^q$ and $s_D^q = b_D^q$ for any interior solution. Here one need only consider the seven partitions: (AAAA) = 1,

(AABB) = 2, (BABB) = 3, (BAAA) = 4, (ABAA) = 5, (AABA) = 6, and (AAAB) = 7.

Suppose crowding sets in for jurisdictions with more than 2 residents and the public good is pure otherwise. Then the per capita cost of producing z must increase as the number of residents in a jurisdiction increases, and the production coefficients must satisfy:

$$\frac{b_A^1}{n_A(1)} > \frac{b_A^4}{n_A(4)} = \frac{b_A^5}{n_A(5)} = \frac{b_A^6}{n_A(6)} = \frac{b_A^7}{n_A(7)} = \frac{b_B^3}{n_B(3)} > \frac{b_A^2}{n_A(2)} =$$

$$\frac{b_B^2}{n_B(2)} = \frac{b_A^3}{n_A(3)} = \frac{b_B^4}{n_B(4)} = \frac{b_B^5}{n_B(5)} = \frac{b_B^6}{n_B(6)} = \frac{b_B^7}{n_B(7)}$$

where $n_B(q)$ is the number of consumers assigned to jurisdiction B by partition q. Let $b_A^1 = 3$, $b_A^4 = 1.6$ and $b_A^2 = 1$.

Suppose there are 4 consumers with identical utility functions $u_i = x_{1i}\, y_i + .000001\, x_{2i}$ and private good endowments $\omega_i = (0,5)$. Solution of this system yields:

$$U_1(q=1) = U_2(1) = U_3(1) = U_4(1) = \frac{25}{3}$$

$$U_1(q=2) = U_2(2) = U_3(2) = U_4(2) = \frac{25}{2}$$

$$U_1(3) = U_3(3) = U_4(3) = \frac{375}{32} \text{ and } U_2(3) = \frac{25}{4}$$

$$U_1(4) = U_3(4) = U_4(4) = \frac{375}{32} \text{ and } U_2(4) = \frac{25}{4}$$

$$U_1(5) = \frac{25}{4} \text{ and } U_2(5) = U_3(5) = U_4(5) = \frac{375}{32}$$

$$U_1(6) = U_2(6) = U_4(6) = \frac{375}{32} \text{ and } U_3(6) = \frac{25}{4}$$

$$U_1(7) = U_2(7) = U_3(7) = \frac{375}{32} \text{ and } U_4(7) = \frac{25}{4}$$

From these outcomes, partition 1 = (AAAA) is an equilibrium since no consumer can do better by unilaterally moving:

$$U_1(1) > U_1(5) = U_1(BAAA), \quad U_2(1) > U_2(4) = U_2(ABAA)$$

$$U_3(1) > U_3(6) = U_3(AABA), \quad U_4(1) > U_4(7) = U_4(AAAB) \; .$$

But partition 2 = (AABB) is Pareto superior to partition 1; everyone gets more utility when $q = 2$. The inability of consumers to coordinate their moving decisions makes it impossible to create (AABB) directly from (AAAA) since 3 and 4 would have to move simultaneously; neither 3 nor 4 has any incentive to move by himself. Incidentally, notice that although (AABB) is globally Pareto optimal, it is unstable. Consumer 1 will move because $U_1(3) = U_1(BABB) > U_1(AABB) = U_1(2)$.

5. Conclusion

This paper has examined economies with local public goods and endogenous consumer migration. Wedding an underlying market process with a game theoretic migration model capable of analyzing consumer indivisibilities and crowded public goods yields several results. When public goods are locally pure and jurisdictions are far apart, consumers will voluntarily form one jurisdiction to share their consumption of public goods. On the other hand, if jurisdictions are close together like a city and its suburbs, migration might continue through time as consumers chase each other from place to place. One might in this case expect unstable location patterns in urban areas. However, consumers in an urban area will generate a globally optimal equilibrium with everyone living togehter if public goods are pure and the economy has a Leontief technology or consumers are identical.

When public goods are crowded, an integrated consumer partition with people of all types in each jurisdiction generates a Pareto optimal global equilibrium. Furthermore, a segregated partition will almost never be sustained as an equilibrium because any consumer with a lower income living in a low income jurisdiction can always increase his utility by desegregating a wealthier jurisdiction. Some non-market restrictions on migration are required if the society wants to maintain segregated communities.

A four consumer economy is constructed which has a non-optimal global equilibrium. The inability of consumers to agree on a partition in several circumstances and the non-optimality of some equilibria suggest that a planning authority in the form of a consolidated metropolitan government or an interregional agency might be useful in helping consumers coordinate their moving decisions.[15] Finally, observe that restrictions on preferences and technology which generate global equilibria also define a class of n-person non-zero-sum games which have pure strategy Nash equilibria.

FOOTNOTES

*This paper is part of my Ph.D. dissertation written at Northwestern University under the guidance of John Ledyard who I wish to thank for several valuable comments.

[1] A local public good is a commodity for which the total quantity supplied is consumed by each resident of a jurisdiction while non-resident consumption is zero.

[2] The technology for producing a public good is subject to crowding if the cost of producing any fixed quantity in a jurisdiction increases as the number of residents increases.

[3] Utility is transferable between consumers if some commodity is linearly separable in everyone's utility function. Such a commodity can be used for side payments between consumers. See Owen [11] pp. 132-133 for a brief discussion.

[4] These results are sensitive to the use of Lindahl pricing for allocating public goods. An interesting problem is how property taxation and majority rule might change behavior.

[5] In general one could have two types of private goods -- one consumed only by residents and the other consumed by residents and non-residents. Fixed natural resources are an example of the former.

[6]Employing the usual vector notation, $x > y$ means $x_i > y_i$ for all i; $x \geq y$ means $x_i \geqq y_i$ for all i with $x_i > y_i$ for some i; $x \geqq y$ means $x_i \geqq y_i$ for all i.

[7]One might think prices are in $R^{NJ + MIJ}$, but recall that each i has personal public good prices only for public goods in the community where he lives.

[8]See Luce and Raiffa [7], p. 157 for a discussion of n-person games.

[9]A pure strategy game is one in which each player must pick just one element from his strategy set. There are also mixed strategy games in which a player can assign probabilities to all elements in his strategy set and allow chance to pick the final outcome. Such games always have Nash equilibria but seem inappropriate in the context of my model.

[10]Properties of a Leontief economy when there are no public goods are discussed by Arrow and Hahn [1] pp. 40-48, Malinvaud [8] pp. 117-123, and Gale [6] pp. 294-306.

[11]See, for example, Theorem 9.5 in Gale [6] p. 302.

[12]If the local Lindahl equilibrium for each partition is unique, then it must be symmetric by Theorem 2.3. In this case (A.19) need not be made since equals will be treated equally as a natural outcome of the market process.

[13]If different public goods vary as to their degree of crowding then separate jurisdictions for each good would be appropriate. "Purer" public goods requiring more consumers to achieve a minimum cost per person would require larger jurisdictions, creating a federal or hierarchical system of government. Such systems are not dealt with in this paper.

[14]For a discussion of homothetically separable preferences and their implications for individual demands, see "Demand and Supply Under Separability" in [10], especially pp. 100-103.

[15]It is possible but by no means obvious that other schemes for allocating public goods would generate more stable location patterns.

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