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Glenn M. MacDonald

Chris Robinson

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# THE CENTRE FOR DECISION SCIENCES AND ECONOMETRICS

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Glenn M. MacDonald and Chris Robinson

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The University of Western Ontario  
London, Ontario N6A 5C2



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AN OPERATIONAL THEORY OF MONOPOLY UNION-COMPETITIVE FIRM INTERACTION:  
NEW PREDICTIONS, A SIMPLE FRAMEWORK FOR INTERPRETING  
EXISTING EMPIRICAL DATA, AND PROPOSED TESTS\*

Glenn M. MacDonald\*\*

and

Chris Robinson\*\*\*

April, 1985

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\*\* Centre for Decision Sciences and Econometrics, U.W.O.; Economics Research Center/NORC, University of Chicago; and Center for Economic Research, University of Rochester.

\*\*\* Centre for Decision Sciences and Econometrics, U.W.O.

## I. INTRODUCTION

The study of labor unions was at one time the primary focus of labor economists' efforts. But interest faded following the development of human capital theory, the new home economics, and a variety of other structures which altogether greatly expanded the set of topics upon which systematic analysis could be brought to bear. However, recent years have witnessed a resurrection of the labor union as a topic worthy of effort, and a broad collection of new questions have emerged.<sup>1</sup>

Two observations may be made regarding both the older and recent contributions:

i) Though linked by a common focus on unions, the literature is highly fragmentary; models in which a variety of issues may be addressed simultaneously are notable by their absence. Rather, the tendency has been towards utilization of distinct models to analyze each separate topic. Compare, for example, the models used to explain the pattern of inter-temporal incidence of unions (Ashenfelter and Pencavel, 1969; Freeman, 1984), union-nonunion wage differentials by industry (Parsley, 1980), and strikes (Hayes, 1984). This disparate approach has ruled out a potentially fruitful source of predictions--namely restrictions on the covariation of several endogenous variables considered together--in addition to impeding the development of a coherent view of the whole set of union issues.

ii) Very few testable propositions have been established (Pencavel, 1984). Even in the simple terms of partitioning the economy into endogenous and exogenous factors, and then deducing the influence of changes in the economic environment on the endogenous variables, surprisingly little progress

has been made. The difficulty is not that investigators have ignored this goal, for attempts abound. Rather, the models typically contain a set of offsetting factors making clean predictions, or even "leading case" scenarios, hard to come by. As a consequence, though much empirical work on unions has been done, there is no sense in which well-formulated models have been subjected to test.

This paper makes headway in the direction of remedying the situation. In the following section a simple model of the interaction of workers, firms, consumers and a union is presented. Section III displays the model's equilibrium, a wide variety of predictions being derived in Section IV. The model is first studied in a very elementary environment; that it accommodates and makes predictions regarding a wider variety of issues is established in Section V. It is shown that the theory is of some assistance in understanding some existing empirical "facts".

All economic predictions are to some degree open to the objection that all claims to providing real assistance in understanding the data are defeated in the process of empirical implementation. Section VI provides an empirical methodology which is implementable and provides experiments which are of sufficient sharpness that the results could conceivably cast serious doubt on the validity of the theory.

Before proceeding, the essential departure of this model from earlier work should be noted: unions are assumed to use resources in their dealings with firms and workers. A consequence is that optimality will not generally imply unionization of all firms in an industry; i.e.

incomplete union coverage. It transpires that this outcome allows the union to handle the market demand for output in a somewhat less restrictive fashion than is typical. With this relaxation, clear predictions are straightforwardly obtained.

## II. THE BASIC MODEL

This section explores the simplest structure which contains the features essential to the analysis. The section following presents a set of elaborations which can easily be understood once the basic model is grasped firmly.

There are four types of agents: workers, consumers, entrepreneurs and a union. As is conventional in the analysis of unions, the equilibrium concept employed here is leader-follower with the union leading all other agents. The implied procedure is therefore to analyze the behavior of consumers, workers, and entrepreneurs conditional on actions taken by the union. Subsequently, the union's optimization problem is studied, taking into account optimal response by the other agents.

Consumers play a passive role in the analysis. Their behavior is summarized by a market demand function  $X = \varphi(p)$ , where  $X$  is the total quantity purchased and  $p$  is the price of output. It is assumed that (i)  $\varphi$  is twice continuously differentiable, with  $\varphi' < 0$ ; (ii) for all  $p \geq \bar{p}$ , where  $\bar{p} < \infty$ ,  $\varphi(p) = 0$ ; and (iii) for all  $p > 0$ ,  $\varphi(p) < \bar{\varphi}$  for some constant  $\bar{\varphi}$ ,  $0 < \bar{\varphi} < \infty$ .

Workers are identical in all respects. They find all production activities in this industry equally distasteful and work a fixed number of hours in any firm. The employment options workers face are as follows. The best alternative to participation in this industry generates utility

$\hat{V} > 0$ . Work in nonunion firms is also available, paying wage  $\hat{w}$  and generating utility  $v(\hat{w})$  where  $v(\cdot)$  is an indirect utility;  $v' > 0$ . Union firms pay wage  $w$  and the union levies dues of  $d$ . For the basic model, it is convenient to assume that union membership confers no consumption benefits or costs on workers. Under this assumption union workers obtain utility  $v(w-d)$ .

Entrepreneurs, all are equally good at operating firms, and have best alternatives valued at  $A$ . The industry is assumed to be small relative to the economy, in which case  $A$  is the constant supply price of entrepreneurs.

The technology used to produce output  $Q$  has the number of workers  $L$  as its sole variable input:<sup>2</sup>

$$Q = f(L); \quad (1)$$

$$f' > 0, f'' < -\epsilon \text{ for some } \epsilon > 0, \text{ and } \lim_{L \rightarrow 0} f' = \infty.$$

At this point, in line with the treatment of workers, it is assumed that the union does not provide any services the firm finds either productive or harmful. Let  $h(Q) \equiv f^{-1}(Q)$  be the labor input required to produce  $Q$ ;  $h' > 0$ ,  $h'' > \zeta$  for some  $\zeta > 0$ , and  $\lim_{Q \rightarrow 0} h' = 0$ . Then for any wage  $\tilde{w}$ , the firm's variable cost function is

$$c(\tilde{w}, Q) \equiv \tilde{w}h(Q), \quad (2)$$

and total costs are

$$F + c(\tilde{w}, Q), \quad (3)$$

where  $F \equiv R + A$ , and  $R$  is the price of fixed factors.

How do the union and firms interact? There are two points. First, union activities must ultimately raise the price of the final product above

that which would prevail under competitive free entry conditions; the increase is the source of the surplus the union obtains. Thus the union must somehow limit entry into the industry. In this paper the union does so in precisely the fashion implicit in conventional models of unions--by threatening potential entrants with unionization. That this threat is credible in equilibrium is established below.

Next, of the firms producing in the industry, the union designates how many are unionized and how many are not. Unionized firms are required to pay their workers the union wage  $w$ , while nonunion firms need not do so, and thus pay  $\hat{w}$ .<sup>3</sup>

Given the union's actions, there are therefore three types of firms--union, nonunion, and potential entrants.

Consider the optimizing behavior of unionized firms. Given the price of output, unionized firms earn profit

$$pQ - F - wh(Q) \tag{4}$$

if they produce  $Q$ . Unionized firms therefore produce either  $Q = 0$  or  $Q = Q^*$ , where

$$Q^* = \operatorname{argmax}_{Q \in (0, \infty)} \{pQ - F - c(w, Q)\},$$

depending on whether  $pQ^* - F - c(w, Q^*) \geq 0$ .

In a similar fashion, nonunion firms face  $F + c(\hat{w}, Q)$  and produce either  $Q = 0$  or  $Q = \hat{Q}$ , where

$$\hat{Q} = \operatorname{argmax}_{Q \in (0, \infty)} \{p\hat{Q} - F - c(\hat{w}, \hat{Q})\}$$

depending on whether  $\hat{\pi} \equiv p\hat{Q} - F - c(\hat{w}, \hat{Q}) \geq 0$ .

Before proceeding further, one additional minor restriction is required. The assumption is merely that



$$\bar{p} > \frac{F + c(\hat{w}, \bar{Q})}{\bar{Q}},$$

where  $\bar{Q} = \operatorname{argmin}_Q \frac{F + c(\hat{w}, Q)}{Q}$ .

That is, if there were no union, demand is such that the good would indeed be produced in a free entry competitive equilibrium.

Two preliminary results which simplify presentation of the union's problem can now be established. First of all, since workers are freely mobile, it is obvious that

$$\hat{w} = v^{-1}(\hat{v}) \quad (5)$$

and

$$w - d = \hat{w} \quad (6)$$

must hold.

Next, for given  $w$ , union firms must earn zero profits. The argument is that non-negative profits are required to induce union firms to produce, while nonpositive profits are necessary if the union's threat to unionize entrants is not to be regarded as an invitation.

This result and the definition of  $Q^*$  imply unionized firms produce

$$Q^* = q(w, F) \equiv \operatorname{argmin}_{Q \in (0, \infty)} \frac{F + c(w, Q)}{Q},$$

and the equilibrium product price is

$$\begin{aligned} p^*(w, F) &= \frac{F + c[w, q(w, F)]}{q(w, F)} \\ &= wh' [q(w, F)] \end{aligned} \quad (7)$$

Also, nonunion output  $\hat{Q} = \hat{q}(\hat{w}, w, F)$  is the unique solution in  $Q$  to

$$p^*(w, F) = \hat{w}h'(Q)$$

Finally, assuming potential entrants regard the union's threat as credible, their behavior is simply abstention.<sup>4</sup>

To reiterate what has been obtained so far, union firms produce  $q(w,F)$ --the output which minimizes their average cost given  $w$  and  $F$ --and must obtain zero profits in equilibrium. If there are any nonunion firms, they earn nonnegative profit, and do so by producing  $\hat{q}(\hat{w},w,F)$ . Potential entrants remain "potential", and the price of output is  $wh'[q(w,F)]$ .

The union is treated in a very straightforward manner. First of all, what does it maximize? In the basic model the union's only activity is the collection of dues. As such it is reasonable, and as it turns out fruitful, to suppose the union to simply maximize the excess of dues over costs of collecting them. Letting  $N$  represent the number of union firms, and recalling that each employs  $l(w,F) \equiv h[q(w,F)]$  workers, the union seeks to maximize

$$\pi(w,F) \equiv (w-\hat{w})Nl(w,F) - u[N, l(w,F)] \quad (8)$$

where (6) has been used to eliminate  $d$ , and  $u[\cdot]$  is the cost of collecting dues from  $l(\cdot)$  workers in each of  $N$  distinct locations.  $u[\cdot]$  is assumed to be monotonically increasing, strictly convex, and twice continuously differentiable;  $u(0,0) = u[0, l(\cdot)] = u(N,0) = 0$ .<sup>5</sup> Moreover, it is required that the cross effect  $u_{Nl}$  not be "too large":

$$u_l - Nu_{Nl} > 0. \quad (9)$$

What tools can the union use to maximize  $\pi(N,w)$ ? It is assumed that the union partitions the set of all firms into union, nonunion (numbering  $M$ ) and potential entrants, the latter group being threatened with union

status. Of course the union faces constraints on its activities. One is that the market for output must clear. The other is, as shown above, that union firms must earn zero profits. Recalling (7), and letting  $\hat{q}(\hat{w}, F, p)$  denote optimal output for a nonunion firm, the union's maximization problem is

$$\begin{aligned} \max_{M, N, w} \quad & \pi(N, w) \\ \text{S.T.} \quad & Nq(w, F) + M\hat{q}(\hat{w}, F, p) = \varphi(p) \\ & p = wh' [q(w, F)] \\ & N \geq 0, M \geq 0. \end{aligned}$$

Since  $N > 0$  is necessary for  $\pi > 0$ ,  $N \geq 0$  is satisfied provided the union chooses to operate, which is assumed. Further, non-negativity of  $M$  and  $p = wh'$  can be included in the first constraint. Proceeding thusly, the programming problem becomes

$$\max_{w, N} \pi(N, w) \quad \text{S.T.} \quad Nq(w, F) \leq \varphi\{wh' [q(w, F)]\} \quad (10)$$

The next section presents the solution to this problem.

### III. EQUILIBRIUM

In this section, the programming problem stated in (10) is studied. It is first checked that enough assumptions have been made to guarantee that the problem in fact has a well-defined solution. Conditions characterizing the solution are then presented.

In order to proceed, a couple of derivatives are required. Recall that

$$q(w, F) = \operatorname{argmin}_Q \frac{F + c(w, Q)}{Q},$$

in which case  $q(w, F)$  satisfies

$$\frac{1}{q} \frac{\partial c}{\partial Q} - \frac{1}{q^2} [F + c(w, q)] = 0 \quad (11)$$

and

$$\frac{\partial^2 c}{\partial Q^2} > 0.$$

It follows that (recall  $c(w, Q) \equiv wh(Q)$ )

$$\frac{\partial q}{\partial w} = \frac{h/q - h'}{wh} < 0, \quad (12)$$

and since  $\ell(w, F) \equiv h[q(w, F)]$ ,

$$\frac{\partial \ell}{\partial w} = h' \frac{\partial q}{\partial w} < 0. \quad (13)$$

A rise in the union wage rate reduces the output and therefore the labor input in unionized firms. Moreover, for future reference, the approximation  $0 = h(0) \cong h(q) - h'(q)q + h''(q)q^2$  gives the elasticities

$$\eta_w^q \equiv \frac{w}{q} \frac{\partial q}{\partial w} = -1, \quad (14)$$

and

$$\eta_w^\ell \equiv \frac{w}{\ell} \frac{\partial \ell}{\partial w} = -\eta_q^h < -1, \quad (15)$$

where  $\eta_q^h$  is the elasticity of  $h(Q)$  with respect to  $Q$ ;  $\eta_q^h > 1$  due to the convexity of  $h(\cdot)$ .<sup>6</sup>

Turning to the constraint in (10), consider its boundary  $Nq - \varphi = 0$ .

For given  $w$ , reductions in  $N$  always cause  $Nq < \varphi$ , and conversely for increases in  $N$ .  $Nq - \varphi = 0$  therefore defines a unique  $N$  for each  $w$ :  $N(w) \equiv \frac{\varphi}{q}$ . Under the assumptions made above,  $N(w)$  is twice differentiable. As  $w \rightarrow 0$ , (11) implies  $q \rightarrow \infty$ , so  $N \rightarrow 0$  is required to satisfy  $Nq - \varphi = 0$  provided  $\varphi$  is bounded, as is assumed;  $N(0) = 0$ . On the other hand, since  $[F + c(w, q)] \div q$

is an increasing function (with derivative bounded away from 0) of  $w$ , raising  $w$  eventually generates  $[F+c(\bar{w},q)] \div q = \bar{p}$  for some  $\bar{w} < \infty$ , in which case  $\varphi(\bar{p}) = 0$  implies  $N(\bar{w}) = 0$ . Thus, under assumptions,  $N(w)$  is a continuous function of  $w$  with  $N(0) = 0$ ,  $N(\bar{w}) = 0$  for some  $\bar{w}$  and  $N(w) > 0$  otherwise. The set of  $(N,w)$  pairs from which the union might pick,  $\{(N,w) \mid Nq - \varphi \leq 0, N \geq 0, w \geq 0\}$  is therefore compact.

Since  $\ell(w,F)$  is a continuous function of  $w$ ,  $\pi(N,w)$  is continuous too. Thus the problem in (10) involves maximization of a continuous function on a nonempty (given the restriction on  $\bar{p}$ ,  $w \geq \hat{w}$  is feasible) compact set, in which case it has a solution.

For future reference,  $N(w) = \varphi/q$  can be thought of as the locus of  $(N,w)$  pairs for which aggregate nonunion output is 0. More generally,  $N(w,\alpha) \equiv (\varphi - \alpha)/q$  is the locus of  $(N,w)$  pairs for which nonunion output equals some  $\alpha \geq 0$ . The slope of this locus is

$$\frac{\partial N(w,\alpha)}{\partial w} = \frac{1}{q^2} [\varphi' h - Nq \frac{\partial q}{\partial w}], \quad (16)$$

which may take on either sign. That is, an increase in  $w$  lowers each firm's output, and thus  $Nq$  for given  $N$ . But the price of the product must rise too, and so quantity demanded is reduced. Whether the number of union firms required to produce  $\varphi - \alpha$  rises or falls depends on whether the impact through the output of each firm exceeds or falls short of the product demand effect.

The Lagrangian for problem (10) is

$$\mathcal{L}(N,w) = (w - \hat{w})N\ell(w,F) - u[N, \ell(w,F)] + \lambda[\varphi - Nq(w,F)],$$

where  $\lambda \geq 0$  is an undetermined multiplier. First-order conditions for a maximum are (asterisks denoting optimal values)

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial w} = N^* \ell + (w^* - \hat{w}) N^* \frac{\partial \ell}{\partial w} - u_\ell \frac{\partial \ell}{\partial w} + \lambda^* [\varphi' \cdot (h' + w^* h'' \frac{\partial q}{\partial w}) \\ - N^* \frac{\partial q}{\partial w}] = 0 \end{aligned} \quad (17)$$

$$\frac{\partial \mathcal{L}}{\partial N} = (w^* - \hat{w}) \ell - u_N - \lambda^* q = 0 \quad (18)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \varphi - N^* q \geq 0 \quad (19)$$

where  $\lambda^* > 0$  only if (19) is an equality. Second-order necessary conditions are long expressions with straightforward interpretations. If  $\lambda^* = 0$ ,  $\pi(N, w)$  must be strictly concave for  $(N, w)$  in a neighborhood of  $(N^*, w^*)$ . For  $\lambda^* > 0$ , the locus of  $(N, w)$  pairs for which  $\pi(N, w) = \pi(N^*, w^*)$  must have "more curvature" than  $N(w)$  in the neighborhood of  $(N^*, w^*)$ .

Interpretation of (17) and (18) is straightforward. In (17), an increment to the wage generates greater dues from  $N\ell$  workers. However the number of workers hired by each union firm falls, reducing both dues collected and union costs. If there are no nonunion firms, in which case  $\lambda^* > 0$ , an increment to the wage either tightens or slackens the constraint. From (16), an increase in  $w$  slackens the constraint if  $\partial N(w, 0) / \partial w > 0$ , hence the last term in (17).

In (16), an increment to  $N$  yields dues from  $\ell$  workers and raises union costs. If there are no nonunion firms, an increment to  $N$  also tightens the constraint.

One issue which emerges immediately is that of whether the union will choose to unionize all the firms in the industry; will there be complete union 'coverage'? Perhaps the most obvious way to answer this question is to point out that coverage will be incomplete if and only if the global maximum of  $\pi(N, w)$  with respect to  $(N, w)$  lies inside the constraint.

For this condition to hold, it is necessary that  $\pi(N,w)$  indeed have a global maximum for finite  $(N,w)$ . The possibility of infinite  $N$  for given  $w$  is ruled out by  $u_{NN} > 0$ . The possibility of perpetual wage increases, for given  $N$ , is best analyzed by rewriting  $\pi(N,w)$  as

$$\pi(N,w) = wN\ell - [u(N,\ell) + \hat{w}N\ell],$$

whence the usual monopoly condition that  $\ell(w,F)$  be elastic in  $w$  emerges. Thus, referring back to (15), sufficient convexity in  $h(\cdot)$  gives  $\pi(\cdot)$  a global maximum for finite  $(N,w)$ , in which case the existence of nonunion firms depends on the location of the constraint.

Finally, is the union's threat to unionize any potential entrant credible in equilibrium? The assumptions required for the existence of a zero profit competitive equilibrium with a determinate firm size are also sufficient to guarantee the answer to be in the affirmative. Loosely, to achieve zero profit competitive equilibrium when firms possess U-shaped average cost curves and demand is arbitrary, the minimum point on union firms' average cost curve must occur at a "small" level of output (see Sonnenschein, 1982); the output for which price equals marginal cost for nonunion firms must also be small. Given these assumptions, small increments to demand can be accommodated by entry of nonunion firms producing at the output for which price equals marginal cost or unionized firms producing at minimum average cost. When such is the case, as is implicit above, in the union's problem the numbers of firms designated union and nonunion are appropriately treated as continuous variables. It follows that at the union's optimum  $(N^*,w^*)$  either

- (i) the union is indifferent about whether to unionize a small number of potential entrants given  $w^* - \lambda^* = 0$  and  $\pi_N = 0$ ;

or

- (ii) the union is indifferent about whether to unionize a small number of potential entrants and adjust  $w^*$  slightly-- $\lambda^* > 0$ ,  $\pi_N > 0$  and  $\pi_w \lesssim 0$ .

In either case the threat to unionize any individual firm, or a small coalition for that matter, is entirely credible. That the union is confronted with competitive firms gives it power not necessarily available otherwise.

At this point the reader may well wonder whether any progress has been made. When the constraint (19) holds with equality, the model is, given  $N$ , for all intents and purposes the usual monopoly unionism model which is known to have virtually no predictions. Further, when (19) holds as a strict inequality, the problem can be written out so that it looks much the same as the standard model. But there is an important difference between the two situations. When the constraint does not bind, the presence of nonunion firms frees the union from having to adjust the number of unionized firms to clear the market for output. Demand merely determines the number of nonunion firms.

This additional freedom yields a large class of new predictions that could not be obtained from the standard model. First, it permits predictions on the relation between level of unionization and the wage differentials in competitive industries. By contrast, in the standard monopoly model the unionization level is 100%. Second, it predicts radically different responses in union behavior to changes in industry demand when unionization is less than 100%, compared with 100% coverage. Since the incomplete coverage case provides most of the new predictions, this case is considered in more detail below.



#### IV. PREDICTIONS FROM THE BASIC MODEL

Though the basic model is a sparse setting, it is nonetheless possible to obtain a variety of predictions from it. In this section changes in the following aspects of the economic environment are considered: the demand for final output ( $\varphi(p)$ ); workers' alternatives ( $\hat{v}$ ); entrepreneurs' alternatives and technology; and union costs. As indicated above, few results are available for the complete coverage case, and many under incomplete coverage. Consequently, the bulk of the discussion is devoted to incomplete coverage.

##### Incomplete Coverage Equilibrium

In the incomplete coverage equilibrium, (19) may be ignored and (17)-(18) simplified to:

$$\pi_w = N^*l + (w^* - \hat{w})N^* \frac{\partial l}{\partial w} - u_l \frac{\partial l}{\partial w} = 0 \quad (20)$$

and

$$\pi_N = (w^* - \hat{w})l - u_N = 0 \quad (21)$$

Second-order conditions require  $\pi_{ww} < 0$ ,  $\pi_{NN} < 0$  and  $\pi_{ww}\pi_{NN} - \pi_{Nw}^2 > 0$  when all are evaluated at  $(N^*, w^*)$ .

First consider changes in  $\varphi(p)$ . Since  $\varphi(p)$  does not appear in (20) or (21), neither  $w^*$  nor  $N^*$  depends on it, from which it follows immediately that  $p^*$  does not vary either. Accordingly, all changes in  $\varphi(p)$  are fully captured by the increment to  $\varphi(p^*)$ , and the sole response is in terms of entry or exit of nonunion firms:

$$dM^* = \frac{1}{q} d\varphi(p^*) .$$

The result stands in striking contrast with the traditional literature, in

which characteristics of product demand figured prominently in the determination of union wages and employment (e.g. Marshall (1896), p. 246).

Next, consider variations in the value of workers' alternative  $\hat{v}$ . Since  $d\hat{v} = v' d\hat{w}$ , such a change translates into  $d\hat{w}$ .

Application of the usual calculus to (20) and (21) gives

$$\frac{dw^*}{d\hat{w}} \propto \pi_{wN} \pi_{N\hat{w}} - \pi_{NN} \pi_{w\hat{w}}.$$

$\pi_{NN} < 0$  by the second-order conditions. Also,

$$\pi_{w\hat{w}} = -N^* \frac{\partial l}{\partial w} > 0 \text{ from (13),}$$

and

$$\pi_{N\hat{w}} = -l < 0.$$

Further

$$\begin{aligned} \pi_{wN} &= l + (w^* - \hat{w}) \frac{\partial l}{\partial w} - u_{Nl} \frac{\partial l}{\partial w} \\ &= \left[ \frac{u}{N} - u_{Nl} \right] \frac{\partial l}{\partial w} \text{ from (20)} \end{aligned}$$

$$< 0 \text{ from (13) and (9).}$$

Accordingly,

$$\frac{dw^*}{d\hat{w}} > 0.$$

Similarly,

$$\frac{dN^*}{d\hat{w}} \propto \pi_{Nw} \pi_{w\hat{w}} - \pi_{ww} \pi_{N\hat{w}} < 0$$

since  $\pi_{ww} < 0$  from the second-order conditions.

Thus a rise in the value of alternative opportunities generates an increase in the union wage and a decline in the number of union firms. The intuition is just that the resultant rise in  $\hat{w}$  operates as a factor price increase for the union, which therefore responds by scaling back operations directly via reducing  $N$  and indirectly through causing a reduction in  $l$  by raising  $w^*$ .

The basic results  $d\hat{w}^*/d\hat{w} > 0$  and  $dN^*/d\hat{w} < 0$  immediately imply a variety of other results about other attributes of the union sector of the industry.

Since unionized firms produce  $q(w^*, F)$ ,

$$\frac{dq}{dw} = \frac{\partial q}{\partial w^*} \frac{dw^*}{d\hat{w}} < 0,$$

and

$$\frac{dl}{d\hat{w}} = h' \frac{dq}{dw} < 0.$$

In addition, total output of, and employment in, unionized firms are  $Nq$  and  $Nl$  respectively, in which case

$$\frac{d}{d\hat{w}} Nq = q \frac{dN}{d\hat{w}} + N \frac{dq}{dw} < 0$$

and

$$\frac{d}{d\hat{w}} Nl = l \frac{dN}{d\hat{w}} + N \frac{dl}{dw} < 0.$$

Moreover, from (14) and (15) and minor manipulation,

$$\frac{\hat{w}}{Nl} \frac{d}{d\hat{w}} Nl - \frac{\hat{w}}{Nq} \frac{d}{d\hat{w}} Nq = [\eta_w^l - \eta_w^q] \frac{\hat{w}}{w} \frac{dw}{d\hat{w}} < 0.$$

In percentage terms, the union employment response to a proportional increase in  $\hat{w}$  is greater than the effect on the aggregate output of unionized firms.

Furthermore, since  $l(w,F)$  is elastic with respect to  $w$ , payments to union workers  $w^*Nl$  fall in total:

$$\frac{d}{d\hat{w}} w^*Nl = N\left(\frac{d}{dw} w^*l\right) \frac{dw^*}{d\hat{w}} + w^*l \frac{dN^*}{d\hat{w}} < 0,$$

as well as when measured as a fraction of total factor payments in unionized firms,

$$\frac{d}{d\hat{w}} \left(\frac{w^*l}{F+w^*l}\right) \propto \left[\frac{1}{w^*l} - \frac{1}{F+w^*l}\right] \frac{d}{d\hat{w}} w^*l < 0.$$

Finally, consider the union-nonunion wage differential  $D \equiv w - \hat{w}$ . A tedious derivation indicates that  $D$  will rise with an increment to  $\hat{w}$  unless the increase in  $w$  generates a sharp absolute increase in the elasticity of  $l(w,F)$  with respect to  $w$ . The leading case is clearly  $dD/d\hat{w} > 0$ .

This result offers sharp contrast with the usual monopoly union model, which has inherently ambiguous predictions regarding the union differential. Indeed, the existing literature almost entirely ignores the determinants of the wage differential.

Two final predictions for changes in  $\hat{w}$  concern union profits and the equilibrium product price. First, the envelope theorem implies  $d\pi/d\hat{w} < 0$ --the amount of resources agents might be willing to expend to acquire the union monopoly position or organize workers' declines as  $\hat{w}$  rises. Second,  $dw/d\hat{w} > 0$  implies an increase in the level of minimum average cost, and hence

$$\frac{dp^*}{d\hat{w}} > 0,$$

where  $p^*$  is the equilibrium price of the product. It is nevertheless true that although  $p^*$  rises, both the revenue of each unionized firm ( $p^*q$ ) and the

revenue of the union sector ( $N^*p^*q$ ) fall when  $\hat{w}$  rises. Such must occur simply because factor payments ( $w^*l$  and  $w^*N^*l$ ) fall and union firms must earn zero profits.

Turning to the individual behavior of nonunion firms, when  $\hat{w}$  rises, marginal cost rises, as does price. No simple restriction yields a unique sign for  $d\hat{q}/d\hat{w}$ . Similarly, the change in  $\hat{w}$  increases both costs and returns (through  $p^*$ ), and no prediction regarding nonunion profits is obtained.

The above results constitute unambiguous predictions for a variety of aspects of union behavior both at the individual union firm and at the aggregate unionized firms' level, in response to changes in  $\hat{w}$ . An additional feature of interest in the union literature, however, is the extent of union organization in the industry as measured by the fraction of total employment which is unionized. In order to consider changes in this entity, predictions on the nonunion component of the industry are required. Consider first, the aggregate behavior of nonunion firms. Recall that these firms simply fill in the difference between total union output and quantity demanded at the equilibrium price. When  $\hat{w}$  rises, total union output falls; however the equilibrium price rises so that total quantity demanded also falls. Consequently, for a given reduction in union output, the change in nonunion output depends on the price elasticity of demand for the product. When product demand is sufficiently inelastic, the effects of changes in  $\hat{w}$  on union employment  $Nl$ , translate into effects on fraction unionized  $Nl/(Nl+M\hat{l})$ , where  $\hat{l} \equiv h(\hat{q})$ .

Unambiguous predictions involving changes in  $F$  are somewhat more difficult to obtain. In contrast with changes in  $\hat{w}$ , which operated essentially as a factor price increase for the union, alterations in  $F$  act on  $\ell(w, F)$ . For given  $w$ , increments to  $F$  increase the minimum average cost level of output, hence  $\partial \ell / \partial F > 0$ . This effect operates to raise  $w^*$ , for given  $N$ . But under plausible restrictions  $\ell$  becomes more sensitive to increases in  $w$ , working to lower  $w^*$ , again given  $N$ .<sup>7</sup> A sufficient condition for complete comparative statics is that the latter effect dominates at  $(N^*, w^*)$ . Specifically, at  $(N^*, w^*)$ , it is assumed that  $\pi_{wF} < 0$ .

In terms of interpretation,  $F$  can change as a result of alterations in either the entrepreneur's opportunity cost  $A$ , or the cost of fixed factors  $R$ . The latter interpretation is only strictly appropriate when  $Q = f(L)$  remains unaltered. Consequently, changes in  $R$  should be thought of as representing any movements in the prices of fixed factors ( $r_i$ ) which generate increments to  $R$  on net, and leave  $f(L)$  unchanged.<sup>8</sup>

Proceeding as usual,

$$\frac{dw^*}{dF} \propto \{ \pi_{wN} \pi_{NF} - \pi_{NN} \pi_{wF} \}.$$

$\pi_{NN} < 0$  holds by virtue of the second-order conditions, and  $\pi_{wN} < 0$  was established above. Moreover,

$$\begin{aligned} \pi_{NF} &= (w^* - \hat{w}) \frac{\partial \ell}{\partial F} - u_N \ell \frac{\partial \ell}{\partial F} \\ &= \frac{1}{\ell} \frac{\partial \ell}{\partial F} (u_N - \ell u_{N\ell}) \\ &> 0 \end{aligned}$$

given the cross effect restrictions imposed on  $u(\ )$  and (21).<sup>9</sup> Coupled with the assumption  $\pi_{wF} < 0$ ,

$$\frac{dw^*}{dF} < 0.$$

Similarly,  $\pi_{wN} < 0$ ,  $\pi_{wF} < 0$  and  $\pi_{NF} > 0$  yield

$$\frac{dN^*}{dF} \propto \{ \pi_{Nw} \pi_{wF} - \pi_{ww} \pi_{NF} \} > 0,$$

when the second-order condition  $\pi_{ww} < 0$  is taken into account. The intuition is that as  $F$  rises,  $l(w,F)$  becomes more responsive to increases in  $w$ , in which case it pays to expand labor sold to each union firm by lowering  $w^*$ . In addition, since increments to  $F$  raise  $l$  directly, greater unionization is called for provided the increase in  $l$  does not raise the marginal cost of an additional unionized firm ( $u_N$ ) too much, as is assumed.

Other predictions follow readily. Since  $\hat{w}$  is constant,

$$\frac{dD}{dF} < 0.$$

Also, increases in  $F$  raise  $q$  both directly and because  $w^*$  falls

$$\frac{dq}{dF} = \frac{\partial q}{\partial F} + \frac{\partial q}{\partial w} \frac{dw^*}{dF} > 0,$$

in which case,

$$\frac{d\ell}{dF} = h' \frac{dq}{dF} > 0.$$

Given these results, it is immediate that

$$\frac{d}{dF} N^*q = N \frac{dq}{dF} + q \frac{dN}{dF} > 0,$$

$$\frac{d}{dF} N^*\ell = \ell \frac{dN}{dF} + N \frac{d\ell}{dF} > 0,$$

and

$$\frac{d}{dF} w^*N^*\ell = N \left( \frac{d}{dw} w^*\ell \right) \frac{dw^*}{dF} + w^*\ell \frac{dN}{dF} > 0$$

since  $l(w,F)$  is elastic in  $w$ .

Whether  $p^*$  rises or falls as  $F$  increases is ambiguous. The union partially affects the firms' cost increase by reducing  $w^*$ , in which case the effect on average cost is unclear.

As regards union profits, the envelope theorem provides

$$\begin{aligned} \frac{d\pi}{dF} &= \frac{\partial \pi}{\partial F} \\ &= (w^* - \hat{w})N^* \frac{\partial l}{\partial F} - u_l \frac{\partial l}{\partial F} \\ &= \frac{\partial l}{\partial F} [(w^* - \hat{w})N^* - w] \\ &= \frac{\partial l}{\partial F} \cdot - \frac{N^* l}{\partial l / \partial w} > 0; \end{aligned}$$

that is, the first-order effect of  $F$  on  $\pi$  is just the demand shift induced by  $F$ .

Turning to  $\hat{q}$  and  $\hat{l}$ , since  $dp^*/dF$  is unsigned, the only restriction is

$$\text{sign } \frac{dp^*}{dF} = \text{sign } \frac{d\hat{q}}{dr} = \text{sign } \frac{d\hat{l}}{dF} = \text{sign } \frac{d\hat{\pi}}{dF} .$$

In a similar fashion, it is possible to consider changes in the quantity of fixed factors, holding constant total expenditure  $F$ . The most straightforward way to do so is to assume that such a change can be represented by a change in  $\gamma$ , where (1) is replaced by

$$Q = F(\gamma L).$$

Proceeding in this fashion, increments to  $\gamma$  operate much like increases in  $F$ . At the level of the unionized firm, each demands more labor,

$$\frac{\partial l}{\partial \gamma} > 0,$$



and becomes more responsive to changes in  $w$ :

$$\frac{\partial^2 \ell}{\partial w \partial \gamma} < 0.$$

It follows immediately that all the predictions regarding  $w^*$ ,  $N^*$ ,  $D$ ,  $q$ ,  $\ell$ ,  $N\ell$ ,  $Nq$ ,  $w^*N^*\ell$ , and  $\pi$  are identical to those derived for increments to  $F$ , and have the same basic explanation. Sufficient for these predictions is  $\pi_{w\gamma} < 0$ , very similar to  $\pi_{wF} < 0$ , both intuitively and formally.

In contrast to increases in  $F$  though, increments to  $\gamma$  produce a determinate effect on  $p^*--dp^*/d\gamma < 0--$ because labor becomes more efficient and  $w^*$  falls. Both operate to reduce average cost. (Effects via changes in  $q$  itself are ignored by the envelope theorem.) Turning to  $\hat{q}$ ,  $\hat{\ell}$ , and  $\hat{\pi}$  the decline in product price and increment to  $\gamma$  again work in opposite directions and no clearcut prediction emerges.

A final set of experiments involves the union cost function  $u(N, \ell)$ . Implicitly  $u(N, \ell)$  is the solution to a cost minimization problem wherein the union uses factors to collect dues. If the price of the  $j^{\text{th}}$  factor used by the union is denoted  $r_j$ , and both  $u_{Nr_j} > 0$  and  $u_{\ell r_j} > 0$ , it is straightforward to obtain

$$\frac{dw^*}{dr_j} > 0$$

and

$$\frac{dN^*}{dr_j} < 0.$$

That is, an increment to  $r_j$  causes a scaling back of union operations directly, by reducing  $N$ , and indirectly through  $\partial \ell / \partial w < 0$ .

Since the change in  $r_j$  has no direct impact on union or nonunion firms (i.e. given  $w^*$ ), the predictions obtained for this experiment are qualitatively identical to those following from an increase in  $\hat{w}$ .

### Complete Coverage Equilibrium

When the constraint  $M \geq 0$  is binding,  $\lambda^* > 0$  in (17)-(19) provided  $M=0$  strictly dominates  $M=\epsilon$  for any  $\epsilon > 0$ . In this case (19) can be solved for  $N(w) = \varphi/q$ , and  $\pi[N(w), w] \equiv \bar{\pi}(w) \equiv (w - \hat{w}) \frac{\varphi \ell(w, F)}{q(w, F)} - u[\varphi/q(w, F), \ell(w, F)]$ .

Necessary conditions for a maximum are

$$\begin{aligned} \frac{\bar{\pi}}{w} &= 0 \\ \text{and } \frac{\bar{\pi}}{ww} &< 0 \end{aligned} \quad (22)$$

for  $w = w^*$  solving (22).

The complete coverage model is vastly more difficult to analyze when compared to the incomplete coverage setting. Most of the predictive content of the latter model obtained because  $N$  and  $w$  could be manipulated independently, which fails in the former. Referring back to (16), and imposing the complete coverage restriction  $\alpha = 0$ , the sign of the required  $N$ - $w$  relationship is simply not available. Consequently, the major building block underlying the predictions provided above--a firm handle on movements in  $w^*$  and  $N^*$ --is no longer in place.

Nonetheless, at least some headway can be made for one parameter. In the usual way,

$$\frac{dw^*}{dw} \propto \frac{\bar{\pi}}{w\hat{w}}.$$

After some manipulation,

$$\frac{\bar{\pi}}{w} = \frac{\varphi \ell}{q} + (w^* - \hat{w}) \frac{\varphi}{q} \frac{\partial \ell}{\partial w} - u_{\ell} \frac{\partial \ell}{\partial w} + [(w - \hat{w}) \ell - u_N] \frac{dN}{dw},$$

in which case

$$\begin{aligned} \frac{\pi}{w\hat{w}} &= -\left\{\frac{\varphi}{q} \frac{\partial l}{\partial w} + l \frac{dN}{dw}\right\} \\ &= -\left\{\frac{\varphi}{q} \frac{\partial l}{\partial w} + \frac{l}{q^2} \left[\varphi' l - \varphi \frac{\partial q}{\partial w}\right]\right\}. \end{aligned}$$

Now  $-\varphi' > 0$ , and

$$\begin{aligned} &-\left\{\frac{\varphi}{q} \frac{\partial l}{\partial q} - \frac{l}{q^2} \varphi \frac{\partial q}{\partial w}\right\} \\ &= -\frac{\varphi}{q} \frac{\partial q}{\partial w} \left\{h' - \frac{h}{q}\right\} > 0 \end{aligned}$$

by convexity of  $h$  and (12). Consequently,

$$\frac{dw}{d\hat{w}} > 0.$$

Given this result,

$$\frac{dq}{d\hat{w}} < 0$$

and

$$\frac{dl}{d\hat{w}} < 0$$

follow as before, as does  $d\hat{\pi} < 0$  and  $dp^*/d\hat{w} > 0$ .

The prediction  $dD/d\hat{w} > 0$ , to be obtained under relatively mild restrictions in the incomplete coverage case, does not appear to follow here even as a "leading case". Effects on the rest of the endogenous entities require knowledge of  $dN/dw$ . If  $dN/dw < 0$ , which could in principle be checked, the predictions for  $N^*q$ ,  $N^*l$ , and  $w^*N^*l$  continue to hold.

In summary, for increases in  $\hat{w}$ , the complete coverage case provides either the same predictions as does the incomplete coverage case, or no predictions apart from  $dq/d\hat{w} < 0$  and  $dl/d\hat{w} < 0$ .

At this point it appears that the above are all the operationally meaningful restrictions, which do not depend on specific parameter values, that the complete coverage model places on the data. This is in sharp contrast with the incomplete coverage case. For changes in demand, in particular, the independence results for most of the endogenous variables in the model may be contrasted with the general non-independence in the complete coverage case. This difference is an additional prediction not available in the standard literature.

## V. ELABORATIONS AND DISCUSSION

The basic model presented above is easy to manipulate and generates a variety of predictions. However, the setting assumed is a somewhat rarefied one, and there are numerous other issues on which the model can shed light. In this section some of these elaborations are explored. Further, discussion of the effect of perturbing some of the model's assumptions is called for, as is information on whether the model assists in understanding existing empirical work. Finally, it is shown that at some cost, the model's prediction can be stated in other ways, and that doing so may yield a return in reduced informational requirements for testing the theory.

### 1. A More Active Role for the Union

In the simple model presented above, unions were viewed purely as dues collection agents. This simple view provided clean analysis but is less than adequate for several reasons. One is that the intra-industry union-nonunion wage differential is predicted to equal the dues ( $w - \hat{w} = d$ ) and comparison of estimates of the differential with typical levels of dues would reject that hypothesis. Second, unionized firms seem to organize work differently, and in a fashion which is not

readily explicable as a simple response to higher wages (see the data in Duncan and Stafford for example).

The point of this subsection is to show that the analysis is easily augmented to allow unions a role in the structure of production. The extension can be made more complicated, and presumably a study focusing on this issue would do so, but the simple route taken here suffices to make the point.

Suppose the union provides firms with services converting  $L$  units of labor input into  $\gamma L$  units,  $\gamma \neq 1$ --possibly at a cost of raising  $F$  to  $\delta F$ ,  $\delta > 1$ --and in so doing generates services to union workers valued at  $s$ . (Here  $\gamma\delta$  and  $s$  are taken as exogenous, but they need not be.) The monitoring type activities making large assembly lines efficient that are frequently discussed (again see Duncan and Stafford) can be treated by specifying  $\gamma > 1$ ,  $\delta > 1$  and  $s < 0$ . On the other hand,  $\gamma < 1$  and  $s > 0$  may represent the "social chit" aspect of union membership. In either case, the union is thought of as the agent who internalizes external economies which prevent individual firms from offering this service on their own.

Proceeding in this fashion, the modifications required in the above analysis are simply the replacement of (6) by

$$w + s - d = \hat{w},$$

and the union firm's cost function by

$$F + (w/\gamma)h(Q).$$

The union's problem is then analogous to (10)<sup>10</sup>

$$\max_{N,w} (w + s - \hat{w})N\ell(w,F,\gamma) - \tilde{u}[N,\ell(w,F,\gamma) \gamma,s]$$

where

$$l(w, F, \gamma) \equiv h[q(w, F, \gamma)],$$

$$q(w, F, \gamma) = \operatorname{argmin}_Q \frac{F + (w/\gamma)h(Q)}{Q},$$

and  $\tilde{u}[N, l(\cdot), \gamma, s]$  is the cost of providing the  $(\gamma, s)$  package to  $l$  workers at  $N$  firms.

This richer structure allows consideration of a new set of issues; for example, the effects of the union on labor productivity and relative profitability and size of union and nonunion firms. Moreover, new predictions arise. For example, changes in  $s$  can be analyzed essentially as alterations in  $\hat{w}$ .<sup>11</sup>

## 2. Strikes<sup>12</sup>

The model analyzed above does not admit the possibility of strikes. The setting is one of complete information, and incomplete information appears to be a prerequisite for strikes to emerge as equilibrium outcomes.

An incomplete information environment in which strikes may be analyzed is as follows. Suppose that having decided to enter an industry, each firm learns the value of a firm-specific cost parameter--a location advantage, for example--the knowledge of which is private information. Suppose further that the union chooses a collection of firms to unionize, as above. Having done so it is to the union's advantage to attempt to learn the firm's private information. A strike can be used to pursue this end. Specifically, equilibrium can involve the union offering two wage rates,  $w_1$  and  $w_2$ ;  $w_1 > w_2$ . Union firms can choose to pay  $w_2$ , but only by inducing a strike of length  $\tau$ . The

triple  $(w_1, w_2, \tau)$  can be chosen along with  $N$  to maximize union profits subject to the strike length-wage combination yielding truthful revelation of cost parameters by firms.

Proceeding in this fashion, union wages, number of firms and strike length are all endogenous and jointly determined. Predictions regarding the response of each to changes in the economic environment may be obtained.

### 3. Union Effects on Skill Accumulation

That the presence of unions might alter the worker's training choice in various ways has been pointed out in a series of recent papers; see Weiss (1983) and the references therein. Though the model used here is not a dynamic one, it can still prove to be a useful framework for addressing the type of questions raised in those papers.

One example will suffice. The model set out above treats the union as in some way interacting with  $l$  workers at each of  $N$  locations. Unless dealing with skilled workers renders this process much less costly, it is not hard to construct a situation in which the union finds the possibility that union firms will seek to substitute more skilled (though still unionized) workers a real constraint on its behavior. Much like a product market monopolist, the union would prefer not to see a decrease in demand for its product. Here the product is "bodies", and the decline in demand comes via substitution of skill for bodies. In such a situation, the possibility that workers' skill might be augmented generally causes the union to lower the union wage and raise the number of union firms.<sup>13</sup>

#### 4. Discussion of Assumptions

Of the assumptions made above, there are two which particularly deserve some comment.

First of all, the equilibrium concept used here is leader-follower, with the union leading. Moreover, given the total rents extracted by the union, there is no redistribution of the rents between workers and the union which would make both workers and the union better off. It has become fashionable (see McDonald and Solow (1981), MaCurdy and Pencavel (1983) and Oswald (1982, 1984)) to include firms in this coalition. That is, wages as well as the allocation of labor are taken to be "efficient" from the standpoint of workers, firms and the union, with rents being extracted from consumers (their role again being followers). The point to note here is simply that the results presented above depend only in detail on the exclusion of firms from the coalition in the above analysis. If unions are still modelled as using resources, though hard to obtain, conclusions much like those presented above arise in an "efficient contracts" setting, and for the same reason--the number of union workers and the union wage are not so constrained by the precise structure of demand. In particular, the very strong separation of union variables from  $\varphi(p)$  continues to hold.

The second assumption which is worthy of further comment is that the union unionizes firms rather than workers. It is straightforward to demonstrate that if the union instead designates some workers "union", labels others "nonunion", and threatens all other workers with union



status if they work in the industry, then the equilibrium outcomes are precisely those analyzed above for all union variables. The difference is that firms hiring nonunion workers bid their price up from  $\hat{w}$  to  $w$ , thus transferring the profits earned under the previous scheme by nonunion firms to nonunion workers. The union variables are not sensitive to this part of the specification because all that changes is the manner in which entry is restricted and one method is just as useful as the other in that regard. To put the point differently, in the equilibrium of the model presented above, if the union simply labelled the  $N^*\ell(w^*)$  workers unionized, it would not seek to alter  $w^*$  because the formal problem simply involves replacing  $N$  with  $L^u/\ell(w)$ , where  $L^u$  is the total number of workers labelled unionized.

##### 5. Other Ways to State the Predictions, and Existing Empirical Work

The set of predictions derived above can be stated in several different ways. Pursuing the alternate representations is usually not costless, but may reduce the information required to test the model.

To proceed, note that the conventional route of defining

$$w^* = \omega(\hat{w}, F, \dots),$$

$$N^* = \eta(\hat{w}, F, \dots),$$

$$\vdots$$

$$p^* = \rho(\hat{w}, F, \dots),$$

has been followed. These equations represent the reduced form of the model, and its predictions are in terms of the partial derivatives of  $\omega(\cdot)$ ,  $\eta(\cdot)$  etc. These predictions can be stated in an alternative form because

all exogenous variables in the model influence  $w^*$  and  $N^*$  in opposite directions. From this result it might be tempting to assert that the unconditional covariance of  $w^*$  and  $N^*$  (where the variation in  $w^*$  and  $N^*$  is induced by variations in underlying exogenous variables) is negative. In general this statement is correct only if the exogenous variables are drawn from a distribution independently of one another. But given this restriction, which is the cost of reducing informational demands,  $w(w^*, N^*) < 0$  is implied. Moreover, since the results on  $w^* - \hat{w}$ ,  $q$ ,  $l$ ,  $N^*l$ ,  $N^*q$ ,  $w^*N^*l$ ,  $\hat{q}$ ,  $\hat{l}$ ,  $p^*$ ,  $\hat{\pi}$  and  $\pi$  are obtained from the changes in  $w^*$  and  $N^*$ , the theory has implications for most unconditional correlations between arbitrary pairs of these variables.

Viewing the predictions this way is useful for interpretation of existing empirical results. For example, that  $\text{Cov}[w^*, w^*l/w^*l + F] < 0$  is predicted provides an explanation for the result (see, for example, Rosen (1970)) that union labor and other factors appear to be good substitutes; that is,  $\text{Cov}[ ] < 0$  is usually interpreted as evidence of a substitution elasticity in excess of unity.

A second example is that the standard Lewis (1963) approach to estimation of union-nonunion wage differentials can be given a consistent interpretation in terms of underlying parameters. Letting  $\bar{w}$  equal the industry average wage rate

$$\ln \bar{w} \approx \ln \hat{w} + \mu \ln \frac{W}{\hat{w}},$$

assuming the geometric average approximates the arithmetic average.

Then, using

$$\ln \frac{\bar{w}}{\hat{w}} = \ln 1 + \left( \frac{\bar{w} - \hat{w}}{\hat{w}} \right) \\ \cong d$$

where  $d$  is the percentage union-nonunion differential,

$$\ln \bar{w} \cong \ln \hat{w} + \mu d.$$

The theory implies  $d$  should vary with the exogenous features of the industry (which it seems to; see MacDonald (1981)), and that given  $d$  (i.e. holding all exogenous factors relevant to  $d$  fixed), the coefficient of  $\mu$  in an estimated version of the above equation should yield an estimate of  $d$ . This occurs because for constant  $d$ , variation in  $\mu$  and  $\ln \bar{w}$  is induced by variation in  $\varphi(p)$ .

A third example is the empirical work on unions and firm or plant size. There is little theoretical work that provides a framework within which to generate predictions in this area. Parsley's (1980) survey links discussion of firm size primarily to its relation with the degree of price competition in product markets. Lazear's (1983) model does not have an explicit prediction for the size of union versus nonunion firms. It would appear that this would depend on the sign of the correlation between entrepreneurial abilities that lead to large firm sizes with the abilities to resist unionization. If these were positively correlated, for example, nonunion firms would also be relatively large. In the present model, the relative size of union and nonunion firms could depend on union-nonunion differences in  $\gamma$  and  $\delta$ . If the productivity effects of unions do operate by making large scale production line processes efficient by "solving" the monitoring problems, this would be captured by  $\gamma > 1$  and  $\delta > 1$ . The

size difference across firms will then depend on the relative magnitude of  $\gamma$  and  $\delta$ , and hence the capital intensity of union versus nonunion firms. The larger is  $\delta$ , the more likely union firms will be larger than nonunion firms. The implications for firm size differences and the differential would then follow from an analysis of the effect of  $\gamma$  and  $\delta$  on the differential, analogous to that presented above.

A final example concerns observed wage rigidities. The theory presented in this paper is a partial equilibrium model. Thus it is not in its present form appropriate for discussion of macroeconomic issues. However, individual industry evidence of wage rigidity is often alluded to in macro discussions of unemployment (e.g., Taylor, 1983). There are also micro studies of wage rigidities stemming from union behavior (e.g., Grossman, 1984). Wage rigidities follow from the product market independence results of the present model in cases where unions have less than 100% coverage. Ironically it is the "less strong" unions, in the sense of less than full coverage that yield the wage rigidities. Since the model is partial equilibrium in nature, there are no implications for aggregate unemployment.

#### 6. Unions, Productivity and Profits

An influential area of recent empirical analyses of unionism has been the "Harvard School" productivity studies (Freeman and Medoff, 1979, 1984 (Ch. 11); Clark, 1980). The implications of the findings of some of these studies for firm profits in the unionized sector has also generated discussion around the paradoxical result of lower costs under unionism being associated with lower firm profits. (Freeman and Medoff, 1984; Ruback and Zimmerman, 1984). The hypothesis that unions contribute positively to the production process implies  $\gamma > 1$  in the expanded version of the model in Section V.1 above. Imposing this

restriction, the relationship between profits and productivity across union and nonunion sectors is readily examined in our model. Ruback and Zimmerman (1984) provide some evidence from changes in equity value of firms following union elections that unionization lowers the equity value. Freeman and Medoff (1984, Table 12.1) also conclude that unions reduce profitability of the unionized firms. This is always implied by our model since a hitherto nonunion firm in the industry is making positive profits. Any change that causes the union to unionize a larger fraction of the industry (say, a reduction in union costs) will imply lower profits for the firm. The traditional monopoly union model in which a union takes over a competitive industry would predict no change in the profit levels of on-going firms before and after unionization since they would be making zero profits in both situations. Both Freeman and Medoff (1984) and Ruback and Zimmerman (1984) refer to casual evidence that firms always resist unions. This would not be a prediction of the traditional monopoly model since the firm knows it will be unionized and hence it would not pay to use resources in a vain attempt to prevent this. On the other hand in our model, firms in an industry to be unionized (or in which unionization is increasing) will have an incentive to resist unionization since there will, in equilibrium, be nonunion firms making positive profits. It will in general pay to improve the firms' chances of being one of these nonunion firms.

Lazear (1983) presents an explicit analysis of firms spending resources to prevent their becoming unionized. Because of the complexity of the Lazear model, the amount spent to combat unionism was considered exogenous. In particular, no attempt was made to relate this to the size of the union differential. In the simpler model of the present paper, the amount of resources spent--and more particularly its relationship

to other variables in the model--could readily be derived. All that is required is a mechanism for the allocation of firms to the nonunion sector based on resources spent. The basic analysis would not change provided that the resources could not be captured by the union.

#### VI. HOW CAN THE MODEL BE TESTED?

The proposed model, as illustrated in the previous section, has predictions for a wide variety of empirical phenomena connected with unions. It may also be used to interpret much of the existing empirical literature. In this section some tests are proposed that concentrate on the major predictions of the model. More especially, the tests concern predictions that most easily differentiate the present model from others in the literature. These predictions concern the basic independence result concerning union behavior and industry demand conditions in cases where unionization is less than 100% of an industry.

The standard monopoly models predict some response of union employment levels and wage rates in response to changes in product market demand conditions. Measures in the data that divide "industries" between union and nonunion members or firms would have to be interpreted either as different industries aggregated, or as union and nonunion types of work in the same industry. Either way, the union-nonunion differential should be sensitive to product market conditions. Similarly union employment levels, and hence coverage, would also typically react to product market changes. In the present model neither the union wage, nor employment are predicted to be sensitive to product market conditions where unionization is less than 100%. Union coverage, which is well defined in this model, is predicted to be inversely related to product market conditions. Thus a test which most readily discriminates the present model from other models in the literature would

be a comparison of the reaction of union wage rates and employment (or coverage) when product market conditions change for industries which are 100% unionized with those that are not.

The model is a long-term model. The product market changes that are used for a test should therefore be permanent. In addition they should be separated from changes in other exogenous variables--especially the alternative wage, or union costs--that would affect union behavior. Thus, an appropriate time period for the test would be one where there was general wage rate stability and where there was no recent trade union legislation. Under these conditions, the absence of a different reaction of union wages and employment across the two coverage "regimes", and in particular, a finding of sensitivity of union variables to product market conditions in the less than full coverage case would be major evidence against the model proposed in this paper.

## FOOTNOTES

<sup>1</sup>Apart from the extensive literature that reopened the debate on the size of union wage differentials by allowing in one way or another for endogeneity of union status (see Parsley, 1980, section III for a summary), there have also been new debates on other union topics. One issue has been whether union-firm contracts are "inefficient" in the usual monopoly sense--i.e. is the wage-employment combination obtained by the union setting a wage and the firm choosing a point on its demand curve; or is the wage-employment combination on the contract curve? (See, for example, McDonald and Solow, 1981; MaCurdy and Pencavel, 1983; Oswald, 1982, 1984). A related issue concerns the objective function of the unions and whether we can learn anything from observed data (see Pencavel, 1984). Unions' membership in recent years has declined quite substantially. Attempts have been made to explain this pattern (Freeman, 1984; Neumann and Rissman, 1984). Strike activity has received some attention, in the form of both theoretical and empirical studies (Hayes, 1984; Reder and Neumann, 1980; Kennan, 1980). Finally, some authors have focused more on the members that make up a union and the firms that make up the industry, rather than dealing directly with the aggregate entities "union" and "industry" (Lazear, 1983; Oswald, 1982, 1984).

<sup>2</sup>This assumption can be relaxed at the cost of including more algebra and minor restrictions on the expansion paths of the multiple input technology.

<sup>3</sup>This specification rules out many types of union-firm interaction-- lump-sum payments for example. The point of doing so is to force logical



distance between a model of a unionized industry and what becomes essentially (though not exactly, due to the presence of  $u(\cdot)$  below) a producer's cartel. That is, given enough freedom in its interaction with firms, the union more or less owns them.

<sup>4</sup>Note that if the union permits any nonunion firms to operate in the industry, those firms will earn positive profits if  $w > \hat{w}$ , as is shown to be the case below. That these profits are not eroded by competition from outside is ensured by the union's threat. That they do not accrue to the union is implied by the restriction that the union only obtains revenue through collection of dues from union members. It is shown below that the union will not in general go after these rents by choosing full union coverage (100% unionization). An implication of all this is that nonunion firms will be willing to devote much effort to retaining that status.

While initially disturbing, the positive profits earned by nonunion firms are just a manifestation of the presence of the monopoly "inefficiency" assumed in the model, and are that part of the total "monopoly profit" which the union does not obtain owing to the limited means through which the union is permitted to earn revenue.

<sup>5</sup>The predictions derived below are in terms of  $N$  and  $w$ . They can equally be produced for  $w$  and  $E \equiv N\ell$ , which is more easily observed. Note though that in doing so  $u[E/\ell, \ell]$  is specified. The multiplicative form  $u(N, \ell) = g[N\ell]$  is not convex in  $N$  and  $\ell$ .

$$\begin{aligned}
\frac{w}{l} \frac{\partial l}{\partial w} &= \frac{w}{h} h' \frac{\partial q}{\partial w} \\
&= q \frac{h'}{h} \frac{w}{q} \frac{\partial q}{\partial w} \\
&= - \frac{qh'}{h} \quad (\text{from (14)}) \\
&= - \pi_q^h
\end{aligned}$$

<sup>7</sup>The restriction utilized is

$$\frac{\partial^2 c}{\partial q^2} - q \frac{\partial^3 c}{\partial q^3} > 0 \quad (*)$$

(\*) is a sufficient condition for  $\partial^2 q / \partial w \partial F < 0$ , itself sufficient for  $\partial^2 l / \partial w \partial F < 0$ . (\*) is by no means necessary.

<sup>8</sup>Change in F and/or in the labor productivity parameter  $\gamma$  introduced below can be given several interpretations. One interpretation is that production in unionized plants is organized differently than in nonunion plants, involving both larger F and  $\gamma$ . This is discussed in more detail below. Another interpretation is that F (and  $\gamma$ ) change for both union and nonunion firms over time--i.e. technical change. Whether this interpretation is helpful outside of a dynamic setting is debatable.

<sup>9</sup>Substitution of  $w^* - \hat{w} = u_N / l$  from (21) into (20) implies  $Nu_N - lu_l > 0$ . The cross effect restriction is  $u_l > Nu_{Nl}$ . So

$$Nu_N - lu_l > 0 \Rightarrow Nu_N - lNu_{Nl} > 0 \Rightarrow N(u_N - lu_{Nl}) > 0$$

Hence  $\pi_{NF} > 0$ .

<sup>10</sup>Implicit here is the restriction that union firms remain at a cost disadvantage. If this relation fails, the model can still be analyzed, but the nonunion firms earn zero profits, union firms earn non-negative profits, and the union threatens potential entrants with nonunion status. If the union-nonunion production difference was purely in terms of  $\gamma$ , the restriction implies  $w/\gamma > \hat{w}$  at the optimum. More generally, if  $\gamma \neq 1$  also implies a different  $F$  for union firms,  $w/\gamma > \hat{w}$  is not required.

<sup>11</sup>These experiments are cost function experiments whether the cost of providing  $s$  and  $\xi$  to  $l$  workers at  $N$  firms remains fixed. (\*) in footnote 7 is used here as well.

<sup>12</sup>This material is from a U.W.O. thesis prospectus. See Stirling (1985).

<sup>13</sup>The model used here is much like that presented above. In equilibrium, workers are indifferent about their level of skill accumulation, firms optimally choose skilled or unskilled workers given the configuration of wages, and the union leads all other agents.

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